THE PROCEEDINGS OF THE OPTICAL CONVENTION

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THE PROCEEDINGS OF THE OPTICAL CONVENTION

Vol. II

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CONTENTS

		PAGE
I.	Inaugural Address, by the President, Professor Silvanus P. Thompson, B.A., D.Sc., F.R.S.	1
II.	Sector Shutters. By Cyril F. Lan-Davis, F.R.P.S.	24
III.	On Transmission of Visible Light by Photographic Lenses. By R. W. Cheshire, B.A.	34
IV.	The Shape of Scales Required for Reflecting Instruments with Concave Mirrors. By E. H. Rayner, M.A.	41
V.	Measurements and Notes on the Visibility of Point Sources of Light. By Clifford C. Paterson and B. P. Dudding, A.R.C.Sc.	49
VI.	Some Recent Advances in the Measurement of Light and Illumination. By J. S. Dow and V. H. Mackinney	70
VII.	Some Alloys Suitable for Instrument Work. By W. Rosenhain, B.A., D.Sc.	89
VIII.	A New Spectral Colour-Mixing Apparatus. By F. W. Edridge-Green, M.D., F.R.C.S.	99
IX.	On Mirror Spectroscopes and Spectrographs. By R. A. Houston, M.A., Ph.D., D.Sc.	100
X.	The Scotomagraph. By J. H. Tomlinson, M.R.C.S.	104
XI.	On Aspherical Surfaces for Spectacle Lenses (Gullstrand's Cataract Lenses). By M. von Rohr, Ph.D.	108
XII.	Improved Indirect Ophthalmoscopy. By Lionel Laurance and H. Oscar Wood	115
XIII.	Notes on Spectrographs. By W. H. Eagle, B.Sc.	120
XIV.	The Variations of a Binocular Testing Bench. By William Shackleton, F.R.A.S., A.R.C.Sc.	138
XV.	Note on the Calculation of the Spherical Errors of an Optical System of Centred Lenses, by means of the Seidel Formulæ. By James P. C. Southall	145
XVI.	A New Method of Testing Objectives. By S. D. Chalmers, M.A.	156
XVII.	Demonstration of Laboratory Apparatus and Experiments. By G. F. C. Searle, Sc.D., F.R.S.	161
XVIII.	Diffraction Images. By J. W. Gordon.	173

PAGE

XIX. Principal Surfaces of a Lens. By P. Everitt . . 204

XX. The Measurement of Colour. By A. C. Jolly and A. J. Bull 206

XXI. The Design and Construction of Large Polariscopes. By Professors E. G. Coker and S. P. Thompson . . . 212

XXII. Colour-mixing Apparatus. By Thomas H. Blakesley . 218

XXIII. Method of Tracing Caustic Curves. By Archibald Stanley Percival, M.A., M.B. Cantab. 223

XXIV. A Telescope Objective Achromatized with Water. By Thomas H. Blakesley 234

XXV. On Specific-gravity Balls. By Thomas H. Blakesley . 236

XXVI. A New Optical Method of Scale-recording for Ophthalmological Instruments. By John H. Sutcliffe . . 240

XXVII. The Ophthalmic Significance of Facial Asymmetry. By Harry L. Taylor 245

XXVIII. Errors of Observation. By T. Y. Baker and G. B. Bryan, D.Sc. 252

XXIX. The Teaching of Mathematics for Students of Elementary Visual Objects. By Dr. Max Coque . . . 264

XXX. Some Graphical Methods, including those that determine the Situation of Cardinal Points. By Archibald Stanley Percival, M.A., M.B. Cantab. . . . 271

XXXI. The Micro-spectra Method of Colour Photography. By Julius Rheinberg, F.R.M.S. 276

XXXII. Some Points in the Use and Design of Refractometers. By Thomas Martin Lowry, D.Sc., F.C.S. . . . 291

XXXIII. The Trend of Geometrical Optics. By Silvanus P. Thompson, D.Sc., F.R.S. 297

XXXIV. The Ideal Photographic Lens. By T. Smith, B.A. . . 307

XXXV. The Great Observatories of America. By Professor H. H. Turner, D.Sc., F.R.S., of Oxford 315

XXXVI. Report of Lecture by Professor William Stirling on Optical Illusions 325

Appendix. Tables for the Calculation of Spherical Aberration calculated by H. C. Beck, and arranged by H. M. Elder, M.A. 337

List of Members 353

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1912

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PROCEEDINGS

OF THE

OPTICAL CONVENTION, 1912.

INAUGURAL ADDRESS OF THE PRESIDENT,

PROFESSOR SILVANUS P. THOMPSON, F.R.S.

(*Wednesday, June 19th*, 1912.)

Introductory.—Seven years have elapsed since the first Optical Convention assembled in 1905, under the presidency of Dr. Richard T. Glazebrook. Both that gathering and the second one, in which we are now met, witness to the efforts which are being made, not less by those concerned in the industries than by scientific men, to promote the progress of optical science and of optical trade. Like all other industries which depend on the application of scientific discoveries, the optical industry has felt the pressure of the times; and a widespread sense of need that science and manufacture must be associated in an alliance more intimate and more active than heretofore, has been the moving cause of both Conventions.

We are met here to exchange views, to deliberate, to discuss: to learn, from one another, and from the material objects which we have been able to bring together in our Optical Exhibition, anything and everything which can stimulate our thought, widen our information, or concentrate and harmonize our activities in matters optical.

Let our first words be to acknowledge the deep debt which we owe to His Majesty's Government, to the President of the Board of Education, and to the administrators of his staff, who have greatly promoted our enterprise by granting us the use of the galleries of the Science Museum for our Exhibition. The aid thus accorded to us by Government has been a powerful stimulus to our efforts, and an important material addition to our resources. Also our thanks are due to the Governors of the Imperial College of Science and Technology for the use of suitable rooms for our

formal meetings, including the theatre in which we are now assembled. We tender our thanks also to the Presidents and Councils of the various learned societies and technical bodies who have supported us, and who will co-operate with us during the business of the Convention. Nor must we forget our great indebtedness to the generous individuals and institutions, including several of the great City Companies, who have contributed of their wealth to our guarantee fund.

Development.—Seven years is but a brief span in the development of any industry, or in the history of any science. It may well be that in the seven years which have fled since our first Convention we have no obvious great discovery to chronicle. When the telescope was invented, at the beginning of the seventeenth century, the fame of it, and of Galileo's discovery of the satellites of Jupiter, ran round all Europe. When in 1815 the kaleidoscope—a mere toy—was produced, three hundred thousand were sold within six months. When in 1838 the stereoscope astonished the world, half a million were sold in half a decade. When in 1897 Röntgen described his X-rays the whole scientific world was filled with wonder. Certainly no such sensational novelty as any of these has appeared during the last seven years, though the developments of the kinematograph have drawn from the public a thousand times as much money as any of the inventions or discoveries just named did in their respective eras. But if no optical invention of first magnitude, nor discovery of fundamental importance, has been announced, it must not be assumed that there have been no advances. Progress there has been; progress solid and real, all along the line. No branch of physical science can in the present day remain stationary. The workers are too numerous; the rewards of success, whether in the joy of scientific discovery, or in fame, or wealth, are too alluring to permit stagnation. Moreover, the increase of knowledge, the mastery of principles over phenomena, the conquest of the forces of Nature, are cumulative. Every attempt at wider generalizations, even if unsuccessful in itself, provokes new researches, and extends the foundations for further advance. To this truth the science of optics furnishes no exception. Progress is continuous, even though the workaday world hears little of it, and heeds it not. For the true pioneer halts not to listen for the sound of the plaudits: he toils on, content in the faith that some day he, or those after him for whom his labours will avail, will arrive at the goal.

Methods: Abstract and Practical.—The history of the development of Optics presents many notable features. Combining, as it does, the practice of a highly skilled craft in the construction of instruments, with the exercise of refined and elaborate theory, it affords continually recurring proofs of the truth that practical applications and abstract science, so far from being antagonistic, afford to one another the most powerful support and stimulus. Probably no other branch of physics—not even magnetism, with all its mysteries—has had a greater effect in stirring the scientific imagination. The very phenomena appeal to the eye, as do those of no other science, by their beauty and intrinsic charm: and the variety of them has taxed the strongest powers of analysis to devise consistent theories for their explanation. Newton might declare *hypotheses non fingo:* but that renunciation on his part has

not prevailed with others to abandon tentative theorizing. The history of Optics is scarred with the battles of rival theories, of which the end is not yet determined. It may, indeed, almost be taken as axiomatic that in all efforts to reach the unknown, to advance human knowledge, it is better to set before one's self some directive hypothesis than to work aimlessly. Every great pioneer in physical science has to frame conjectures, and to keep them, as it were, in a state of solution until either confirmed or disproven. He may even have half a dozen rival and mutually destructive hypotheses before him as he works. Truth is not infrequently reached by a process of exhaustion, by honestly following clues that ultimately prove false; since when they are proved to be false the path to truth has been more closely delimited than before. Even positive error in theory has been known to lead to new and valuable results; as when Euler, arguing from the false premiss that the human eye is achromatic, deduced the conclusion that it must therefore be possible to construct by optical means a lens that should be achromatic. Of the theories of light now in vogue, something must indeed presently be said, but it is perhaps more important here to point out how very far it is possible to push the study of optical phenomena and laws, without committing oneself to any theory at all* as to the vexed question of the physical nature of light itself.

Abstract Methods and Theories.—To gain a clearer perception of the meaning of those numerical and geometrical laws which govern the phenomena of light, it is certainly permissible, and on occasion highly useful, to consider these relations apart from the concrete physics involved. Little advance would have been made in any of the exact sciences had not the relations of number and form been studied in the abstract sciences of algebra and geometry. There has been a distinct gain in separating the study of the geometry of position from the geometry of motion, and, further, in separating kinematics, the science of pure motion, from kinetics, the science of forces producing motion. In like manner in that part of Optics which is concerned only with the directions in which light is propagated, namely, Geometrical Optics, there is distinct advantage in laying aside all considerations as to the physical nature both of light itself and of the media through which it is propagated, so gaining freedom from the embarrassment of conflicting hypotheses.

No doubt there is, even here, room for rival theories. The old hypothesis of Plato, of Euclid, and of Empedocles, that light consisted of visual rays, which proceeded from the eyes to the objects viewed, is no longer admissible: but it is not in itself inconsistent with the geometrical laws admitted to be true.

The medieval hypothesis, apparently originating with Pythagoras, that light consists of corpuscles shot out from shining bodies and travelling in straight lines, subject to reflexion and refraction on their way to the eye, is equally inadmissible, but equally consistent with the geometrical laws. The kindred theory that light consists of rays—a theory still in being, so far as language is concerned—is little

* It was in this spirit that in 1849 Lord Kelvin framed his Mathematical Theory of Magnetism, in which he strove to free the subject from all hypotheses as to the nature of magnetism itself, and so to reach a more general view than Poisson had attained.

more than an admission of the observed fact of the rectilinear propagation of light and, if the use of the word *ray* is not taken to imply any physical hypothesis as to the nature of light itself, is a step toward that clarity of treatment which has just been advocated. The wave-theory of light, adumbrated by Lionardo da Vinci, and by Hooke, but first shaped by Huygens, and triumphantly established by Young and Fresnel, may equally claim to be regarded as a kinematics of light, provided the use of the term *wave* is not held to import any preconceived notion as to the physical nature of light waves themselves. It is certainly difficult to think of waves in the abstract, unless they are waves of something—some medium capable of executing and conveying undulations—but their geometrical properties as to direction, and as to all those phenomena, both of Nature and of optical instruments, which depend on direction, may be studied in absolute independence of any physical constitution, whether mechanical or electromagnetic, that may be assigned to them. It was Huygens who by repeatedly insisting that a " ray " of light was merely the path travelled by the " wave," was able to clarify thought, and initiate a new departure. In themselves, the terms " ray " and " wave " are both kinematical terms, and both are generally deemed to imply physical concepts. If we were forced to choose between them, it is the term " ray " that must be abandoned as connoting a physical hypothesis contradicted by the facts of diffraction. On the other hand the term " wave " may be adopted without assigning to it any other than a purely kinematic significance. Moreover, it is just as possible to build up a purely Geometrical Optics on the conception of wave-motion as on that of ray-motion ; and Huygens's definition of ray reconciles the two. In this connection it is significant to remark that Professor Schuster, though in the preface to his *Theory of Optics* he deprecates, as a retrogression, the substitution of equations representing mathematical conceptions for mechanical or electromagnetic theories, nevertheless himself introduces at an early stage a kinematical discussion, leading him to say that " a wave-front is best defined as a surface such that the disturbance over it originally came from the same source, and started from that source at the same time," thus carefully removing from the definition any implication as to its physical nature.

Geometrical Optics.—In fact, whether founded physically on the notion of the projected corpuscle, the ray, or the wave, Geometrical Optics, that section of the science which deals solely with the *directions* of the light paths (either free in space or as controlled by some optical instrument) necessarily deals with geometrical conceptions and expresses itself in geometrical terms. Without returning to the primitive work of Alhazen on Reflexion, we owe to Kepler first, and to Descartes, Barrow, Cotes, Euler, and Lagrange * afterwards, the formulation of the laws of Geometrical Optics, as relating to lenses, mirrors, telescopes and microscopes. The simple formulæ that every student knows so well for the determination of the position of the principal focus, for conjugate foci, and for the magnification of the image by

* Lagrange succeeded in solving the problem of the passage of rays through a system of successive lenses by finding expressions in the form of continued fractions, which, though cumbrous, can be solved by sufficient labour.

the lens, mirror, or telescope, have come down from these men, though more familiar to us in the forms used by Coddington or Parkinson and the current University textbooks. Unconsciously we have become accustomed to think of them, and more particularly the expressions relating to the passage of light through lenses, as essentially optical formulæ, and as having something to do either with the glass of the lenses to which they are applied, or with the properties of light. The fact that they are usually derived trigonometrically, by assuming certain simplifications, such as that the angles of incidence and emergence, or those of the pencils of rays, are small (so small that sines and tangents may be written simply as angles), or that the lenses themselves are infinitely thin, gives a certain arbitrariness to them; while the mixing up of the monochromatic or so-called "spherical" aberrations, which are a purely geometrical affair, with the chromatic aberrations that arise from the physical properties of the glass, tends to further confusion. The first stage toward deliverance from this is due to Gauss, who, in 1840, in his *Dioptrische Untersuchungen*, laid the foundations of a new treatment. With rare insight he succeeded in reducing the performance of any optical combination of centred lenses—aberrations apart—to a geometrical system of "principal" and "focal" points and "principal" and "focal" planes, whereby the formulæ for lenses of any thickness, and for instruments compounded of several lenses, became as simple as those formerly employed for the formation of images by a single infinitely thin lens. Independently, Moebius had, in 1830, discovered the same properties of the "principal" points, though he missed a part of their significance for determining the refracted rays. They had, indeed, been foreshadowed by Harris in 1775. To the system of cardinal points and planes of Gauss, Listing in 1845 added the "nodal" points, which coincide with the "principal" points of Gauss only if the first and last media of the lens-system are alike (both air). Toepler, in 1870, added the "negative principal points" (or "symmetric" points *), and indicated the existence of a series of image-planes, lying beyond one another at distances apart equal to the focal length, in which planes the linear magnification has values of successive whole numbers. Blakesley, in his suggestive little book on *Geometrical Optics* (1903) has extended this conception.

While the method of Gauss gave a clear and consistent account of the generalized optical system in producing images, the very simplicity of the method depended on assumptions unrealizable in practice for simple lenses, viz., that the images should be free from the aberrations of distortion and curvature, as well as from those which result in indistinct definition. It was, therefore, an artificial and highly abstract method of treating lens problems, of great power for its own purpose, but incapable of dealing with the phenomena of aberration. In the hands of Martin, Gavarret,

* In passing I may remark on the extreme convenience in graphical construction of one property of the two symmetric planes, namely, that if any oblique ray passes through one of the planes, through a point having co-ordinates, y, z, in that plane, it will on emergence from the optical system pass through a corresponding and conjugate point $-y$, $-z$, in the other symmetric plane. I made use of this property in a mechanical model (now in the Exhibition of the Convention) which illustrates the finding of conjugate points of a lens-system. The Focometer (also in the Exhibition), which I described to the Royal Society in 1891, depends on the use of the symmetric planes.

Reusch, Helmholtz, Gariel, and Pendlebury, it received various enlargements, including extension to the formation of images by curved mirrors. Later Casorati and his distinguished pupil, Galileo Ferraris, developed Gauss's method by use of determinants. Ferraris's treatise was translated into German and found wide acceptance on the Continent.

But earlier than any of these expounders of Gauss's method, Maxwell, in 1858, published in the *Quarterly Journal of Mathematics*, a short but pregnant paper, which has remained all too little known to English physicists, comprising investigations intended to show " how simple and how general the theory of optical instruments may be rendered by considering the optical effects of the entire instrument without examining the mechanism by which these effects are obtained." He assumed that a " perfect " instrument would fulfil the three conditions of freedom from astigmatism, curvature, and distortion. On these assumptions he established, with elegant simplicity and economy of symbols, a theory, geometrically complete, of the formation of images; while he revenged himself upon his own simplicity by conclusively proving that no instrument, depending on reflexion or refraction, could possibly be " perfect "—that is, free from the aforesaid aberrations—except the plane mirror ! *

But British optical science needed not to wait upon Gauss for the revelation of a method. Embedded in the notes in the second volume of Robert Smith's ponderous *Compleat System of Opticks*, of 1728, there stands (pp. 76–78) a delightful digression setting forth a " noble and beautiful theorem," which he states to have been " the last invention of that great mathematician, Mr. Cotes, just before his death at the age of 32 : upon which occasion, I am told, Sir Isaac Newton said, ' If Mr. Cotes had lived we might have known something.' " This theorem, which is too elaborate to quote at length, is an elegant solution of the proposition how to find the situation and apparent magnitude with which an object is seen through any number of lenses of any sort, at any distances from each other and from the eye and the object. Of this general proposition many cases were worked out by Huygens and others. But the beauty of Cotes's demonstration was overlooked.† One of Huygens's particular cases, communicated to the Royal Society in 1669, is worthy of being quoted : namely, that if an eye looking at an object through a lens sees the image of apparently a certain size at a certain distance, it will still seem to be of the same size and distance if the places of the eye and the object are interchanged. Cotes demonstrated this still to be true for an optical system of several lenses. Another particular case of the theorem is known as Hemholtz's tangent law of magnification.

An Epoch-making Book.—In 1893 there appeared a book destined to produce a profound impression on the study of Optics, Czapski's *Theorie der Optischen Instru-*

* See also Klein, *Zeitschrift für Math. u. Phys.* xlvi., 376 (1901); or, Whittaker, *The Theory of Optical Instruments* (1907).

† Apparently after Smith's time this was forgotten until, in 1886, Lord Rayleigh drew attention to it. It is now to be found in the recent textbooks of R. A. Herman, M. von Rohr, J. P. Southall, and others.

mente nach Abbe. Of the originality and power of Abbe's teaching, and of the ability with which his follower expounded his views, there can be no question. Alas ! that both the distinguished master and his accomplished pupil are no longer with us. One of the many notable features* of that book is the chapter on the Formation of Images. It dawned on Abbe, more fully than on Gauss, Moebius, or even Maxwell, that the true theory of the Formation of Images is not only independent of the particular optical mechanism of instruments, but is, in fact, purely a question of projective geometry. Let all space be divided into two regions (which may or may not overlap), one containing the object, the other containing its image. For the formation of the perfect image it is required that the image shall correspond with the object, point for point, line for line, plane for plane. This implies that to every point in the object space there shall correspond a conjugate point in the image space, in such wise that to all points ranged on any line in the object there shall be formed points correspondingly ranged on a corresponding line in the image-space. This correspondence, termed by Moebius a *collinear relation*, is a purely geometrical question, to be treated by methods familiar to the student of such works as Cremona's *Projective Geometry*, or Salmon's *Geometry of Three Dimensions.*† The best account in the English tongue of Abbe's theory is to be found in Southall's recently published *Principles and Methods of Geometrical Optics*, a work invaluable to every teacher. By Abbe's intuition that the formation of images is a pure question of the geometry of rays, that is of straight lines, without any ascription of optical properties or any question whatever of physical ways and means, he completed the last step in that clarification alluded to at the outset. It may, perhaps, perplex old-fashioned devotees of Optics to learn that principal foci are simply the vanishing points of anharmonic ranges; that conjugate foci on the principal axis are homographically related; or that foci in general are points such that conjugate lines meeting in them cut orthogonally. These phrases seem to fill the air with their sweet jargoning; nevertheless they express abstract relations which every student of optics ought to know. For if this collinear relation is realized in the performance of any instrument, the images which it forms will at least be free from the defects of curvature of image and distortion.

But freedom of the image from curvature and distortion can scarcely be said to have a definite meaning unless the optical system of the instrument is, at least approximately, so constructed that points of the object are rendered with good definition literally *as points* in the image. Mr. Gordon has given us the useful term *anti-point*, to denote the image, whether well-defined or blurred, of a point-object. But unless the anti-point be itself a well-defined point, what avails collinearity ? The perfect optical instrument, lens, mirror, or combined system, must not only be collinear but stigmatic. Correspondence of image to object, point for point, line for line, plane

* Another is the thorough consideration of the effects of limiting the rays in an optical instrument by diaphragms and stops, a matter developed in the more recent writings of Dr. M. von Rohr.

† The latest contribution to the question of abstract collinear relations is the *Elemente der Projektiven Dioptrik*, by Professor Arthur von Oettingen, Leipzig (1909).

for plane, is the ideal sought. Want of collinearity results, as said, in two defects called " curvature " and " distortion "; the image of a flat object being formed not as a plane, but as a curved surface and unequally magnified at the margins as compared with the central parts—defects particularly obnoxious in photographic lenses. But there are also defects in the definition of focus, which are no less serious, and are known as aberrations. Given freedom from aberrations, and fulfilment of collinear relations, then everything is reduced, even in the most complicated system of centred lenses to the sweet simplicity of Gauss,* wherein distances of foci, both principal and conjugate, are measured from the respective equivalent planes. Before, however, we enter upon aberrations in detail, we must deal with the work of two of the giants of past days.

Newton and Huygens.—For more than two centuries the names of Newton and Huygens have been pre-eminent. These two men tower above their contemporaries and followers, dominant figures even to-day. The influence of Newton in science has been immense. His great genius, shown in his *Opticks* in the unravelling of the puzzle of the colours of the prismatic spectrum, and in his *Principia*, in laying the foundation once for all of the laws of motion and of the doctrine of universal gravitation, won for him an almost idolatrous regard. We may recall Alexander Pope's couplet—

> Nature and Nature's laws lay hid in night;
> God said, " Let Newton be," and all was light.

Even his mistakes—and they were few—were accepted as dogmas, as when he pronounced the dispersive power of prisms of different kinds of glass to be proportional to their refractive power; involving the impossibility of ever obtaining an achromatic lens. Even after a hundred years the Newtonians out-Newtoned Newton in their antipathy to anything that seemed counter to his views; and their hostility to Thomas Young's doctrine of interference is a matter of history. It is patent to anyone who will read the Queries at the end of Newton's *Opticks* (1704), that while he held to the emission theory of light he was quite alive to the probability that emission was performed in shining bodies by the vibrating motions of their parts (query 8); and he was prepared to admit that the several sorts of rays of different colours make vibrations " of several bignesses . . . make after the manner that the vibrations of the air, according to their several bignesses, excite sensations of several sounds " (query 13). Newton, " who looked all Nature through," who " surpassed the human race in power of intellect," who refused to frame hypotheses, must not be judged by the indiscretions or bigotries of his school.

If we would truly appreciate his greatness, we must go back to his own writings,

* One system of Geometrical Optics forms an exception. It is that of Biot (1844) and of Bosscha (1877–86), who base their equations on measurements of the distances of object and image not from equivalent planes, but from the respective refracting surfaces, and introduce into their expressions the angular magnitude as seen from the pupil of the eye, and four constants, viz., two principal focal lengths also measured from the first and last refracting surfaces respectively, and two ocular points which are the conjugates of the summits of the refracting surfaces. Sissingh has adopted this plan of measurement, but its advantages are doubtful.

and see how he worked. It is in the hope of reviving some interest in his actual achievements, that the promoters of this Convention, aided by the liberality of the Ancient Guild of Spectacle Makers of the City of London, whose Charter was granted thirteen years before Newton was born, have organized in our Exhibition an Isaac Newton Room. In this apartment are set up simple arrangements of apparatus to repeat the particular experiments by which, in 1670, using beams of sunlight through holes or slits in a shutter, and triangular prisms of glass, he demonstrated the resolution of white light into colours, and clinched the demonstrations by recombining the colours into white light. If, following the modern views of Lord Rayleigh and of Arthur Schuster, we now have to admit that periodicities characteristic of the several rays are in reality imposed by the prism, and are not antecedently present in the white light, that does not derogate from Newton's achievement. The mathematical arguments which lead logically to the view that the prism manufactures the colours from the white light, and does not merely sort them out, seem unanswerable; but to some of us those arguments seem to be of that class which compel assent rather than satisfy the mind.

Christiaan Huygens, Newton's great contemporary, propounded his wave-theory of light in 1678, though his famous *Traité de la Lumière* appeared only in 1690. Few British students have ever read that rare work, but none can read it without being impressed with the genius of its author. Every one knows of Huygens as the inventor of the wave-theory of light; but how few are familiar with the contents of the Treatise! He expounds the analogy of the propagation of light with that of sound, then points out the essential differences, and develops the geometrical notion of movements spreading in spherical waves. He had, in fact, to take into account seven fundamental facts *: (1) The rectilinear propagation of light; (2) the mutual penetrability of two beams where they cross one another; (3) the law of reflexion; (4) the law of refraction (which he had learned from Des Cartes); (5) atmospheric refraction; (6) the finite speed of light, discovered by Roemer in 1676; and (7) the double-refraction of Iceland spar, discovered by Bartholinus in 1669. The insight with which, by aid of his conception of elementary waves building up an enveloping wave-front, he succeeded in giving a consistent theory, is a matter for wonder and admiration.

He availed himself of Fermat's principle of least time, deduced from it the law of sines for refraction, and based on it the geometrical construction for his wave-fronts which now appears in all books on Physical Optics. It is true that he had no conception of transversality in the movements of his waves, or of the principle of interference, or even of the existence of trains of waves or of wave-length. His wave-theory was far from being the complete doctrine of Young and Fresnel, and belongs to geometrical rather than to physical optics. But the exquisite skill with which he unravelled the intricacies of double-refraction in crystals and the anomalies of atmospheric refraction must excite the admiration of every reader. His speculations as to the æther of space, his suggestive views of the structure of crystalline bodies,

* See List of Phenomena of Optics in the Appendix.

and his explanation of opacity, slight as they are, surprise one by their seeming modernness. He detected the double-refraction of quartz, and discovered the phenomenon of polarization, while frankly unable to explain it. Another section of his book deals with aspherical forms of lenses for focusing light when one surface is prescribed.

That the Treatise of Huygens should never have been translated into English* is strange : perhaps the omission is due to the mistaken zeal with which everything that seemed at variance with the teachings of Newton was denounced. Now, after more than two centuries, it has seemed to me that the time has come to make reparation. Accordingly, a complete English version of the *Treatise on Light* has been published by Messrs. Macmillan in time to lay it before the present Convention.

Huygens was in many respects a notable personage. A Dutch aristocrat, a friend in his youth of Des Cartes, he, like the philosopher Spinoza, was a grinder of lenses. With his own hands he built in 1655 the telescope with which he discovered the ring of Saturn and the fourth of his satellites. In 1663 he was elected a Fellow of the Royal Society, at the second meeting after the granting of the Charter. The Royal Society possesses three telescope objectives ground and polished by him, one having 122 feet focal length, presented in 1691 by himself, and two others of 170 and 210 feet focal length, the former presented by Sir Isaac Newton. In 1666 Huygens was called to Paris, by Louis XIV, to aid in the formation of the Académie des Sciences. There he resided till 1678, and there he wrote the famous *Traité de la Lumière.* He was again in England in 1689; for his brother records that on July 10 in that year he went with him in company with Newton to wait on King William III for the purpose of recommending Newton for the vacant Mastership of one of the Colleges in Cambridge !

Huygens's Rejoinder.—It has been objected, and by no lesser an authority than Sir George Stokes (*Burnett Lectures on Light,* Vol. I, p. 19), that Huygens's " grand attempt " to explain the rectilinear propagation of light by the superposition of undulations failed. " Notwithstanding all that Huygens has done, the existence of rays and shadows, one of the most obvious properties of light, had received no satisfactory explanation on the theory of undulations such as it came from the hands of Huygens." " This principle does not by itself alone suffice for the explanation of rays. It proves, or at least appears to prove, too much. It is as applicable to Sound as, on the supposition that Light consists in undulations, it is to Light : and if Huygens's explanation of rays were complete, there ought equally to be rays of Sound, and Sound ought to present the same sharp shadows as Light." Newton, in fact, had made the same objection in his *Principia* (Book II, Proposition XLII, and Scholium to Proposition L), where he notes how Sound spreads round obstacles, and that Light travels as in rays. We know now that sounds can cast shadows if the wave-length is small compared with the size of obstacles. We

* I do not forget the fragment translated by Professor Henry Crew in the collection on the Wave-Theory, published at New York in 1899. Here, however, the most interesting chapters on Atmospheric Refraction and Double-Refraction are omitted.

also know, and partly from the experiments of Newton on " Inflexion," though he understood it not, that light can spread to a small extent into the shadows of objects; the sharpness of the shadow resulting from the minuteness of the wave-lengths. But Sir George Stokes, in repeating Newton's objection, does not seem to have been aware that Huygens had himself answered Newton. This answer is not given in the *Traité de la Lumière*, but in the *Discours de la Cause de la Pesanteur*, which was printed in 1690, along with the *Traité de la Lumière*. As the passage is not generally known, and as the Discourse on Weight has never been translated into English, I here supply a version of it, from the original—

" He [Newton] will perhaps say to me that when one shall have granted me that for the transmission of Light the ethereal matter consists in particles which touch one another, one will not see how Light would observe the rule of being transmitted only in a straight line, as it is; because this is contrary to his Proposition XLII of Book II, which says that the movement which spreads in a material fluid, after having passed through some aperture, is not propagated only straight from its origin, but that it diverges also laterally. To which I reply in advance that what I have alleged, in order to prove that light (save in reflexion or in refraction) is propagated directly only, does not cease to be valid notwithstanding the said Proposition. For I do not deny that when the sun shines through a window he does not transmit movement aside of the space illuminated; but I say that these deviating waves are too feeble to produce Light. And though he will have it that the emission of Sound proves that these lateral divergencies are perceptible, I hold for certain that it rather proves the contrary. Because if Sound, having passed through an aperture spread also laterally, as Mr. Newton will have it, the equality of the angles of incidence and reflexion would not be so exactly observed as they are in an Echo; in such a way that when one is placed in a spot whence no perpendicular can be drawn to the reflecting plane of a somewhat distant wall, one hears no Echo whatever to the sound that one makes at that spot, as I have very often found experimentally. I do not doubt also that the experience which he adduces of the sound which is heard, notwithstanding an interposed house, would be found quite different provided that house were placed in the middle of some large lake, or so that there should be round about it nothing which could send back by reflexion any part of the sound.

" And as for what he says that in some place, being in a room the window of which is open, one then hears the sound from outside, not by reflexion from the walls, but coming directly from the window; one sees how easily one may be deceived because of the multitude of repeated reflexions which occurs as in an instant, in such wise that the sound which is heard as if coming immediately from the open window, may come from thence, or from places quite near, after a double reflexion. I confess, then, so far as regards the undulations or circles which are formed on the surface of water, that things occur almost exactly as Mr. Newton declares: that is to say, that a wave, after having passed the aperture, spreads successively at both sides, and always more feebly there than in the middle. But as for Sound, I say that these lateral emanations are almost insensible to the ear: and so far as concerns Light, they produce no effect at all on the eyes.

" I have thought that I ought to meet in advance these objections which the book of Mr. Newton might suggest, knowing the high esteem accorded, and rightly so, to that work; because there is none known more learned in these matters, nor any attesting a greater penetration of mind."—*Discours de la Pesanteur*, 163–165.

Aberrations.—The enormous focal lengths adopted by Huygens for his telescopic object glasses were adopted because with such slight curvatures as these lenses possess there is comparative freedom from aberrations. No actual lens ever gives perfect stigmatic results; and every beginner knows that aberrations are of two classes: those that arise from the polychromatic nature of light, and those which, even when monochromatic light is used, are due to the form of the surface of the lens, and are often—though not very happily—termed spherical aberrations. Newton calculated (*Opticks*, pp. 84, 89) that in a 100-ft. telescope with suitable aperture the aberration of colour would be at least 1200 times as great as the aberration caused by the sphericity of figure of the object glass. We know, in fact, that in despair at making a lens devoid of chromatic aberration he gave up refractors and invented his reflecting telescope. But when in 1757 Dollond, by the invention of the achromatic lens, removed the worst of the aberrations, the correction of the aberrations due to form became the next desirable step. Des Cartes, Deschales, and other writers, suggested various devices for grinding lenses with hyperbolic, elliptical, and other aspherical curves; but practical difficulties prevented their use. In the early part of the nineteenth century, Coddington and Airy, the younger Herschel, and others, investigated in great detail the aberrations of lens combinations, and brought that part of Optics to a high pitch, though much of their work remained unknown outside England. The complexity of the problem of the passage of a ray or pencil through a centred system of spherical lenses has been alluded to already. The complexity arises entirely from the trouble occasioned in continually being obliged to shift the origin from which the angular position of the ray is measured, from one point to another, as the ray passes from one refracting surface to the next. Lagrange, as mentioned above, solved the detailed problem by the use of continued fractions. Trigonometrical formulæ for the purpose were greatly used by Steinheil, and improved by von Seidel. Von Seidel, in 1855, introduced a different method of considering the successive deviations, leading him to define five conditions, each expressed in terms of the sum of a series, which must be severally fulfilled if the performance of the optical system was to be stigmatic and collinear. The conditions were that each sum must be severally zero. In this way the three errors of central aberration, coma and radial astigmatism, and the defects of curvature and distortion, could be successively predetermined in a given lens combination. Abbe introduced another method of dealing with aberrations by the use of the " optical invariant," used throughout in Czapski's Handbook. Earlier in time is the Hamiltonian method of treating the same complex problems by use of a " characteristic function," a development of Fermat's method of least time. Though of great generality, this has been little used. Thiesen and Chalmers have sought to modify it. Recently, Schwartzschild has re-stated Hamilton's function in terms of the variables used by von Seidel, yielding complete formulæ suitable for calculating lenses; while Lord Rayleigh, in the fifth volume of his *Scientific Papers*, the publication of which is one of the milestones of progress, has shown how readily the number and nature of the five constants are deducible from Hamilton's principle. Of the five equations of condition yielded

by von Seidel's method, great stress is justly laid on the second (the condition for the removal of coma), which is equivalent to the sine-condition independently discovered by Abbe. Abbe afterwards found that this condition had been well fulfilled in the famous Heliometer objective of Fraunhofer, and, still more remarkable, that it had been approximately fulfilled by the makers of almost every * good wide-angled microscope objective, by means of corrections which they must have found empirically since the law was unknown to them. They " builded better than they knew." The other four conditions of von Seidel had all been previously worked out on other plans by Airy or Coddington. The recent admirable tract on the *Theory of Optical Instruments*, by Professor E. T. Whittaker, published in 1907, discusses von Seidel's five conditions, with a convenient notation, and using Abbe's invariants. R. A. Herman† and R. A. Sampson‡ have also contributed to the detailed study of aberrations; the latest contribution being that of H. C. Pocklington,§ who has modified Lord Rayleigh's method to obtain expressions suited for the detailed calculation of particular systems. Two important contributions to the study of aberrations come from Scandinavian investigators, Charlier and Gullstrand, the latter of whom, in 1900, published an elaborate monograph dealing both with astigmatic and non-astigmatic pencils. Nor must it be forgotten that, in 1905, C. V. Drysdale showed how the curvature method of geometrical optics can be extended to certain of the aberrations. In 1906 appeared an important work by Mr. H. Dennis Taylor, *A System of Applied Optics*, which is of the more interest because written by a most accomplished practical optician and instrument designer. In it there is worked out, chiefly on the basis laid down by Airy and Coddington, a complete system of formulæ for lens systems, the merit of which is enhanced by the addition of many examples of their practical application : but the value of the work is reduced by the adoption of a method of reckoning signs that is awkward, if consistent with itself.

Comment was made above that the complexity of all the formulæ for computing the path of a ray through an optical system arises from the necessity of carrying on the calculations from one surface to another by a series of transferences. Now the analytical or arithmetical complications involved in these transferences could be dealt with, as remarked by Larmor, quite simply by graphical methods if only those methods were capable of the necessary precision. But where small angles are involved graphic processes are exceedingly rough, hence the preference for analytical processes. This suggests the thought that some adaptation of vector calculus and the use of complex numbers, which in recent years has yielded excellent results in electrical problems as a substitute for clumsy graphic processes, might reasonably be expected to produce equally good results as applied to optics. A new optical calculus of this sort might require the tabulation of new functions, but assuredly is worthy of being formulated; and if carried out in conjunction with the

* Abbe: *Sitzber, Jenaischen Gesellschaft für Medecin and Naturwissenschaft* (1879).
† *A Treatise on Geometrical Optics.* (Cambridge, 1900.)
‡ *Proc. Lond. Math. Soc.*, xxiv. 33 (1898).
§ *The Aberrations of a Symmetrical Optical Instrument*, Proc. Roy. Soc. (Nov. 1909), Series A, Vol. 83, pp. 99–106.

method of convergencies (that is, founded upon the unit of convergency, the *dioptrie*, which has been in vogue in British teaching for nearly a quarter of a century), would prove still more valuable. There is abundant room here for younger workers to strike out a new departure.

Illumination.—One branch of Geometrical Optics has not been touched upon—that of Illumination, involving questions of the distribution of light, and the measurement of it in quantity and intensity by photometers. During the past seven years there has been great activity in the development of this branch, and new societies have grown up for its study. Of this subject the best view will be obtained by the perusal of the masterly book entitled *Illumination* (published 1911), by Mr. A. P. Trotter, himself a pioneer in two directions, the photometric measurement of the amount of illumination which is received upon any surface, whether roadway, table or wall, and the redistribution of the light emitted by any luminous source (whether gas or electricity) by means of dioptric globes or shades, such as those shown in our Exhibition by the Holophane Company and other commercial houses. Though a better standard source of light than either the Harcourt pentane lamp or the Hefner amyl-acetate lamp is still a desideratum, it is satisfactory to know that international agreement upon the unit of light is practically secured, through the collaboration of the four great laboratories at Sèvres, Charlottenburg, Bushy and Washington. Committees have been actively at work on the questions of minimum illumination required in schools, libraries, factories of various kinds, and in roads and streets. Even the House of Commons has awakened to the fact that the illumination enjoyed by its members is only about half a candle-foot, whereas for comfortable reading it should be two or three times that amount. Photometry has indeed grown since the photometric law of inverse squares was first announced by Deschales in 1674, or since the early treatises of Bouguer and Lambert. New forms of photometer have multiplied, and every month sees fresh developments. Amongst the novelties in distributing appliances is the simple and beautiful device of Messrs. Mackinney and Chalmers for the uniform illumination of pictures in such a way that from no standpoint can any reflected view of the lamp be seen.

Physical Optics.—When we turn to the vast subject of Physical Optics we cannot but be struck with the variety of phenomena* which must be taken into account by any one who would deal with the nature of light itself, or with the mechanism of the ethereal medium by which it is conveyed. Dispersion and its anomalies,† interference, diffraction, the multitudinous effects of polarization, the problems of radiation and luminescence, of opalescence and the blue of the sky, of iridescence and the gorgeous colours of butterflies and humming-birds, to say nothing of radioactivity, or of the

* See list, at the end, of the chief properties.

† Herschel, in 1828, in his article "On Light" (*Encyclop. Metrop.*, p. 450), declared: "The fact is that neither the corpuscular nor the undulatory, or any other system which has yet been devised, will furnish that complete and satisfactory explanation of *all* the phenomena of light which is desirable. Certain admissions must be made at every step, as to modes of mechanical action, where we are in total ignorance of the acting forces; and we are called on, where reasoning fails us, occasionally for an exercise of faith."

chemical, physiological, electrical, magnetic, and mechanical relations of light, furnish whole fields in which knowledge is still in the making.* In Physical Optics, though there are mathematical laws, such as those discovered by Fresnel and Stokes, to be mastered, the chief concern is with physical phenomena; and the study of these would seem to be inseparable from speculations as to the nature of the luminiferous æther, and from consideration of the conflicting theories as to its constitution. Formerly the vexed question was the mechanical explanation of an æther which should behave like an elastic solid a million times more rigid than steel, and at the same time as a mere vapour a billion times less dense than air. Then there was an outstanding quarrel between the followers of Fresnel and those of Neumann and McCullagh as to whether the vibrations of light were executed in or across the plane of polarization. Maxwell dissipated the controversy when in his electromagnetic theory of light he showed that both were present, the electric vibrations taking place across the plane, and the magnetic ones in it. To-day, and ever since Maxwell propounded the electromagnetic theory, the main interest has been transferred to the question how the æther is related to electricity and to ponderable matter, and whether the motion of matter in space affects or is affected by the æther. Is it a fact that the æther is stagnant, fixed, " while the molecules constituting the earth and all other material bodies flit through it without producing any flow in it " ?† Or is the æther speeding along with the earth and the whole solar system in headlong and enormous flight ? That singular doctrine, now in fashion, called the " Principle of Relativity," invites us first to deny that we can ever detect or measure the absolute velocity of the earth in space, and then to admit that, therefore, since we cannot regard the æther as filling space or fixed in it, we must abolish the notion of the æther as a conveying medium, and must explain the finite velocity of light in some other way depending on electro-magnetic principles inherent in the light impulse, and expressed in terms of co-ordinates whose origins are to be only relatively, and not absolutely fixed. Without pursuing these anarchical ideas,‡ we may remark that for all useful purposes it suffices to admit that no terrestrial optical phenomena have any relation to the direction of the earth's motion through the universe.

Matter and Æther.—As for the relation between matter and æther, while for clarity of thought we must frame some idea of the connexion between them, we may accept Sir Joseph Larmor's dictum that " Matter may be, and likely is, a structure in the æther, but certainly æther is not a structure made of matter." His view, that " the motion of matter does not affect the quiescent æther, except through the motion of the atomic electric charges carried along with it," is, of course, bound up

* Let no one suppose that the field of discovery in physical optics is exhausted. I have recently observed a new and beautiful phenomenon, not hitherto described, to be observed when a luminous point is viewed by both eyes through a slab of double-refracting crystal revolving in its own plane.

† Larmor: *Æther and Matter* (1900), p. 163.

‡ I omit all reference to the still more anarchical notion of *Licht-Quanten*, which requires us to consider light as being emitted in atomic quantities.

with the further conception that the æther is a plenum in which " vortices or other singularities of motion and strain " are the nuclei of which matter consists.*

It was pointed out how in Geometrical Optics there is some advantage in stepping aside from physical considerations to deal with the geometrical laws in the abstract. So also in Physical Optics, there are mathematical laws governing the phenomena, the formulation of which in no way depends on the vexed questions of the æther or its connection with matter. A brave attempt has been made by Duhem† to perform this task. Confining himself to monochromatic light, and to isotropic media of perfect transparency, he has with rare and critical analytical skill built up a complete and general theory of Optics, including interference, diffraction, and polarization, without making any assumptions as to the nature of light itself, except that it is *a quality* capable of transmission; so that Physical Optics becomes here a system of symbolic equations, the aim of which is to present rather than to explain the features observable, by experiment, in that quality.

Interference Phenomena.—But Duhem's *tour de force* was eighteen years ago, and the trend of physical optics at present is rather in the direction of pure experiment. Fraunhofer's invention of the diffraction grating is commemorated in our Exhibition by a piece of his original apparatus, containing six gratings made by himself, a loan from the Royal Society. Of modern grating apparatus, we have several examples: a photographic grating made by Lord Rayleigh in 1874; and some of those perfect reproductions in celluloid made by Mr. T. Thorp from Rowland's ruled gratings. Michelson (*Astr. Phys. Journal*, 1910) has succeeded in ruling gratings of a high degree of perfection, no less than 10 inches in diameter, from which great things are expected. A notable contribution to objects of this class is R. W. Wood's echelette grating, stamped, not ruled, with grooves of scalene section. We are further promised a study of diffraction patterns, by Mr. J. W. Gordon. Interference continues a subject of fruitful research. New forms of interferometer have been devised—for example, that of Fabry and Perot—and good work has been done by various workers with the echelon spectroscope of Michelson, and the interference-plate apparatus of Lummer and Gehrcke. In the construction of such instruments the utmost precision is needed; and it may justly be stated that never before in the world's history has such marvellous optical workmanship been shown. Messrs. Adam Hilger and Co., who construct Michelson's echelons, have no easy task in fulfilling the requirement that each glass plate of which the echelon is built up shall be of the same thickness—usually about a centimetre—with an accuracy of one-twentieth of a wave-length. I have lately seen in Professor Lummer's laboratory at Breslau a glass plate 15 centimetres in diameter and $4\frac{1}{2}$ centimetres thick, with faces guaranteed to be accurate all over to within one-tenth of a wave-length; and the accuracy attained over the central portion within a diameter of four centimetres was at least double as great. That is, there was no error of thickness so great as

* Particular reference may be made to Sir Joseph Larmor's *Æther and Matter* (1900) and to Professor E. T. Whittaker's *History of the Theories of Æther and Electricity* (1910).

† *Fragments d'un Cours d'Optique.* (Bruxelles, 1894.)

one part in two millions; or as if one had succeeded in measuring off a mile without an error greater than one three-hundredth part of an inch!

Allusion was made already to Lord Rayleigh's fifth volume of *Scientific Papers*. Time will not permit me to enumerate even the titles of many important memoirs in Physical Optics to be found therein; but those on Motion through the Ether, on the Spectrum of an Irregular Disturbance, on the Passage of Waves round Spherical Obstacles, on the Origin of Prismatic Colours, on the Measurements of Wave-lengths, on the Dynamical Theory of Gratings, on the Colours of Sea and Sky, proclaim him as our unquestionable master of to-day. Perhaps we live too near him to appreciate the permanent importance of his magnificent contributions to Optics, of which these are the latest instalment.

Polariscopy.—Polarizing apparatus will be found in various forms in our Exhibition, including sundry new large reflecting polariscopes, designed for and with the aid of my colleague, Professor Coker, for the study of optical phenomena in strained transparent material. Brewster's discovery in 1815 of the optical effect of stress of glass led him to suggest that it might be applied to the useful and important object of ascertaining the state of strain or compression of the different parts of structures, such as bridges or framings, by the use of glass models. Professor Coker, imbued with the same idea, has made models, not in glass but in transparent celluloid, of many kinds of structures, from a crane-hook to the deck of a torpedo boat, and particularly of springs of different kinds, and has explored and measured the distribution of strains in them. To eliminate in such observations the "black cross" effect inevitable in the dark field of the polariscope, we have employed quarter-wave plates of mica of large size. Some of these are 12 inches square. Others are circular, up to 20 inches diameter; others rectangular, 40 inches by 10. For these largest sizes it has been necessary to join together two pieces. Messrs. Wiggins, who furnished the sheets of fine quality from which these pieces were split, are showing a noteworthy collection of mica specimens in our Exhibition. Phosphorescence is illustrated in our exhibition by two exhibitors of luminous preparations. One of these, Mr. A. F. Landau, has propounded a theory of the constitution of phosphorescent bodies somewhat differing from those of Lenard and Kowalsky. He confirms an observation to which years ago I drew the attention of Sir George Stokes, that a specimen of the white phosphorescent material (essentially calcium sulphide) when in the luminous state, assumes a slightly brown tint; a question recently investigated by Mr. J. B. Burke.

The properties of ultra-violet light, as well as those of the infra-red rays, have received much attention in recent years. No researches in this field are of greater intrinsic interest than those of Professor R. W. Wood, whose shadowless mid-day photographs of natural scenes are truly surprising. Professor Wood's book on *Physical Optics*, now happily in its second edition, is a wonderful repertory of the whole round of phenomena in this domain. There could be no better antidote to the poison of examinational Optics, as found in the common textbooks, than the whole-hearted possession of the contents of this book.

Lastly, in this hasty survey of this branch, there is the vast subject of Radiation by incandescent bodies. The law of Stefan connecting the total amount of radiation with the absolute temperature, has been supplemented by the further relations discovered by Wien, Paschen, Lummer, and other investigators. Lummer's work on the radiation of the ideal black body, or as we now call it, " full " radiation, is at last being properly recognized: it suggests much that still calls for further research.

Optical Instruments.—Time fails for comment on the progress of instrument manufacture. Our Exhibition speaks for itself in the array of microscopes, telescopes, spectroscopes, binoculars, and other instruments. If the advances of the last few years have been in the improvement of detail, they are none the less advantageous. English microscope makers, who have never lost the traditions of good design and fine workmanship since the days of Lister, Ross, or Powell and Lealand, have nothing to fear in criticism of their designs or their quality. They have certainly to face the competition in prices with factories organized for large-scale production. No one who really knows the excellence of their work can help deploring the disadvantageous commercial position in which they hold their own against the world.

The researches of Mr. J. W. Gordon, on means for further magnification of microscopic images, on microscope illumination, and on the diffraction theory of the formation of images, will be fresh in the minds of those who follow the proceedings of the Royal Microscopical Society.

In camera lenses there is little to chronicle, except the new lens designed by Messrs. Horace and Conrad Beck, and called the Isostigmar; the peculiarity of which is that while its design departs considerably from that which would fulfil the Coddington-Petzval condition for the anastigmatic flattening of the field, it nevertheless gives an excellent flat field up to 35°, with an aperture of f/5·8.

A new instrument, the Cystoscope, a special species of telescope for examining the cavities of the human body, has appeared; the chief forms being those designed by Nitze and by Dr. M. von Rohr. The periscopes used on submarine boats for observation purposes are akin in their optical design.

As to telescopes, Sir Howard Grubb's exhibit demonstrates the continual progress in that field. It is a matter of regret that the abiding difficulty of procuring discs of optical glass of sufficient size is seriously retarding the construction of astronomical telescopes as needed for modern stellar research. A Royal Society Committee has recently been appointed to inquire into the manufacture of optical glass.

Spectacle Optics.—It seems a far cry from " trunk spectacles," as our forefathers quaintly named their telescopes, to visual spectacles; but the oldest application of Optics is undoubtedly that of aiding vision. The fixing of two lenses together to form a pair probably dates from the thirteenth century; but history is obscure. Raphael, in 1517, painted Pope Leo X wearing concave spectacles. But not all pictures are good as evidence, for there is, or was, in the Chiesa de' Ognissanti, in Florence, a picture, attributed to Sandro Botticelli, depicting St. Jerome in his cell, with a pair of spectacles beside him. This does not prove that spectacles existed

in the fourth century; and the presence of the spectacles may be as great an anachronism as, in another picture of the same Saint, is the presence, on the wall of the cell, of a pendulum clock. Coming down to the present day, few persons probably are aware of the rapid rate at which that branch of the subject is developing into a severe scientific study. Perhaps they think that the only progress in spectacle making has been the introduction of lighter spectacle frames or ingenious dodges for grinding bifocal glasses, or for fusing one kind of glass into another for a bifocal lens, or appliances for grinding toric lenses. This would be quite a mistake. It may be that the teaching in the medical schools has remained much as it was; but the training of spectacle opticians to deal with the problems of astigmatism, both of eyes and of lenses, has taken great strides, and under the stimulus of the system of certification by the Spectacle Makers' Company and other optical bodies is assuming an important development. Improved instruments for retinoscopy and keratometry are in evidence, and the young opticians are very keen to be in the forefront.

But, apart from actual practice, an exceedingly important advance in theory has been initiated by the genius of Allvar Gullstrand. In the year 1903 he pointed out that the centre of rotation of the eyeball does not coincide with the nodal point which is its optical centre. It is in fact from two to three millimetres behind it; and therefore in all those uses which the eye makes of its power of turning about in its socket the mathematical treatment which assumes it to be fixed is inadequate. The assumptions of the Gauss system are no longer fulfilled, and modifications have been introduced. For precise work this affects the efficiency of spectacle lenses and introduces new sources of aberration. For this reason spectacles should be so designed that the particular point at which they are corrected for radial astigmatism should lie at the centre of rotation of the eye. The importance of correcting this source of error is greater in the case of eyes whose refractive power differs greatly from the normal, and, in especial, for eyes that have been operated upon for cataract. The very thick lenses thereby necessitated, if ground with the usual spherical curves, are, as Gullstrand has shown, quite unsuitable for any oblique vision. He has shown that special forms are needed, and has devised aspherical lenses to meet that need. About these we are to hear at this Convention from Dr. Moritz von Rohr.

But while it would be difficult to overrate the importance of this advance, care s needed on the other hand that no false impression is created. For all those optical purposes where the eye is fixed to gaze through an eye-piece, and not rolling about in free vision, the new theorems are of little importance. Gauss's system has not been overthrown. Collinear relations never did in fact apply strictly to the formation of images* in the eyeball, for the images there are not required to be focused (as in a camera) upon a flat plate, but are thrown on the interior surface of a globe. It does not appear that any mathematician has ever even considered the conditions necessary in an optical system, in which the curvature of the image is regarded as an advantageous result. Possibly some future calculator will discover how much better adapted

* The fact that the parallel lines of a wall opposite which one stands seem to bend toward one another on both sides has been known for a hundred and fifty years.

to its own work the optical system of the eye is than if it had been designed to be free from the so-called aberration of curvature.

The Dioptric System.—One other point in spectacle optics needs attention. Thirty years or more have passed since British opticians ceased to denominate their lenses in terms of inches of focal length, and adopted the dioptric system of numbering, in which a lens having a focal length of one metre is described as having a power of one dioptrie, and a lens of twice that power as of two dioptries. The *dioptrie*, the international unit of lens-power, was adopted in 1875 on the proposal of Monoyer, at the Brussels Conference. Nearly thirty years ago it was pointed out that the dioptrie, being the reciprocal of a length, is in reality a unit of curvature, and may be applied to express curvatures of wave-fronts, and of surfaces, as well as the power of a lens, which is, in fact, merely the expression of the convergency which it imposes on the light passing through it. A special dioptrie spherometer, described before the Physical Society in 1889, is on view in our Exhibition. For over twenty-five years, at the City and Guilds Technical College, Finsbury, and for nearly twenty at the Northampton Institute, Geometrical Optics has been taught on the dioptric or curvature method. But, strangely enough, the impression made in some quarters by Gullstrand's work is so great that the legend is growing up that he is the discoverer of the curvature-method, and the framer of the dioptric system. Two recent German textbooks have lately been published, in which—implicitly in one, explicitly in the other—it appears that the method of numbering lenses in dioptries instead of inches is due to the famous Swedish ophthalmist! It may be, though it is scarcely credible, that the dioptric numbering of lenses was unknown in Germany from 1875 until 1899.

Optical Illusions.—From this Address are deliberately excluded all references to the immense subjects of Photography and Physiological Optics, not because there are no advances to record—such as the improvements in colour-photography, or the recent investigations of Sir William Abney and of Dr. Edridge Green in colour-vision, or Sir W. F. Barrett's Entoptiscope—but simply for want of adequate space. The question of persistence of vision, with the array of instruments depending upon it, the stroboscope, the thaumatrope, the animatograph and the kinematograph, would furnish matter for a whole discourse. The vexed question of the sensations of colour, and which of them are really primary sensations, and how many are the truly primary, is another physiological topic of wide extent.

But one section of Physiological Optics, namely, that of Optical Illusions, must claim notice, since our Exhibition contains various examples, and a special lecture by Professor W. Stirling is devoted to these matters. One of the newest, by Dr. James Fraser, in which concentric circles drawn on a background of diaper pattern appear like spirals, is also one of the most surprising. All optical illusions are not, however, physiological; for some of them depend purely on the geometry of projection; and others, such as the anamorphotic telescope, on the properties of non-homocentric pencils. Amongst the latter will be shown some singular cases of distortion, the shadow of a straight wire appearing as an oval ring; while telescopes which invert

without perverting, or which magnify unequally in different directions, are simple combinations of astigmatic lenses.

If any excuse be needed for introducing the frivolity of illusions into a scientific exhibition, it is afforded by the valuable illustrations these toys afford of the principles that underlie the whole.

Optical Education.—To the optical industry as a whole the question of the scientific training of young men who shall hereafter become technical leaders and pioneers is a very serious one, in view of the stress of the times. Men are wanted who can undertake mathematical calculations with a first-hand practical knowledge how these calculations are applied in the design of instruments, and who have a thorough acquaintance with the whole range of Optics. That training at present they cannot acquire at any of the Universities. It is a melancholy fact that now, when this need is sorest, the pursuit of Optics at our Universities and Colleges is in a deplorable state. Except in the Northampton Polytechnic, and one or two other institutes, the study of Optics for its own sake is entirely ignored. Not one of the Universities of Great Britain has created a chair of Optics, though there are professorships and extensive laboratories for electrical engineering, for metallurgy, for brewing, and for various branches of technical chemistry. In the Universities and Colleges the only people who are learning Optics are merely taking it as a part of Physics for the sake of passing examinations for a degree, and care nothing for the applications of Optics in the industry. They are being taught Optics by men who are not opticians, who never ground a lens or calculated even an achromatic doublet, who never worked with an ophthalmoscope, or measured a cylindrical lens.

Again and again, as might be demonstrated by many instances, advances in Optics have come about through the association of the highly trained mathematician with the practical workman, and most effectually when these are combined in one individual. But where is England to look for the training up of such men? For twenty-five years some of us have urged the need of an Institute of Technical Optics, where students of Optics will be trained in Optics by men whose work is Optics. The need grows year by year. Deputations from the trade have waited on the London County Council, and questions have been asked in Parliament, yet in vain. It has been suggested that two separate schools are needed—one for optical workmen, the other for optical calculators, the latter to be a mere small department in one of the Universities or Colleges. Such a divorce of practice and theory would be futile. What is wanted is an establishment where the whole atmosphere is one of optical interest; where theory and practice go hand in hand; where the mathematician will himself grind lenses and measure their performance on the test bench; where brain-craft will be married to handcraft; where precision, whether in computation or workmanship, will be a dominating ambition.

As yet the only attempt made towards this ideal is the Optical Department of the Northampton Polytechnic, in Clerkenwell, where a handful of students are housed in wholly inadequate surroundings. In the future Institute the teaching must be thorough, and independent, and free from all ulterior domination of

examinations. The examination blight, which has cramped education in so many ways, has brought us to this pass, that outside the centre just named there is not a college student in Great Britain who is being trained in *Optics for its own sake.* The moral is obvious. The future Optical Institute must be properly housed and equipped as a self-contained Monotechnic, concentrating all its energies on the one aim. On no consideration whatever ought it to be under the baneful influence of a University, where its students would be diverted from whole-hearted devotion to progress by the temptation of degree-hunting. Would that this Convention might make it clear to those in authority that the optical industry is in deadly earnest in demanding the establishment of such a centre of optical training.

There remains only the pleasant duty of inviting your presence in the Exhibition, whose portals, now unbolted, give us access to the most beautiful of the wonders of Nature.

Claustris ergo reseratis, aditus apertus est nobis in pulcherrima rerum naturæ.

APPENDIX

LIST OF PHENOMENA OF OPTICS

1. Rectilinear Propagation; Shadows.
2. Mutual Penetrability of Beams.
3. Reflexion.
4. Refraction.
5. Atmospheric Refraction.
6. Finite Speed.
7. Double Refraction.
8. Illumination.
9. Aberration (Astronomical).
10. Mirage.
11. Dispersion (Rainbow, Prism).
12. Absorption.
13. Diffraction.
14. Interference (Newton's Rings, etc.)
15. Polarization.
16. Mechanico-optical strain.
17. Optical Rotation (Quartz).
18. Reflexion and Refraction (Quantitative).
19. Scintillation of Stars.
20. Opalescence and Blue of Sky
21. Infra-red (Calorific) Light.
22. Ultra-violet (Actinic) Light.
23. Anomalous Dispersion.
24. Radiation and Incandescence.

25. Luminescence (Firefly, etc.).
26. Phosphorescence (Insolation).
27. Fluorescence.
28. Magneto-optic Rotation.
29. Velocity of Electro-magnetic Waves (" V ").
30. Refractivity of Dielectrics.
31. Opacity of Conductors.
32. Electric Double Refraction.
33. Magnetic Double Refraction.
34. Magnetic Duplication of Spectrum Lines.
35. Mechanical Pressure of Light.
36. Röntgen Radiations.
37. Constitution of White Light.
38. Radioactivity.
39. Photographic and Photo-chemical Effects.
40. Physiological Effects.

SECTOR SHUTTERS

By Cyril F. Lan-Davis, F.R.P.S.

(*Thursday, June 20th,* 1912.)

One of the earliest between-lens shutters was the Central, invented by Dallmeyer in 1888, and described in one of his old catalogues. Examples are still to be met with occasionally. Eight blades were used; seven or eight speeds adjusted by spring tension were provided, and the shutter opened in the course of setting.

The Bausch and Lomb shutters were introduced about 1890, the Unicum being the best known. It was a serviceable instrument, though now rather scorned on account of the discrepancies between the actual and marked speeds. The case was circular, and two leaves worked by pivoted bars were used, an arrangement which did not permit the power of the springs to be well applied.

A variety of other forms have been designed, the best known of which are the Compound and Koilos. These, though made in separate factories, are similar in general design. Three and sometimes five blades are employed, pivoted at one end, and connected by projecting pins to a zone plate which is capable of being rapidly moved by a powerful spring. In action the plate is rotated by the full strength of the spring sufficiently to open the shutter. The plate is then held in this position either instantaneously or for such longer period as the pneumatic brake arranged to give timed exposures may permit, and then the plate is drawn back to its original position by the spring, the opening and closing of the shutter occupying very short periods.

The shutter blades open from the centre towards the edges and close in the opposite way, the centre being thus open for a longer time than the edges. The ratio of total diameter to the diameter of the opening is from 3 : 1 to 2·5 : 1.

In order to ascertain what improvement can be made in this class of shutter I propose now to discuss the best shape of leaf and the number of leaves to be used.

I consider in each case that the full aperture is circular and that the entire aperture must be uncovered when the shutter is open.

The simplest type of rotating shutter is shown in Fig. 1. It is that adopted for lantern flashers. Only one leaf is used and the opening is at the side of the shutter.

A disc AC is pivoted at C and rotates to CA′ for full opening.

Total diameter of the shutter AA′ = twice opening diameter AC.

In Fig. 2 the case is shown of a single leaf shutter giving central opening. Here total diameter = 3 times opening diameter.

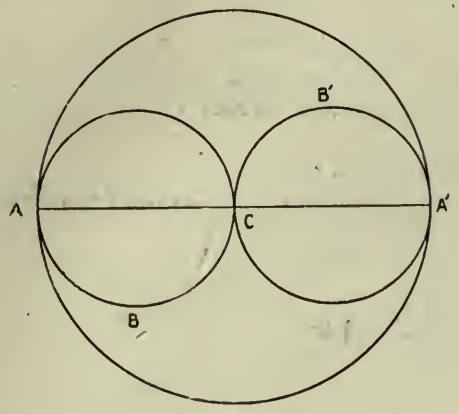


Fig. 1.

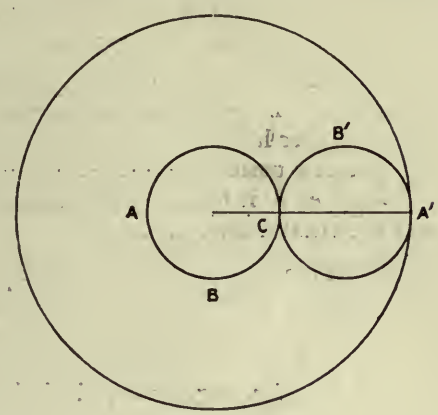


Fig. 2.

Two Leaves.

The next case is that of two leaves of identical shape. We may distinguish the cases of the pivot at the opening and the pivot on the extreme edge of the shell and leaf.

Two Leaves.
Pin at opening.

Fig. 3 shows the case of two leaves pivoted at B and D on the circumference of the opening.

The $\odot^s$ AE′F and BC′D are the bounding edges of the shell, so that

AF is the total diameter = 2R (say)

and BD is the diameter of opening = $2r$ (say).

Clearly the inner edge of each leaf should be an arc of a circle of the same radius as the opening, so that this inner edge will be S-shaped.

The outer edge will also be a circle of the same radius as the opening and the shape of the visible portion of the leaf will be as shown in BC′DC.

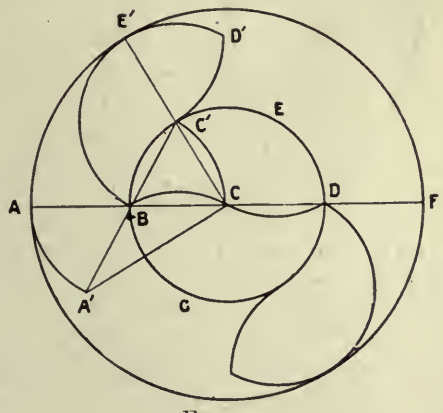


Fig. 3.

Let now the leaf BC′D rotate round B until the edge BC′ coincides with the circle of opening, and the leaf assumes the position BC′D′E′.

C will swing to C′ and the edge BE′D′ is found by describing a $\odot$ with C′ as centre and CB as radius.

Clearly for the least total diameter, the ⊙ AE′F must touch ⊙ BE′D′ and the point of contact E′ must be collinear with the centres C and C′—

i. e. $CE' = 2CC'$

and total diameter = twice opening diameter.

In practice it is usual to revolve the leaf round B by fitting a small projection or pin at A, and moving this by a revolving plate the shape of the zone ABDF as in the actual shutter here.

The new position of this pin A will be A′ lying along BC′.

Then ACA′ is the angle at the centre through which the plate must move in order that the shutter may open.

From the Δ^s BCC′, BA′C

$B\hat{C}A' = BA'C$

$= \frac{1}{2}C\hat{B}C' = \frac{1}{2}\,60°$

$= 30°$

i. e. angular travel of plate = 30°.

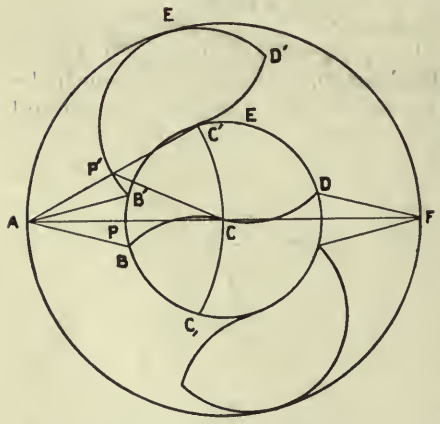


FIG. 4.

Pin at extremity. Fig. 4.

Here the shape of the visible portion of the leaf must be the same as before.

We have, however, to find the position for the pivot.

Let us take this to be at A.

Then we know that the new position of C will be C′ at the intersection of the ⊙ with A as centre and AC as radius, and the ⊙ of opening.

Also the inner edge of the leaf must be along the ⊙ BC′D of opening.

That is, the centre of the ⊙ of this inner edge must coincide with C, and this lies on the ⊙ with A as centre and AC as radius, and in the original position of the leaf it lies at C_1, the intersection of this ⊙ with the opening ⊙.

The general shape of the leaf is then ABCDE.

In its new position AB′C′D′E′ it touches opening ⊙ at C′ and the shell at E.

∴ C_1 C′ and E′ are collinear and as before,

Total diameter = twice opening diameter.

If now we wish to compare the angular travel of the plate necessary to open and close this shutter with the result obtained previously, we must take the pivot and pin in corresponding positions.

The pivot is now at A and the pin at P, the intersection of AC with opening ⊙.

The new position of P is P′, so that we must find $P\hat{C}P'$.

From the figure $C\hat{A}C' = 2\sin^{-1}\frac{CC'}{2CA} = 2\sin^{-1}\frac{1}{4}$

$= 29°$ nearly;

and from $\Delta ACP'$

$P'C^2 = 5r^2 - 4r^2\cos C\hat{A}C'$

and $\therefore$ $P\hat{C}P' = \sin^{-1}\frac{\sin 29°}{\sqrt{5 - 4\cos 29°}}$

$= 23°$ nearly;

i. e. angular travel of plate $= 23°$ as compared with $30°$ in the case of the pin at the opening. By bringing P close to A, this angular travel may be made quite small.

There is therefore a definite advantage in pivoting the leaf near the edge of the shell, and this is done in those shutters with which I am acquainted. There is, however, no shutter that I know of using two leaves of this shape. There is a well-known American shutter with two leaves, but their shape is quite different.

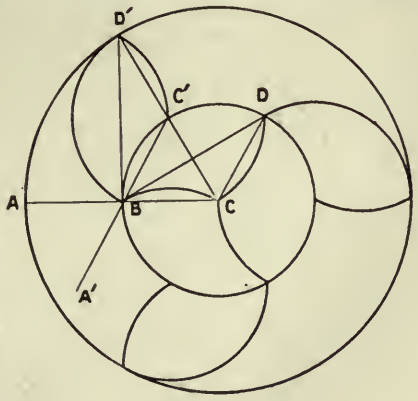


FIG 5.

THREE LEAVES.

This is an interesting case from the fact that the Compound and Koilos shutters have three blades.

Pivot at opening. Fig. 5.

The bounding edges of the leaf are circles as before, so that the shape of the leaf is ABCD with the pin at A and the pivot at B.

In the new position A'BC'D', C has swung to the circumference of the opening. We must prove that C, C' and D' are collinear and find CD'.

We have $B\hat{C}'D' = B\hat{C}D$

$= \frac{2\pi}{3}$

and $\Delta BCC'$ is equilateral

$\therefore$ $B\hat{C}'C = \frac{\pi}{3}$

$\therefore$ $B\hat{C}'D' + B\hat{C}'C = \pi$

i. e. CC'D' is a straight line

and $CD = 2r$

so that the total diameter = twice opening diameter, as in the case of two leaves.

For the shutter to open fully BC must swing to BC' just as in the case of the two-leaved shutter, and thus the angular travel of the plate in the two cases is identical and equal to 30°.

Three Leaves.

Pivot at extremity. Fig. 6.

The visible portion of the leaf is the same shape as before.

By suitably choosing the position for the pivot, the leaf may be arranged to swing to the position AB′C′D′ and as for two leaves.

Total opening = twice diameter opening.

For the shutter to open, C must swing to C′ so that, as before

Angular travel of plate = 23° nearly.

n Leaves.

Let us now take the general case of *n* leaves (*n* greater than 2) and investigate the relation between total and opening diameters.

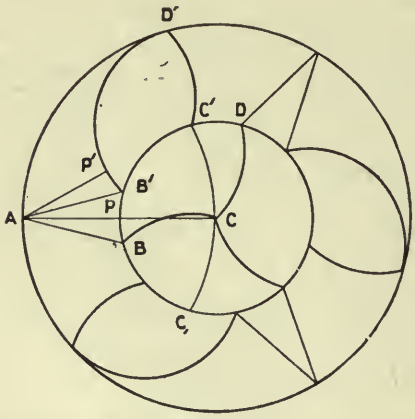


Fig. 6.

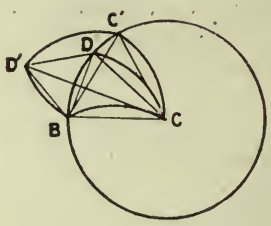


Fig. 7.

n Leaves.

Pivot at opening. Fig. 7.

Let BCD be the leaf rotating around B and let its boundaries be circles as already described, radius BC = r (say).

Then in the new position BC′D′, D′ is the point most distant from the centre and CD′ is then the radius of the shell which we wish to find.

From the figure, BCC′ is an equilateral Δ.

$\therefore$ $C\hat{B}C' = 60°$ and $\therefore$ also $DBD' = 60°$

and $BD = BD'$

$\therefore$ $B\hat{D}D' = BD'D = 60°$

$\therefore$ $BD = BD' = DD'$

and from similar Δ^s CD′ bisects BD at right angles.

$\therefore$ $B\hat{C}D' = \frac{1}{2} B\hat{C}D = \frac{\pi}{n}$

and radius of shell R $= CD' = r\cos\frac{\pi}{n} + r\sin\frac{\pi}{n}\tan 60°$

$= r\sqrt{3}\sin\frac{\pi}{n} + r\cos\frac{\pi}{n}$.

n LEAVES.

Pivot at extremity. Fig. 8.

Let ABCD again be the leaf rotating round A.

Then the centre of the circle of which arc BC forms a part is at C_1 the intersection of the opening circle with circle with A as centre and AC as radius.

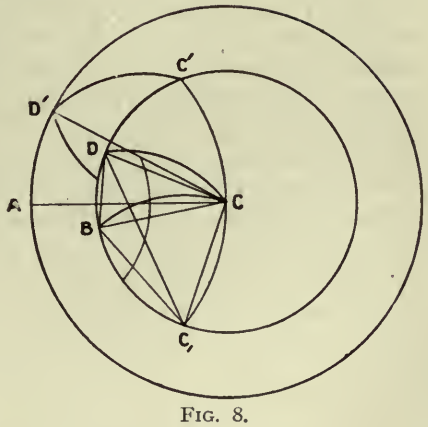


FIG. 8.

Then in the new position of the leaf C_1 has moved to C and D to D′ so that—

$C_1D = CD' = R$ (say)

and the Δ CBC_1 is equilateral,

so that $C_1B = r$

and also $B\hat{C}D = \frac{2\pi}{n}$

$\therefore \quad BD = 2r\sin\frac{\pi}{n}$

and $B\hat{D}C_1 = \frac{1}{2}BCC_1 = 30°$

$\therefore$ in Δ BDC_1

$BC_1^2 = BD^2 + DC_1^2 - 2BD\cdot DC_1\cos BDC_1$

i. e. $r^2 = 4r^2\sin^2\frac{\pi}{n} + R^2 - 4r\sin\frac{\pi}{n}R\cos 30°$

i. e. $R^2 - 2\sqrt{3}\sin\frac{\pi}{n}Rr + r^2(4\sin^2\frac{\pi}{n} - 1) = 0$

i. e. $R = r\sqrt{3}\sin\frac{\pi}{n} + r\cos\frac{\pi}{n}$

the same result as in the case of the pin at opening.

We have then for various values of *n* the following results :—

n Number of Leaves.	R. Total Radius.	*r*. Radius of Opening.
2	$2r$	$\frac{1}{2}$R
3	$2r$	$\frac{1}{2}$R
4	$1{\cdot}93r$	·52R
5	$1{\cdot}83r$	·55R
6	$1{\cdot}73r$	·58R
7	$1{\cdot}65r$	·61R
8	$1{\cdot}59r$	·63R
10	$1{\cdot}49r$	·67R
14	$1{\cdot}36r$	·74R
20	$1{\cdot}26r$	·79R
30	$1{\cdot}17r$	·86R
∞	r	R

Conclusions.

We see then that altering the position of the pivot does not affect the ratio of total diameter to opening diameter, and that by increasing the number of leaves the aperture may be increased and the shutter for a given aperture made smaller.

There is, however, an important disadvantage in doing this which does not seem to have attracted much attention, and that is the effect of the shape of the leaves on the efficiency.

If we are to construct a shutter with blades of identical shape whether there be two used or fifty, we must make the leaf extend from the centre to the edge. And therefore before the shutter is fully open, the part of the blade covering the centre must swing to the edge. That is, for corresponding positions of the pivot the angular travel of the plate is the same. And therefore for equal spring tensions the shutter will open and close equally fast whether there are two leaves or fifty.

But the partial openings of the shutter will be quite different, and this is of great importance for short exposures in which the period of full opening is small.

If we consider a short exposure with a ten-leaved shutter (Fig. 9) we see that as this shutter opens in an expanding circle from the centre to the edge, the edge is only illuminated for a very short period and the plate is practically only illuminated by central rays.

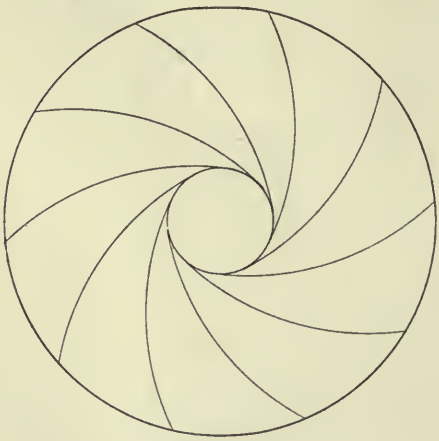


Fig. 9*a*.

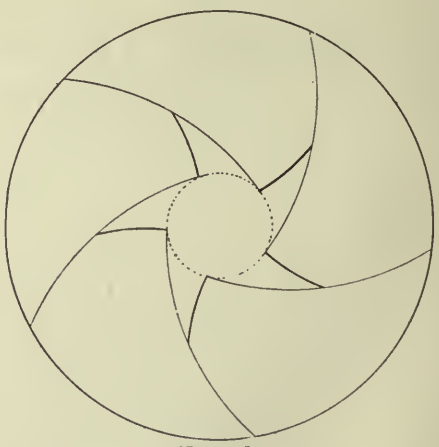


Fig. 9*b*.

As we reduce the number of leaves, we get improvement here. With five leaves you see that a little of the edge is illuminated. With three leaves rather more, and with the two-leaved shutter having the particular form of leaf described by me the illumination is excellent. The shutter opens in a broad band which rapidly expands to the full circle. In Fig. 9 I have sketched shutters having respectively 10, 5, 3, and 2 leaves. In *a* the partial opening of the shutter is a circle. In *b*, *c* and *d* a dotted circle of the same diameter has been drawn, and the additional area uncovered by the shutters with fewer leaves is distinctly shown.

A two-leaved shutter is of course simple and cheap to make, is as light as possible and has the fewest working parts.

Iris shutters (eight-leaved or more) as at present made do not seem to have any advantages. The leaves are not correctly designed, the inner edge being flat instead of circular, and the actuating mechanism takes up more than double the opening diameter. These shutters are no smaller than this two-leaved type, and they give a much less efficient exposure.

If we were able to use an iris diaphragm shutter with each leaf made exactly the correct size and without overlapping, it is possible that this would give as efficient an exposure as a two-leaved type. It is, however, always necessary to make the leaves broader and wider in order that there may be no leak of light with the shutter closed. The greater the number of joints between the leaves, the more overlapping there must be, and consequent inefficient exposure.

A further advantage of the two-leaved type is that the shutter case may be made rectangular or square in shape, the total diameter being only twice the opening diameter

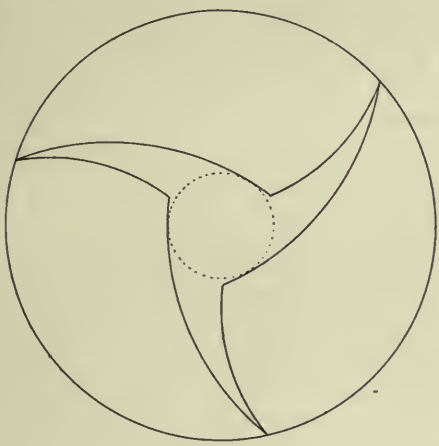


FIG. 9*c*.

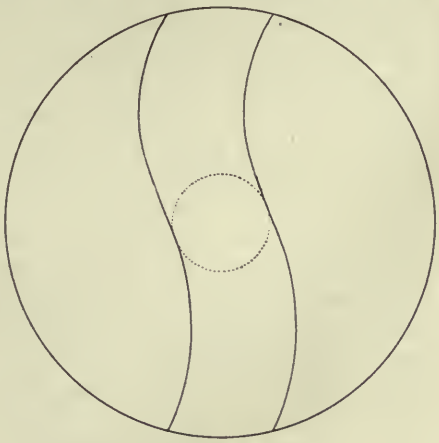


FIG. 9*d*.

or even less, as compared with the three-leaved type in which the total diameter is from two and a half to three times the opening diameter.

This reduction in size is of considerable practical advantage as it permits of a wide aperture lens being fitted to the high-class pocket cameras which become of increasing usefulness year by year. The front of such a camera consists generally of a U-shaped fork carrying the shutter, and the distance between the prongs of this fork is fixed by other conditions. An increase in the ratio of the aperture of the shutter to the total diameter therefore permits lenses of larger diameter and higher rapidity to be used.

In the models which I have here for your inspection the shape of the case is square and the ratio of aperture to total diameter is 2 : 1. The shutters are arranged to

give time and instantaneous exposures, and shutters with these two leaves to give the shortest exposures are also under construction.

The leaves are pivoted at their extremities and are rotated by a zone plate, actuated in turn by a powerful spring in the manner already described.

The formulæ given are applicable also to working out iris diaphragms of the conventional form.

Mr. T. Smith said that since Mr. Lan-Davis had referred to his work he would make a few remarks to open the discussion of this paper. As the author had said tests showed that there was practically nothing to choose between the fastest focal-plane shutter and the fastest between-lens shutter, either as regards the shortest total duration of exposure, or the efficiency when a lens was used having a large aperture ratio. Under these circumstances there was doubtless much to be said for the choice of a between-lens shutter instead of the rival form. But, assuming that the highest possible speed was the first consideration, he thought there might be greater possibilities of improvement in the focal-plane shutter than with the between-lens type. The velocities attained by the blinds of shutters were not very great, and any improvement in that direction would make it possible to increase the efficiency greatly while retaining the same total duration of exposure. In fact, measuring all lengths in terms of the same unit, if w were the width of the opening of the blind, v its velocity, a the distance between the blind and the photographic plate, and n the f-number of the lens aperture, i. e. $n=\frac{1}{4}$, when the aperture was $f/4$, the total duration of exposure would be $(w+na)/v$, and the equivalent exposure be w/v, giving for the efficiency $w/w+na$. Thus the greater w was compared with a, the greater was the efficiency, and w, to secure a properly exposed plate, was proportional to v. For very fast speeds w might be $\frac{1}{12}''$ or less, while a was usually $\frac{1}{2}''$, and so with a lens working at $f/4$ the efficiency would be ·4; if the velocity of the blind were doubled the efficiency at the same speed would be ·57, and if trebled ·67. Some improvement might also be made by reducing a, but there were very obvious limits to improvement in this direction. It did not, at least at first sight, seem likely that very great improvements could be made in the efficiency of between-lens shutters at very high speeds, as this would involve checking the velocity of the shutter blades at the middle of their motion without adding to the total period of duration of the exposure.

While referring to speeds in connexion with these two types of shutter, he would call attention to a point that was often lost sight of, with the result that focal-plane shutters giving the same equivalent exposure as a between-lens shutter were credited with much greater speeds. The speeds of shutters were naturally marked in such a way that the speed values given were independent of the lens-aperture used. With between-lens shutters opening and closing at the centre, this evidently meant that the marked speed would be the total duration of exposure; in this case the efficiency and the equivalent exposure, which determined whether a sufficient amount of light reached the plate, increased when the lens was stopped down, the amount of light reaching the plate at $f/4$, for example, being less than

four times that at *f*/8, and the inequality being more marked the higher the speed. With a focal-plane shutter, on the other hand, the equivalent exposure is the fixed quantity, and the total duration of exposure increased as the lens aperture was increased; consequently in this case it is the equivalent exposure which is stated by the makers. Taking the case of the slit-width just mentioned, with a blind velocity of 7 feet per second, the equivalent exposure would be ·001 second, and the total duration of exposure with an aperture of *f*/4 would be ·0025 second, a value which could easily be attained with at least equal efficiency by a between-lens shutter. He might say in this connexion that Mr. Chalmers had made the excellent suggestion that makers of focal-plane shutters should give the velocity of the blind in addition to the figures they were accustomed to supply; he hoped that they would adopt this suggestion.

Without modifying the motion of the shutter leaves of any given type of between-lens shutter, there appeared to be only one way of increasing its efficiency, and that was by increasing the size of the shutter opening relatively to the full opening of the lens. The very large shutters at present made were not intended for high-speed work, and really large sizes would doubtless not be attempted for this purpose; but it appeared quite likely that there would be some definite gain by fitting a shutter of one or two sizes larger than the smallest which would take the lens. From the diagrams which Mr. Lan-Davis had shown, it appeared that the two-leaved form of shutter gave a larger portion of the full diameter of the lens exposed earlier than did those shutters having more, and especially than those having a great number of leaves. It was evident from these considerations and the results given in the paper that high efficiency and great compactness had opposite tendencies on the size of the shutter, and a compromise had to be made in actual instruments. In his opinion the present tendency was to needlessly sacrifice useful qualities to compactness—a camera, for instance, being something to fill a waistcoat pocket rather than a scientifically constructed instrument made for picture-taking. There was no doubt a sphere for these very compact instruments, but those who wished to have a really reliable and efficient instrument should receive their share of consideration. The most noticeable result, perhaps, of Mr. Lan-Davis's work was the slow rate at which the size of the shutter was reduced by increasing the number of leaves; ten leaves, for instance, being necessary to make the diameter $\frac{3}{4}$ of that required for the leaves. He hoped that Mr. Lan-Davis would continue his investigations and determine the efficiency attainable with various numbers of leaves, assuming in each case the leaves to be moved in the way actually used for fast exposures. Such an investigation might well determine once and for all, what was the form of shutter which would best combine the desirable qualities of high efficiency and reasonable bulk.

Mr. Alfred Watkins mentioned one objection to sector shutters, which Mr. Lan-Davis had not touched upon, namely the pneumatic regulation. In his own experience (and he had employed a skilled mechanic for some time in attempting to overcome the difficulty) he had found it easy to get fairly reliable pneumatic

regulation of the speed from, say, one second down to $\frac{1}{16}$ of a second. But in quicker speeds the momentum of the mechanical parts seemed quite to override the air-brake, and there seemed a sudden jump above $\frac{1}{16}$ second, so that it was quite uncertain in practice whether one would get $\frac{1}{16}$ or $\frac{1}{50}$ second. He had found this difficulty inherent in the method.

Dr. Tempest Anderson, of York, said that he had made an iris diaphragm in 1867 or 1868 and had taken it to Collins, who had brought it out. As an experienced user of sector shutters he would say that they worked well, but the noise of the click was abominable and frightened natives away. Makers might introduce a soft pad or some other device; they paid enormous attention to securing accurate exposures, whilst he considered three speeds sufficient for most cases.

Mr. Chapman Jones remarked that all users of shutters would be gratified and expressed his pleasure at the increased speed obtainable with between-lens shutters; there was at least one such shutter that was so nearly equal in speed to the focal-plane shutters that it was being used for the fastest work. Its other advantages more than made up for the slight difference in speed. He was accepting the experience of Mr. Abrahams, one of the most experienced workers with high-speed shutters.

Mr. Lan-Davis said in reply to Mr. Watkins that the speed was now controlled by means of a cylinder and plunger, and considerable improvement might be obtained by increasing the length of the cylinder. The noise was difficult to avoid, as there must be a click when the shutter was stopped at high speed. As regards the history of the iris diaphragm he might say that Collins was not the first one to introduce it; it was probably a hundred years old and had come from France, but had several times been reinvented. He also replied to Mr. Smith.

ON TRANSMISSION OF VISIBLE LIGHT BY PHOTOGRAPHIC LENSES

By R. W. Cheshire, B.A.

(*Thursday, June 20th*, 1912.)

This paper embodies the results of some experiments carried out at the National Physical Laboratory on the transmitting power of modern photographic objectives for visible light. The fact that visible light only was used in making the experiments might at first appear to detract from the practical usefulness of the results obtained, but it should be remembered that most photographers now-a-days, in order to secure light-and-shade effects in their landscape work which approximate as closely as possible to those presented to the eye, use colour filters before their lenses which cut out, either entirely or in a great measure, the region of the spectrum lying in the ultra-violet, transmitting only in various degrees the light in the visible spectrum from red to violet. The necessity for these precautions is shown in a most striking

manner by the curious reversal of light-and-shade effects occurring in the photographs recently taken by Prof. R. W. Wood in ultra-violet and in infra-red light. The action of such screens is well shown in the transmission curves for three typical screens—Wratten K filters—shown in Fig. 1.

In these curves for the filters K1, K2, and K3, the percentage of the light transmitted for a specified wave length is plotted against that wave length. The positions

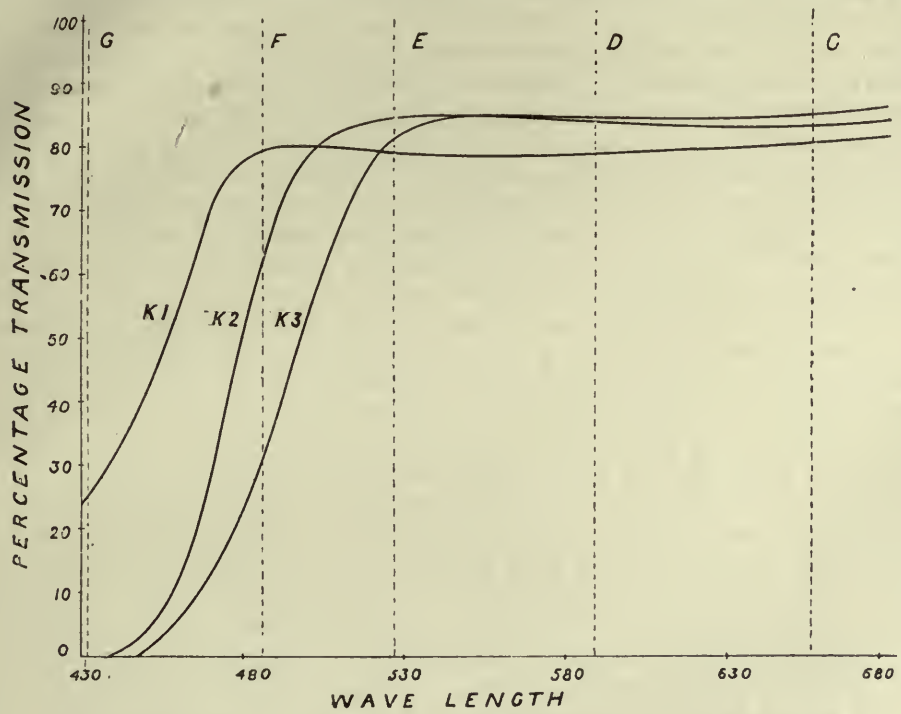


Fig. 1.

of the well-known lines C, D, E, F, and G in the spectrum are also indicated. The curves show that K2 and K3 cut off everything below the blue of wave length about 440 $\mu\mu$, whereas K1 transmits a certain amount of violet, only cutting off the ultra-violet. Under these circumstances, therefore, the quality of the light used by the photographer will not differ markedly from that employed in the following experiments.

When light passes through any lens system, photographic or otherwise, the amount of light which finally emerges from the system is only a fraction of that which entered it. Some of the light is, therefore, said to be lost in passing through the system. This loss, as is well known, is made up of two parts : (1) loss by reflection

and (2) loss by absorption. Assuming for the moment that we are dealing with monochromatic light, a certain definite amount of light will be reflected back at the interface between every pair of media of different refractive indices, μ_1 and μ_2 say; in the case of normal incidence this fraction reflected will be, according to Fresnel's law, $\left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^2$. Lord Rayleigh has verified this formula for a glass-air surface, and has found that it conforms well with experiments, especially if the surface has been recently polished. For a surface which has not been recently polished the amount of light reflected is slightly less than that indicated by Fresnel's formula. For a glass-air surface and for a refractive index of 1·5 this fraction is equal to $\frac{1}{25}$ or four per cent., and for a surface separating media of refractive indices 1·5 and 1·6, the fraction will be $\frac{1}{961}$ or one-tenth per cent. A difference greater than 0·1 between the refractive indices of two consecutive media other than glass and air in a photographic lens is not likely to occur in practice, and consequently it is seen that, neglecting glass-air surfaces, the greatest loss at any glass-glass surface will be about one-tenth per cent. of the incident light. This fraction may be neglected in comparison with losses due to reflection at glass-air surfaces.*

But there is also the loss due to absorption of the light by the glass composing the lens to be taken into account, and it was to determine the relative importance of the loss due to this cause, as compared with that due to reflection, that the experiments described below were specially directed, since there is very little mention made in most of the books dealing with photographic lenses of the quantitative value of this loss. Further, it was hoped that if the absorption loss was at all appreciable it might be possible to develop a general empirical formula by which the transmitting power of a lens might be estimated merely from a knowledge of the number of glass-air surfaces in the lens and the axial thicknesses of the glasses of which the lens is made—two easily ascertainable quantities. Of course it is not to be expected that any very accurate results could be obtained from such a general formula which would apply to all types of lenses composed, perhaps, of very different kinds of glass.

The method employed was to compare photometrically the intensities of two beams of light, in the one case when the photographic lens to be tested is inserted in the path of one of the beams, and in the other case when the lens is withdrawn from the beam. The details of the arrangement adopted are shown in Fig. 2.

The filament of a Nernst lamp, A, is focussed by means of a microscope objective, B, upon a small circular opening placed at the principal focus of an achromatic telescopic objective, C, of focal length about 80 cm. This gives a parallel beam of light which falls upon one side of the diffusing screen of a Lummer-Brodhun photometer, D. The other side of this screen is illuminated by means of a glow lamp, E. The light from this lamp is yellower than that from the Nernst lamp, and, therefore,

* Of course, in a photographic lens the incidence over any given surface will not everywhere be normal, but variations in the reflected light introduced on this account are quite inappreciable, since even at an incidence of 30° the fraction reflected is only ·041 as compared with ·04 at normal incidence.

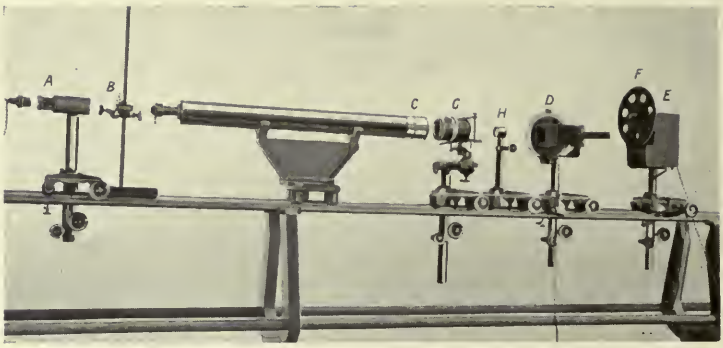


FIG. 2.

[*To face page* 37.

difficulties in matching would be introduced, unless a suitably tinted glass were introduced between the glow lamp and the photometer. Space is provided for a number of these tinted glasses in a wheel, F, rotating before the glow lamp, so that the colours of the two light sources can be well matched. G is the lens to be tested, and H is an auxiliary lens which, although not absolutely necessary, it is convenient to employ so as to be able to obtain a patch on the screen of convenient size when the lens G is removed. The lenses G and H, together with A, D and E, are mounted on suitable supports provided with wheels which run on a pair of parallel rails graduated in millimetres so that the respective positions of the various supports and their mutual distances from one another can be determined. Suitably placed screens are also provided for cutting off any extraneous light.

When taking a reading the lenses G and H are placed on their supports and the lens H is moved along the rails until a conveniently sized circular patch of light is obtained on the diffusing screen of the photometer. The distance of the comparison lamp E is then adjusted until the field of view seen through the photometer is symmetrically illuminated. Several readings are taken and the mean distance of the lamp E from the photometer screen noted. If d_1 is the diameter of the entrance pupil of the lens G, the amount of light incident upon the lens may be taken to be Id_1^2 where I is some constant involving the intensity of the light in the collimated beam. This amount of light is reduced to kId_1^2 after passage though the lens G, where k is some fraction which it is the object of the test to determine, and if d_2 is the diameter of the circular patch of light on the screen of the photometer the intensity of the light forming this patch will be $\frac{kId_1^2}{d_2^2}$. This, of course, will be inversely proportional to the square of the distance r of the lamp E from the photometer screen when a balance has been obtained, so that we may write—

$$\frac{kId_1^2}{d_2^2} = \frac{C}{r^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C is a constant.

The lens to be tested is now removed and a similar set of readings taken with only the auxiliary lens present in the beam. If D_1 is the aperture of the auxiliary lens, or the diameter of the beam incident upon it, if the beam is not large enough to fill the lens completely, and D_2, R the values in this case corresponding to d_2, r in the former, we shall have—

$$\frac{ID_1^2}{D_2^2} = \frac{C}{R^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or, from (1) and (2)—

$$k = \left(\frac{R}{r} \times \frac{d_2}{D_2} \times \frac{D_1}{d_1}\right)^2.$$

This equation gives k, or in other words, the value of the fraction of the incident

light transmitted by the lens G. It will be seen that the effect of the auxiliary lens has been neglected—this is permissible since it is the same in both cases, and therefore disappears from the final result.

The following tables give the results, which are probably correct to about one per cent., obtained for the percentage transmissions of the various photographic objectives considered. The results have been arranged under four headings, according as the number of glass-air surfaces in the lens was 4, 6, 8, or 10. The first column in each case gives the axial thickness of the glass composing the lens, the second the corresponding percentage transmission, and the third the "error," a term which will be explained later.

4 Glass-air Surfaces—

Axial thickness of Glass in cms.	Percentage Transmission.	Error.
·56	77·0	+ 2·0
·94	80·0	− 2·0
1·09	77·5	+0·5
1·18	76·5	+ 1·0
1·59	77·0	− 0·5
1·78	76·0	0
2·57	72·5	+ 2·0
2·59	75·0	− 0·5
2·60	74·0	+ 0·5
2·96	76·0	− 2·5
3·66	69·5	+ 2·0

6 Glass-air Surfaces—

Axial thickness of Glass in cms.	Percentage Transmission.	Error.
1·58	68·5	0
1·58	66·5	+ 2·0
1·87	69·5	− 1·5

8 Glass-air Surfaces—

Axial thickness of Glass in cms.	Percentage Transmission.	Error.
1·09	63·5	− 1·0
1·81	57·5	+ 3·0
1·86	61·5	− 1·0
2·13	58·0	+ 2·0
2·26	59·5	0
2·76	59·0	− 0·5
2·80	58·0	+ 0·5
4·60	55·5	− 1·5

10 Glass-air Surfaces—

Axial thickness of Glass in cms.	Percentage Transmission.	Error.
·98	57·0	− 1·0
1·67	54·5	0

It will be found that, in any particular case, the percentage transmission can be found approximately by the following rule. For each glass-air surface allow, for the loss due to reflection, 5·22 per cent. of the light incident upon that surface, and for each centimetre of the axial thickness of glass in the lens allow, for the absorption loss, 2·4 per cent. of the light incident upon the lens.* An example will make this clear. Consider a lens with 8 glass-air surfaces in which the total axial thickness is 2·8 centimetres. At each glass-air surface 5·22 per cent. of the light is reflected, and therefore 94·78 per cent. only is transmitted. Consequently if reflection were the only cause of the loss of light the amount of light transmitted would be $(\cdot 9478)^8$ of the incident light, or 65·1 per cent. Due to absorption, however, there will be a further loss of $(2\cdot8 \times 2\cdot4)$ per cent., or 6·7 per cent. The final value for the percentage transmission is therefore 65·1 − 6·7 or 58·4 per cent. The value observed was 58 per cent. The "error" in the third column is the difference between the percentage transmission calculated by means of this rule and that actually observed, the difference being reckoned positively when the calculated value is in excess of the observed value.

The expressions $(\cdot 9478)^4$, $(\cdot 9478)^6$, $(\cdot 9478)^8$, and $(\cdot 9478)^{10}$, will be found to be ·807, ·725, ·651, and ·585, and so finally if T denotes the percentage transmission we have the equation

$$T = A - Bt \quad . \quad . \quad . \quad . \quad (3)$$

where, to the nearest half per cent., A is 80·5, 72·5, 65·0 or 58·5, according as the lens possesses 4, 6, 8, or 10 glass-air surfaces, B = 2·4, and t is the axial thickness of glass in the lens, measured in centimetres. An examination of the tables given above shows that the mean "error" in the results obtained by an application of this rule to the 24 objectives tested is only about one per cent. It follows also from the above formula (3) that, so far as loss of light is concerned, two glass-air surfaces in a lens are practically equivalent to four centimetres of glass.

Dr. Moritz von Rohr, of Jena, said that the author was to be congratulated upon the simplicity of his solution of a difficult problem. Two questions occurred to him: Were the experiments made with both new and old lenses? and, Would the formula hold for lenses of different focal lengths and constructions?

* Actually, of course, the relation between the absorption and the glass thickness traversed is an exponential one, but the above is a sufficiently good first approximation.

Mr. T. SMITH had been much interested in the experiments described by Mr. Cheshire. As far as he knew, there had been much talk by photographers on the relative influence of glass-air surfaces, and of absorption on the amount of light lost in passing through photographic lenses, but this was the first occasion on which the results of experiments on actual lenses had been published. These results should enable photographers to determine, within the accuracy attainable by any simple general formula, the probable amounts of light lost in any particular type of lens. The result stated at the end, that two glass-air surfaces are approximately equivalent to 4 cm. axial thickness of glass, was a very convenient one to remember. The objection raised to these results that they were obtained by disregarding ultra-violet light would, of course, be important when they considered light far removed from the visible limits and from those wave-lengths for which glass became an opaque rather than a transparent body, but under such conditions no general result could be expected, and every lens would have to be separately tested if its transmitting power were required. In actual photography these were not the light waves which should be employed, and for smaller excursions into the invisible regions expectations of continuity would probably be justified, and the results given by Mr. Cheshire should be taken to apply until direct experimental results for such wave-lengths were available.

There was one further matter of interest which he would mention : among the lenses tested were a few of the same design, though not all were of the same focal length and aperture, some of which were quite new and others of which were a few years old. The results showed that the older lenses were slightly—it was only a small amount—more transparent than the new. The experiments were not strictly comparative, and the number was too small to establish a rule, and perhaps the differences were of the order of the possible experimental error : still he would mention the difference for what it might be worth, as it was in accordance with the well-known result that the reflecting power of glass was greatest when newly polished.

Mr. CHESHIRE replied that most of his lenses were quite modern objectives, but four of them were twenty or thirty years old. An interesting point of his formula was that the effect of adding two glass-air surfaces was equivalent to an additional glass thickness of four centimetres.

THE SHAPE OF SCALES REQUIRED FOR REFLECTING INSTRUMENTS WITH CONCAVE MIRRORS

By E. H. Rayner, M.A.

(From the National Physical Laboratory)

(*Friday, June 21st,* 1912.)

1. The accuracy of reading certain types of deflecting instruments, which operate without any static friction, can be greatly improved by observing their deflections by the reflection of a beam of light from a concave mirror having a radius of curvature of several feet.
2. When the angular deflection is large the shape of the scale for such an instrument is somewhat similar to a cardioide, varying according to the relative values of the radius of curvature of the mirror and its distance of the source.
3. A special case is discussed when the curve approximates (over a large arc) exceedingly closely to a circle of definite radius.
4. In general the radius of a similar circle may be calculated, which will approximate sufficiently accurately to the true curve.
5. Some suggestions are given as to the use of mica for the window in the covers of such instruments.

Many types of scientific instruments, especially electrical, operate by the forces to be measured causing rotation about a vertical axis, such rotation being opposed and balanced by some elastic control, such as the bending of a spring or the torsion of a wire or flattened strip of metal. The pointer may be a few inches long, and the support of the moving part is often of the nature of pivots such as are used in watch and clock mechanism. The accuracy of reading can be increased by lengthening the pointer, and so increasing the scale length; but a limit is soon reached in practice by the friction of the pivots assisted by the increase in weight and inertia of the pointer. In certain types of instruments, however, in which the whole weight of the moving part is supported by a wire or strip of metal, the friction limit practically vanishes; and if a mirror be attached to the moving part and light from a fixed source be projected on to the mirror, the full advantage of a "pointer" of many feet in length may be obtained, and the accuracy of reading multiplied fifty-fold as compared with the same instrument provided with a pointer moving over a scale in the usual way. Such instruments have intrinsically very little damping, and in practice either air, oil or magnetic damping must be employed to bring the moving part to rest in a convenient time.

The experience which emphasized the importance of the subject dealt with in this paper has been gained by the arrangements adopted at the National Physical

Laboratory for the precision apparatus for alternating current measurements These have to be of the deflecting type, and to enable readings to be made to an accuracy of one or two parts in ten thousand it is necessary to have a scale length o some 20 ft. or more, the instrument being at about 10 ft. from the centre of the scale.

Of the instruments in use the two most important are a Kelvin electrostatic voltmeter of the single quadrant type, and a specially designed quadrant electrometer used as a standard wattmeter. Of these the voltmeter has much the larger torque and has oil damping; and it is interesting to note that there has never been any tendency to " sticktion " as the result of several years' experience of this type of instrument, provided the oil surface be kept dust free.

If there were any friction due to the static oil damping, it would have been necessary to try one of the other forms of damping, as the instrument is of primary importance; but full advantage is obtained of the magnification fifty-fold above that for which the instrument was intended. The other instrument is specially designed with sufficient air friction to bring it to rest in the most suitable time.

There are two methods used for measuring the angular deflection of such instruments: the first is to observe (in a telescope) the apparent motion of a scale reflected from a plane mirror fixed to the moving part of the instrument. For small angular displacements and special research this method is valuable, as the telescope partakes of the nature of a microscope and a very finely divided scale may be used with advantage.

One of its drawbacks is that only one person can make observations at a time; and for routine work often involving observation for several hours a day, the strain renders the method quite impracticable, especially when, as in electrical work, two or even more apparatus have to be observed simultaneously. In such cases the second method has to be used. In this a concave mirror is employed and a real image of an artificial source of light is projected on to a translucent or opaque white scale, on which suitable marks are drawn. A Nernst lamp and a condensing lens with a fine vertical wire in front of it form a suitable source of light, producing on the scale a circular bright spot with a fine line across the vertical diameter.

This paper deals with the shape of the scale required in order to have the vertical wire always sharply in focus, while the concave mirror rotates about a vertical axis.

If the angular deflection be only a few degrees a straight scale is commonly used; but it becomes unsatisfactory if the deflection is considerable. Neither does a circular scale with the mirror at the centre sufficiently approximate to the proper curve for instruments reading by a reflected ray of light whose working deflection may reach 120°.

From the diagrams given it will be seen that the scale has to continually approach the mirror as the angle of incidence increases, and that for deflections of 90° from the incident ray the distance may be reduced to one-half.

Nomenclature.—

u = distance from object to mirror.

v_0 = distance from mirror to scale when the angle of incidence in a horizontal

plane is zero. The angle in a vertical plane is assumed to be zero. In practice it should be kept as small as convenient; v_o is measured along the axis of symmetry of the curve and intersects the scale at its " centre."

v_θ = distance from mirror to scale when the reflected beam makes an angle θ with the incident beam. The angle of incidence $= \frac{\theta}{2}$.

m = magnification $= \frac{v_o}{u}$.

a = radius of curvature of the mirror.

Then $\frac{1}{v_\theta} + \frac{1}{u} = \frac{2}{a \cos \frac{\theta}{2}}$ which, when $\theta = 0$, reduces to the ordinary equation of the concave mirror.

If any convenient value, v_o, has been chosen for the distance between the instrument and " centre " of the scale, the value of u can at once be calculated when the curvature of the mirror is known; and, therefore, also the value of v_θ from point to point along the scale.

For any permanent installation this is worth doing, drawing out the curve full size and bending the scale to it. The scale varies somewhat in shape, according to the relative values of u, v_o and a, departing more from a circle with its centre at the mirror, drawn through the scale " centre " ($\theta = 0$), as the radius of curvature and of the mirror decreases, the distance from mirror to scale " centre " (v_o) remaining constant.

For ordinary purposes, however, a circle may be drawn which will agree sufficiently accurately with the proper curve. The radius of this circle may be taken as the perpendicular to the axis of symmetry from the point where the curve is parallel to that axis, the centre of the circle being on the axis of symmetry at a distance from the intersection of the scale and the axis equal to the above radius.

For about 60° on each side of the axis of symmetry a circle so drawn will lie within or without the true curve according as the magnification is less or greater than about 1·37. When, however, the curve is parallel to the axis of symmetry at the point where $\theta = 60°$, the coincidence between the circle so drawn and the true curve is exceedingly close. In this case the perpendicular to the axis of symmetry, from the point where the curve is parallel to that axis, is exactly equal to the distance from the foot of the perpendicular to the point where the axis of symmetry cuts the scale at right angles. We thus have the circle touching the true curve at $\theta = 0$, and also at two points on each side, where $\theta = 60°$; and intermediately the two curves do not diverge from one another by two parts in a thousand.

Throughout the paper the distance v_0, between the scale ($\theta = 0$) and the mirror, has been taken as 1000. In this special case u, the distance between source and mirror, is $1000\sqrt{3} - 1 = 732{\cdot}05$ and $a = 1000 \times \frac{2}{3}(3 - \sqrt{3}) = 845{\cdot}298$.

The magnification $\frac{v_o}{u} = \frac{3}{2(3 - \sqrt{3})} = 1{\cdot}36603$.

Fig. 1 gives the curve for this case, and the agreement with a circle drawn as described is so near that it is quite impossible to distinguish between them for about 70° on each side of the axis of symmetry, except when drawn on a very large scale.

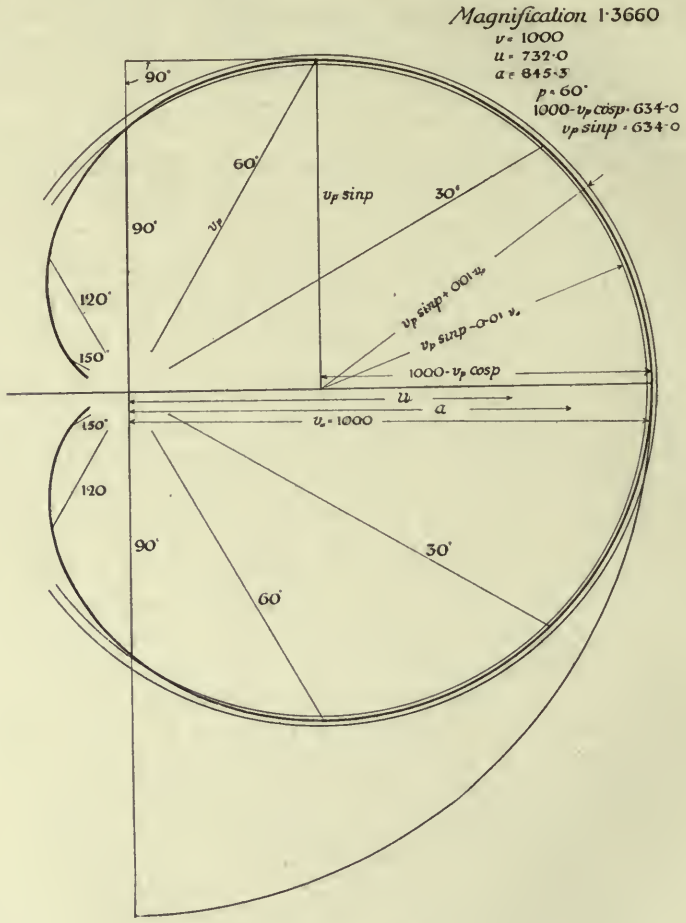


Fig. 1.

Two other circles have been drawn instead, one longer by 1 % of v_o and the other shorter by the same amount, to show how close the coincidence is. It is, therefore, desirable to choose, if convenient, the values for v_o, u and a in the ratio of 1000, 732 and 845, as described above, using a radius of 634 for the curvature of the scale.

The values of many of the quantities given in the table at the end are carried

to a much higher accuracy than is necessary, as the numbers may be used for interpolating; and as many of them have been arrived at by successive approximation, it may be useful to have them on record.

The angle at which the curve is parallel to the axis of symmetry being p, at a distance v_p from the mirror, $v_p \sin p$ is the perpendicular distance to the axis of

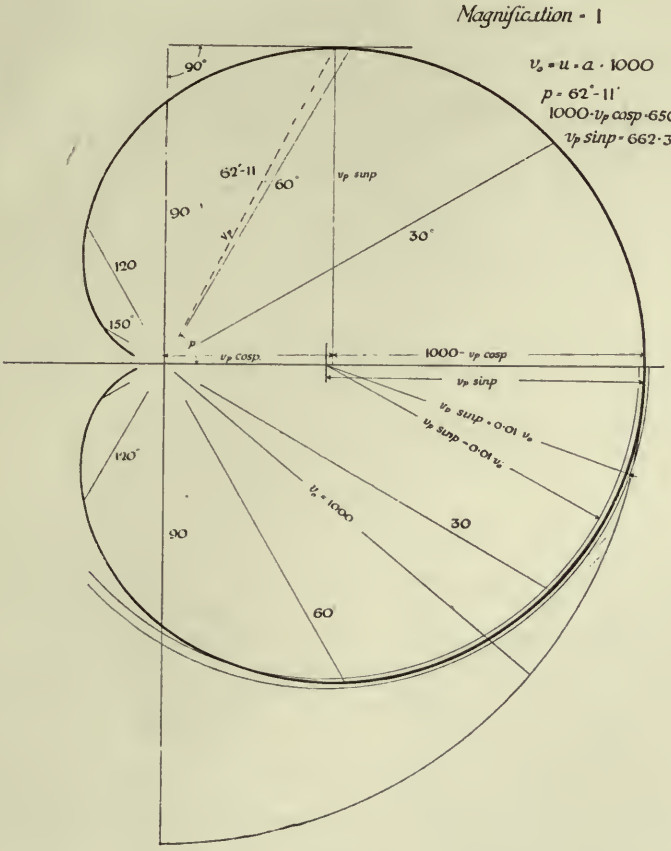


FIG. 2.

symmetry from the point on the curve; and $v_o - v_p \cos p$ is the distance from the centre of the scale to this point. As mentioned above, these are equal when $p = 60°$, and the difference between them increases as p diverges from 60°.

This point on the curve, where the tangent is parallel to the axis of symmetry, is given by the equation—

$$\frac{a}{u} \cos p \cdot \cos \frac{p}{2} = 3 \cos p - 1.$$

The solution of this leads to a cubic equation, and is best solved by successive approximation. Angles are given for p for various values of the magnification.

From the equation it will be seen that if the source is at an infinite distance $3 \cos p - 1 =$ zero, which gives a value for cos p of $\frac{1}{3}$ and $p = 70° 31{\cdot}7'$ as the largest possible angle, and the area enclosed by the curve is then a maximum.

Fig. 2 is the curve for unit magnification when the distances of the source and

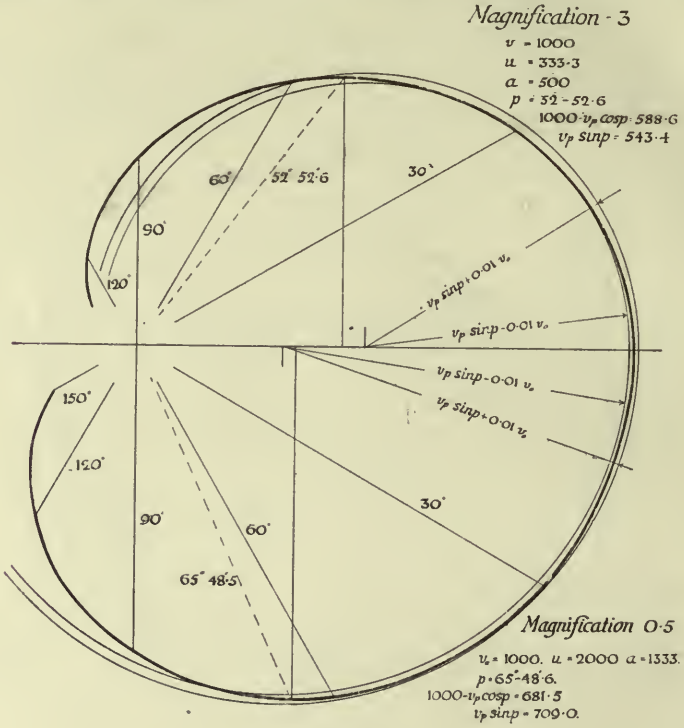


FIG. 3.

image and the radius of curvature of mirror are all equal. The circles with radii, differing from $v_p \sin p$ by 1 % of v_o, are drawn as in the previous figure.

Fig. 3 shows the semi curves for two magnifications between which practical cases will most likely fall. In the upper half is shown the curve for $m = 3$ and the lower or $m = 0{\cdot}5$. The corresponding circles are also shown.

As the mirrors commonly used are small compared with their distance from the source and image, the " depth of focus " is considerable, and it will generally be found quite sufficient to use a circular scale of radius $v_p \sin p$ which will be satisfactory over about 70° on each side of the axis of symmetry. The difference between $v_p \sin p$ and $1000 - v_p \cos p$ will give an indication of the divergence of the circle from the true

curve. If a less angle than 70° on each side of the axis of symmetry be sufficient a slightly different circle will fit the curve somewhat better.

It is generally essential to completely enclose instruments to avoid entrance of dust and disturbance by air currents. Some transparent window must, therefore, be provided, and for moderate deflections plane parallel glass will probably be found most satisfactory. For large deflections a curved window is necessary and mica is the most convenient material.

This should be split into as thin a sheet as possible (thin paper seems to be the best tool to use) and it should not be bent to a small radius. If the material is thick or the curvature sharp the definition will be seriously impaired. In any case curvature will cause the mica to act as a lens and modify appreciably the focal length of the system as calculated from the mirror alone, especially when the latter has a radius of curvature of five or ten feet. The focal length of the combination should, therefore, be determined and used in any calculations.

The following table gives the distances from the mirror to the scale for every 10° of deflection of the reflected beam from its original direction. The values for

m.	0	0·5	1·0	1·36603	1·6	1·8	2·0	2·5	3·0
v_0	1000	1000	1000	1000	1000	1000	1000	1000	1000
u	∞	2000	1000	732·05	625	555·5	500	400	333·3
a	2000	1333·3	1000	845·30	769·2	714·3	666·6	571·4	500·0
θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ
0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0
10	996·2	994·3	992·5	991·1	990·2	989·4	988·7	986·9	984·9
20	984·8	978·4	970·1	964·8	961·4	958·6	955·8	948·9	941·9
30	965·9	949·8	934·1	922·9	916·0	910·1	904·2	850·1	876·3
40	939·7	912·2	886·2	868·1	857·0	847·7	838·5	816·5	795·7
50	906·3	865·8	828·6	805·3	788·1	775·5	763·2	734·3	707·5
60	866·0	811·7	764·3	732·1	713·1	698·2	683·0	648·8	617·7
70	819·2	751·2	693·7	656·9	635·3	618·0	601·5	564·1	531·0
80	766·0	685·8	620·8	580·5	557·4	539·0	521·9	483·3	450·1
90	707·1	617·0	547·2	505·0	481·5	463·0	445·9	408·2	376·4
100	642·8	545·4	473·7	432 0	409·0	391·2	374·9	339·6	310·3
110	573·6	472·8	402·0	362·4	341·0	324·5	309·6	277·6	251·6
120	500·0	400·0	333·3	297·1	277·8	263·2	250·0	222·2	200·0
130	422·6	327·9	268·0	236·3	220·0	207·2	196·2	173·0	154·7
140	342·0	257·3	206·3	180·1	166·6	156·6	147·7	129·3	115·0
150	258·8	188·8	148·6	128·6	118·4	110·9	104·3	90·7	80·3
160	173·6	123·0	95·0	81·6	74·8	70·0	65·5	56·6	49·9
p	70° 31·7′	65° 48·5′	62° 11·6′	60° 0′	58° 44·8′	57° 45·2′	56° 49·2′	54° 43·0′	52° 52·6′
vp	816·5	777·2	748·7	732·1	722·7	715·5	708·8	694·1	681·6
$vp \sin p$	769·8	709·0	662·3	634·0	617·8	605·2	593·3	566·6	543·4
$1000-vp \cos p$	727·8	681·5	650·7	634·0	625·4	618·2	612·2	599·1	588·6

various magnifications (m) are given which will probably cover most of the cases occurring in practice. It will be seen that though to obtain a large length of scale

a magnification of two or three times may be desirable, it would generally be preferable to lengthen the radius of curvature of the mirror, as the scale will then not approach the instrument so much.

Values are also given for $v_\phi \sin \phi$, this being half the breadth of the curve; and this length may often be used as the radius of a circular curve sufficiently accurate for many purposes.

Dr. SCHUSTER said that the paper was of considerable interest to all who had to measure electrical quantities.

Mr. S. D. CHALMERS called attention to the fact that coma of the system might be even more important than the choice of best focus for the system, and therefore it was most desirable to pay attention to this in selecting the magnification; for unit magnification the coma was practically nil, and it might, therefore, be desirable to give up something with regard to the proper curvature for the purpose of eliminating this coma error which might cause a displacement of the line image.

Dr. JOHN MILNE (Edinburgh) asked how the reductions of the observations were performed.

Mr. RAYNER replied that the question was what the actual deflections meant. For their measurements of alternating currents within 1 part in 10,000, the scale divisions were originally fixed by means of continuous currents, and the scale had only the significance of a temporary standard.

Mr. A. P. TROTTER said that he had found a difficulty in using curved scales. In a paper communicated to the Physical Society on February 21, 1890 (see *The Electrician*, XXIV, 428), he had described how, with any reflecting instrument in which the deflections were not proportional to the quantities to be measured, a uniformily divided scale could be bent so that it gave direct readings. Such a scale was rather difficult to set up, and if the relative position of the scale and the instrument were altered, it was difficult to readjust it. Mr. Rayner got over half the difficulty by graduating the scale empirically. This having been done it was important not to disturb the apparatus, or else to take precautions that the exact position should be reproduced.

MEASUREMENTS AND NOTES ON THE VISIBILITY OF POINT SOURCES OF LIGHT

By Clifford C. Paterson and B. P. Dudding, A.R.C.Sc.

(From the National Physical Laboratory)

(*Friday, June 21st,* 1912)

The consideration and observation of point sources of light has generally been concerned either with—

(*a*) The observation of stars; or—

(*b*) The observation of distant lights on land and on sea.

The solution of problems connected with either of these fields of work has been the object of most of the investigations into this subject which the authors have been able to trace.

In attempting to classify stars according to a scale of magnitude astronomers have devised many and various forms of apparatus, but the work is not in any way quantitative, one star being compared against and expressed in terms of another, and thus the results of their work are recorded in terms of an arbitrary standard of magnitude.

Some accounts of the most recent work on this subject can be found in the records of *The Astrophysical Journal*,* and a particularly full account of, and comparison between, German and American results has been given by Prof. Pickering.†

The observation of signal lights at sea has for many years played an important part in the safe navigation of ships, and the appreciation of distant lights in connection with railway work is a matter of great importance. The laws governing the degree of appreciation of these lights by the human eye have been the subject of several researches. In the laboratory small illuminated apertures have been used by most observers both in experiments on the visibility of lights as distinct from their colour, and in experiments devised to test the eye for colour vision. But although so much qualitative work has been done, the authors have been unable to find any reference to work where absolute measurements have been made, in terms of a reproducible standard, of the visibility of a point source of light. From the time of Helmholtz many workers on the Continent have been interested in the visibility of periodic lights, and have experimented with a view to determining to what extent the visibility

* Vols. ii, iv, xiii and xxxi.

† Part II, vol. xiv, *Harvard College Observatory Annual.*

depends on the time of duration of such a periodic light. Some of the more complete investigations are those of Broca and Sulzer,* and since the completion of the authors' experiments a paper has been published by MM. Blondel and Rey † in which is discussed the appreciation by the eye of lights of short duration.

This work is outside the scope of the present paper and deals with a totally different problem, but it is interesting to note that in this paper reference is made to the limit of visibility or perception which is given as one-tenth candle at one kilometre distance for a dark night in the open, or 0·5 to 0·6 of this for observation in a laboratory where all stray light is excluded.

This agrees approximately with the figures obtained by the authors, but it is, of course, a variable quantity depending on the acuity of the eye of the observer.

A short account is given by MM. André Broca and Polack ‡ of some experiments made on illuminated pinholes, with a specially constructed apparatus, with a view to determine how different coloured signal lights might be distinguished even though so faint as to make it impossible to distinguish the colour. Their figures do not enable the value of limiting visibilities of the various coloured signals to be computed, but the generalizations given in this paper have been supported by the author's experiments, and will be further emphasized later.

The Deutsche Seewarte carried out experiments with a view to finding out the minimum candle-power for Ships' Signal Lights, and their report was issued in 1894. Their units for green and red light were a Hefner candle with a green and red glass placed respectively in front of it. No data are given as to the percentage transmission of these glasses, and therefore it has been impossible to compare the results with those of the present authors. It may be deduced from a formula given in the report that for very clear weather 2·1 Hefner candles (white) are required for visibility at two sea miles, and 13·1 Hefner candles for five sea miles.

The suitability of a light for signal purposes can, of course, always be tested by the direct method of setting up the light in the desired position, and ascertaining if it is sufficiently visible at the range of distance for which it is designed. This is the method which has usually been employed to fix a standard of reference, but for the proper comparison of different lights some more universal method is wanted.

The property of light spoken of as its " visibility " needs to be defined in terms of some definite unit, and it should be possible to measure it by comparison with a standard whose value in terms of such unit is known. It has been the object of this work to ascertain if such a unit is generally applicable, and to construct an apparatus by means of which measurements of the visibility of point sources of light in terms of this unit can be made.

* *Journal de Physiologie et de Pathologie générale*, No. 4, juillet, 1902.

† " Sur la perception des lumières brèves à la limite de leur portée "—*Journal de Physique*, juillet et août, 1911.

‡ *Comptes Rendus*, November 11, 1907.

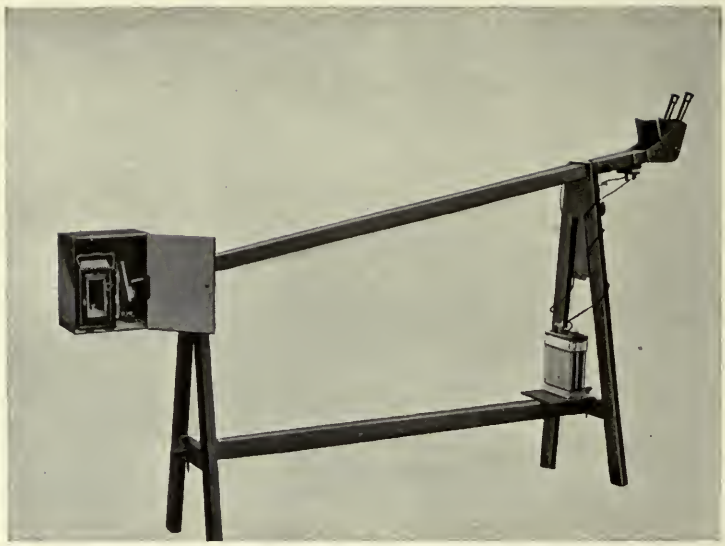


FIG. 1.—Binocular Visibility Photometer.

[*To face page* 51.

Definition of Point Source.—For the purpose of this paper a point source of light is taken as one whose linear dimensions subtend an angle at the eye less than the resolving power of the eye, *i. e.* about thirty seconds of arc for a mean wave length $0{\cdot}5 \times 10^{-3}$ mm. and pupilar aperture 4·5 mm.

The most obvious step in commencing the work was to produce point sources of light whose candle-power could be measured and yet be small enough to be used over the range of the ordinary laboratory dark room, *i. e.* from 2–10 metres.

The pinholes were constructed having diameters varying from 0·01 mm. to 0·20 mm. They were pierced in circular brass plates about 2 cm. diameter and 2 mm. thick. The plate was first deeply countersunk, on one side, and then rubbed down on the other until the metal at the base of the countersink was very thin. This thin metal was finally pierced by means of needles ground under the microscope—the piercing being also done under the microscope.

These pinholes were mounted in front of a flame so as to be fully illuminated. A very satisfactory flame was found in that given by a lamp sold by the Vacuum Oil Company burning a wax known as "Vaclite." This lamp will give a steady constant light for several hours without any tendency to smoke, and the wick chars very little.

The candle-power per square millimetre of the white portion of the standard flame used was obtained by measuring the candle-power of several small areas of the flame, the smallest of which was 13·5 sq. mm. area. From these measurements it was found that the candle-power per square millimetre of the flame was $6{\cdot}6 \times 10^{-3}$.

In the earliest experiments the pinholes were mounted on a metal plate placed in front of the lamp, and held in place by spring clips. This enabled changes of pinhole to be rapidly made. The pinholes were viewed direct, no optical arrangements being used. Later, a visibility photometer was constructed for use with one eye. The principle was the same as a later binocular photometer, although the latter is improved in technical details. The photometer for use with one eye was discarded in favour of the binocular, because the method of employment of the latter more nearly approached the ordinary method of viewing distant lights.

A photograph of the binocular visibility photometer is shown in Fig. 1; and Fig. 2 is a sketch of the general arrangements of the photometer. The instrument consists of a brass tube, A, about two metres long, at one end of which are placed absorption wedges, B, and the pinhole, this latter being held on the wedges by spring clips. The pinhole is illuminated by the standard vaclite lamp, C, which is contained in a box arranged with suitable ventilation. The object of the box is to enable the instrument to be used in the open without the wind causing the flame to flicker. At the end of the tube are placed two adjustable mirrors. In one, D, is seen the image of the standard point source, and the other, E, can be arranged to bring the image of the distant source into the same field of vision. To enable the observer accurately to centre his eyes, the tubes, F, are capable of a sideway adjustment to

suit the distance between the eyes of the observer and of sufficient diameter to enable a full pencil of light to enter the eye.

At G is placed a small metallic filament lamp suitably screened, by which the field surrounding the standard point can be slightly illuminated. This was found necessary for outdoor work and is dealt with later.

The visibility of the standard is varied by sliding the wedges relatively to each other. The motion is given to the wedges by means of the rod H and bevel wheels.

In using the apparatus the points of light were viewed in the mirrors, these being arranged so that the images of the sources were separated by a small angular distance. The observer has to look at one point and carry in his mind the memory

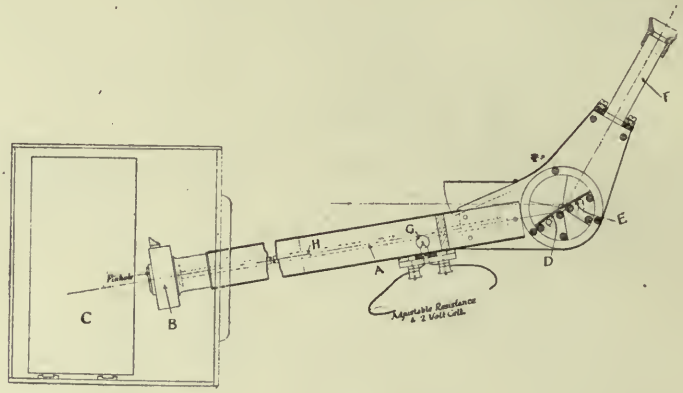


FIG. 2.—Diagram of Visibility Photometer.

of it whilst looking at the other. This procedure was necessary because of the considerable brightening of a point source when it is seen obliquely, and that is to say, when its image does not fall on the centre of the fovea centralis. This effect becomes even more marked when the colours are different; in fact, it was found to be impossible to work with the points close together, so that they were viewed at the same time.

Having arranged the mirrors so that an image of each source can be seen, the standard source is varied by means of the wedges until it matches the distant source for visibility.

Before passing on to enumerate the experiments, it may be mentioned that in all the work hereafter described two point sources of light are matched for visibility. This is most easily done by imagining the points, while matching them, to be distant lights on the horizon. It is found that different observers, provided

their vision is normal, will agree within close limits as to when lights are equally visible, even though the colours are quite different.

The match made here is purely a physiological one, and when of different colours depends partly, but not entirely, on the luminosity curve * for the eye of the observer.

The principal laws which will first be demonstrated are—

I.—That the visibility of a point source of light is independent of its intrinsic brightness and is proportional to its total candle-power.

II.—That the visibility of a given source varies inversely as the square of its distance from the eye of the observer.

I.—Effect of Intrinsic Brightness

Experiment 1.—The smallest illuminated pinhole was set up at about two metres from the eye and others of different sizes in turn were set up at the same distance from the eye, but with an adjustable sector disc inserted in the path of the beam.

The sector opening was adjusted in each case till the visibility of the larger pinhole matched the smaller. The pinholes in this experiment subtended an angle of less than ten seconds of arc, *i. e.* below the resolving power of the eye. They were at the same distance from the eye so that the measurements were not complicated by differences of accommodation. In this way a pinhole of large candle-power has its brightness reduced until it matches the smaller, whose brightness remains constant. The following tables show the results obtained, and it is seen that when the candle-powers of the two holes are equal they are also equally visible, their distances from the eye being equal.

Table I

(*a*) A pinhole for comparison purposes was set up at two metres distance. It had a candle-power $= 5{\cdot}9 \times 10^{-6}$.

Pinhole Normal Candle Power $\times 10^{-6}$.	Angle of Disc open for equal visibility as reference point.	Intrinsic Brightness (Original Brightness = 1·0).	Equivalent Candle Power of dimmed pinhole.	Error. Percentage difference in Candle Power from the reference source.
162×10^{-6}	$13{\cdot}_5$	0·037	$6{\cdot}1_0 \times 10^{-6}$	$+ 3{\cdot}_5$
$84{\cdot}_5 \times 10^{-6}$	$25{\cdot}_5$	$0{\cdot}07_1$	$6\,0 \times 10^{-6}$	$+ 1{\cdot}_5$
$38{\cdot}_2 \times 10^{-6}$	$58{\cdot}_5$	$0{\cdot}16_2$	$6{\cdot}2 \times 10^{-6}$	$+ 5_0$
$22{\cdot}_8 \times 10^{-6}$	$90{\cdot}_0$	0·25	$5{\cdot}7 \times 10^{-6}$	$- 3{\cdot}_5$
$12{\cdot}_2 \times 10^{-6}$	$168{\cdot}_5$	0·47	$5{\cdot}7 \times 10^{-6}$	$- 3{\cdot}_5$

* Sir W. de W. Abney : *Phil. Trans.*, **A.**, 193. (1900.)

TABLE II

(*b*) Smaller pinhole used as comparison source candle-power = $3{\cdot}6 \times 10^{-6}$

Pinhole Normal Candle Power $\times 10^{-6}$.	Angle of Disc open for equal visibility as reference point.	Intrinsic Brightness (Original Brightness = 1·0).	Equivalent Candle Power of dimmed pinhole.	Error. Percentage difference in Candle Power from the reference source.
162×10^{-6}	$7{\cdot}_5$	0·021	$3{\cdot}_4 \times 10^{-6}$	$-5{\cdot}_6$
$84{\cdot}_5 \times 10^{-6}$	$14{\cdot}_5$	0·04	$3{\cdot}_4 \times 10^{-6}$	$-5{\cdot}_6$
$38{\cdot}_2 \times 10^{-6}$	$33{\cdot}_0$	$0{\cdot}09_2$	$3{\cdot}_5 \times 10^{-6}$	$-2{\cdot}_8$
$22{\cdot}_8 \times 10^{-6}$	$57{\cdot}_0$	$0{\cdot}15_8$	$3{\cdot}_6 \times 10^{-6}$	0
$12{\cdot}_2 \times 10^{-6}$	$108{\cdot}_0$	0·3	$3{\cdot}6_5 \times 10^{-6}$	$+1{\cdot}_4$

The last column in the tables indicates the order of agreement between the adjusted candle-power and the candle-power of the point of reference.

Wedges.—The sector disc, although giving the most absolute method of dimming down the intensity of the light without changing its character, is not suitable where a wide range of variation is desired, and in order to exceed the range over which the last experiment was conducted, the sector disc was replaced by absorption wedges. The wedges used were those belonging to a Féry pyrometer, and although slightly selective, the selectivity has not been found in any way to influence the accuracy of the work. These wedges slide relatively to one another and are fitted with a scale of position.

The wedges were calibrated by using them in the path of a beam of light from an arc, the candle-power of the beam being measured. The measurements were made on an ordinary photometer bench using a Lummer-Brodhun photometer head. The wedges were removed from time to time and the candle-power of the incident beam measured. In this way a curve of transmission for the wedges was obtained, connecting the relative position of the wedges to one another, and the percentage transmission of the light.

Experiment.—To prove the validity of substituting the calibrated wedges in the place of the sector disc, as a means of reducing the intrinsic brightness of the sources of light.

Illuminated pinholes of diameters varying from 0·03 mm. to 1·00 mm. were set up one after another behind the wedges in the standard apparatus. They were adjusted by means of the absorption wedges until they matched a comparison source for visibility in the manner described earlier. The results are shown in the following tables—

TABLE III

(*a*) Reference source candle-power = $4{\cdot}1 \times 10^{-6}$.

Area in sq. mm. of Pinhole.	Pinhole Candle Power $\times 10^{-6}$ (for Normal Brightness).	Wedge Transmission Percentage.	Equivalent Candle Power $\times 10^{-6}$.	Error. Percentage difference from the reference source.
$78{\cdot}_5$	4320	$0{\cdot}09_5$	$4{\cdot}1$	0
$18{\cdot}_9$	1040	$0{\cdot}4_3$	$4{\cdot}4_5$	$+ 8{\cdot}_5$
$2{\cdot}9_5$	162	$2{\cdot}6$	$4\ 2$	$- 2{\cdot}_5$
$1{\cdot}5_4$	$84{\cdot}_5$	$4{\cdot}4$	$3{\cdot}7$	$- 9{\cdot}_5$
$0{\cdot}69_5$	$38{\cdot}_2$	$10{\cdot}_4$	$3{\cdot}9_5$	$- 3{\cdot}_5$
$0{\cdot}34_5$	$18{\cdot}_9$	$23{\cdot}_3$	$4{\cdot}4$	$+ 7{\cdot}_5$

TABLE IV

(*b*) Reference source = $1{\cdot}3_5 \times 10^{-6}$ candle-power.

Area in sq. mm. of Pinhole.	Pinhole Candle Power $\times 10^{-6}$ (for Normal Brightness).	Wedge Transmission Percentage.	Equivalent Candle Power $\times 10^{-6}$.	Error. Percentage difference from the reference source.
$78{\cdot}_5$	4320	$0{\cdot}032_5$	$1{\cdot}4_0$	$+ 3{\cdot}_5$
$18{\cdot}_9$	1040	$0{\cdot}13_5$	$1{\cdot}4_0$	$+ 3{\cdot}_5$
$2{\cdot}9_5$	162	$0{\cdot}7_9$	$1{\cdot}2_8$	$- 5{\cdot}_0$
$1{\cdot}5_4$	$84{\cdot}_5$	$1{\cdot}5_2$	$1{\cdot}2_9$	$- 4{\cdot}_5$
$0{\cdot}69_5$	$38{\cdot}_2$	$3{\cdot}6_0$	$1{\cdot}3_8$	$+ 2{\cdot}_0$
$0{\cdot}34_5$	$18{\cdot}_9$	$6{\cdot}_3$	$1{\cdot}2_9$	$- 4{\cdot}_5$

Column 3 gives a measure of the intrinsic brightness of the source, the original brightness being represented by 100·0.

In the above test it is interesting to note that the largest pinhole subtended an angle of about 2° and was, therefore, greater than the generally accepted limit of the resolving power of the eye. But, in spite of this, it appears that no marked change is seen in the law connecting intrinsic brightness and visibility, although the area of one spot was 3075 times the other. Lord Rayleigh* has shown that for areas subtending considerable angles this law does not hold. The corresponding areas of the largest pinhole and the reference pinhole in table (*b*) are shown to scale in the adjacent figure. These had equal candle-power when appearing equally visible.

* Collected Papers.

The object of giving the two tables is to illustrate the improved accuracy obtained by using rather a faint reference point.

The above experiments show conclusively that for sources subtending angles up to two minutes of arc, the intrinsic brilliancy of the source does not affect its visibility, the total candle-power being the only factor to be considered.

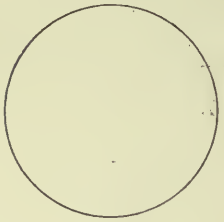


Fig. 3.

Further experiments were devised and carried out which confirmed the previous results. Lenses were used in the path of the beam from one point, about midway between the eye and the point of light. By this means the intrinsic brightness of this point was kept constant, but the apparent size of the point changed by changing the lenses. Matches made with the standard in the photometer showed that agreement to the order of the previous work was obtained between the theoretical visibility, calculated on the assumptions previously outlined, and the measured visibility.

II.—The Inverse Square Law

Experiment 1.—One pinhole was set up at about three metres from the eye and used as a point of reference. The other pinholes in turn were mounted on a travelling carriage and taken to distances up to 20 metres, so as to be equally visible as the point of reference.

Standard of reference—$2{\cdot}5 \times 10^{-6}$ candle-power at $2{\cdot}9_5$ metres.

Visibility assumed $\propto \frac{2{\cdot}5_0}{(2{\cdot}9_5)^2}$, *i. e.* $\propto 0{\cdot}287$.

Pinhole Candle Power × 10^{-6}.	Distance in metres (d).	Visibility (on assumption that Visibility $\propto \frac{1}{d^2}$).	Error. Percentage difference between column 3 and 0·287.
$1{\cdot}2_2$	$2{\cdot}1_1$	0·274	$-4{\cdot}_5$
$6{\cdot}3_5$	$4{\cdot}7_5$	·282	$-2{\cdot}_0$
$16{\cdot}_2$	$7{\cdot}4_6$	·291	$+1{\cdot}_5$
$25{\cdot}_2$	$9{\cdot}0_5$	·308	$+7{\cdot}_5$
$30{\cdot}_5$	$10{\cdot}8_0$	·262	$-8{\cdot}_5$
$52{\cdot}_2$	$13{\cdot}2_5$	·297	$+3{\cdot}_5$
$112{\cdot}_0$	$20{\cdot}_4$	·270	$-6{\cdot}_0$

The agreement of the values in column 3 with that of the comparison source, viz. 0·287, is to be noted.

Experiment 2.—Greater distances were not obtainable in the laboratory dark

room, so that the investigation was continued at night in the 550 ft. gangway of the Ship Tank at the National Physical Laboratory. Here the investigation was more complete, and was carried out for red and green as well as white lights. The method of procedure was slightly changed when working on longer ranges.

A pinhole was set up as standard with the wedges placed in the beam from it, this giving a means of adjusting the visibility of the standard. Another pinhole was placed at various known distances and the standard source varied until it matched the distant point for visibility. The visibility of the distant point could then be measured in terms of the size of the standard and the calibration of the absorption wedges.

It is slightly more difficult to match lights of different colours for visibility than lights of the same colour, the principal reason for this being the unequal behaviour of lights of different colour with regard to oblique vision. The authors found the same effects as those mentioned by Broca and Polack (*Comptes Rendus*, ref. cit.). A green point viewed slightly obliquely appears to have an intensity of four or five times that by direct vision, and a white point about twice. The green loses its characteristic colour and each point becomes less well defined. A red point becomes much less visible when viewed obliquely, and its colour is also indistinguishable when the light is near the vanishing point.

With regard, therefore, to matching green and white lights for visibility, the unequal brightening seen when looking from one point to another bothers the observer considerably and his settings become more erratic. It is for this reason that the authors kept the points always at a considerable angular distance from each other, but this does not entirely avoid the above effect. The red is troublesome for a different reason. Very few people seem to see a red point source as a point unless it is very dull, the image always spreading out somewhat, and appearing to have dimensions, causing the matching of white and red points to be more difficult than matching two spots of the same colour.

As previously mentioned, the authors tried viewing the two points to be matched simultaneously, but finally gave it up as being less accurate than the method of viewing each point separately, and carrying the memory of its visibility when looking at the other.

A method sometimes used was to shut out the point under observation by means of a screen before looking away from it, and to unscreen the other while the eye is focused on the place where it will appear, thus avoiding any oblique sight of the points. When working over these long ranges it was necessary to change the size of the pinhole from time to time, owing to the great difficulty of making efficient matches with very bright points, and also to ensure keeping below the limit of the resolving power of the eye. When changes were made, two readings at that station were taken, one with the larger pinhole which served for greater distances, and one with a much smaller pinhole which was to serve for the shorter distances. In general the visibility at a change like this would be reduced or increased three to eight times, but the readings obtained always checked off within the errors of the

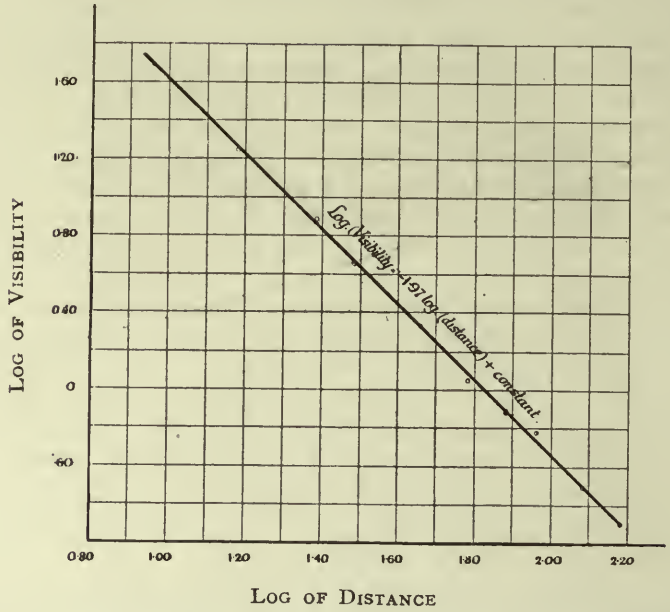


Fig. 4.—Curve showing the variation of visibility with distance (White Light). (This indicates the close agreement with the inverse square law.)

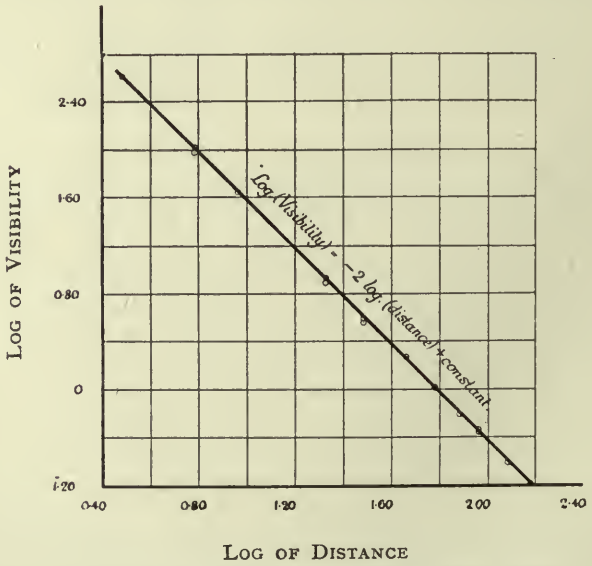


Fig. 5.—Curve showing the variation of visibility with distance (Green Light).

experiments. This, of course, should follow in view of the proof that visibility is proportional to the candle-power alone.

The results of the tests are shown in Tables V, VI and VII, where the visibility obtained by assuming the inverse square law is shown in column three and the percentage error in the last column. These results are also plotted in Figs. 3, 4 and 5, but in order to show the results effectually the candle-power of each pinhole is multiplied up to equal the value of candle-power of the largest hole, and its visibility at the various ranges at which it was observed is multiplied in the same ratio. The log of this derived visibility is plotted against the log of the distance. The line drawn indicates the close agreement to the inverse square law.

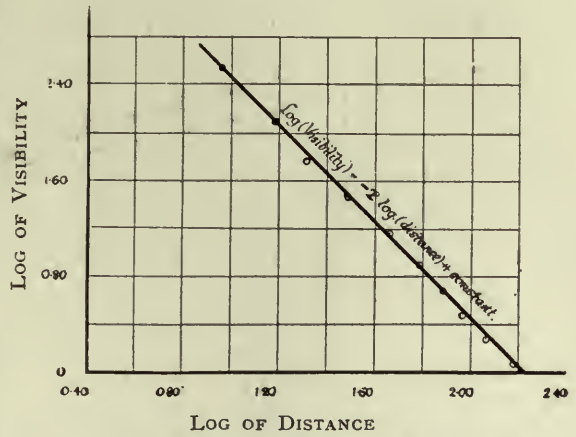


Fig. 6.—Curve showing the variation of visibility with distance (Red Light).

The slight variation from the square law in the case of the white light (Fig. 4) is probably due merely to errors of experiment and, in virtue of the results in Figs. 5 and 6, must not be given weight.

Table V

White light as distant sources; white light standard.

Pinhole Candle Power × 10^{-6}.	Distance in metres (approximately).	Measured Visibility in terms of the Standard.	Visibility in terms of 10^{-6} Candle Power at 1 metre. Calculated assuming inverse square law.	Error. Percentage between columns 3 and 4.
101·5	9·1$_5$	1·1$_6$	0·120	− 4·0
,,	15·2	0·41$_5$	0·44	− 6·5
,,	24·4	0·18	0·17$_0$	+ 5·0
1040	24·4	1·8$_0$	1·7$_5$	+ 3·0
,,	30·5	1·1$_0$	1·1$_5$	− 4·5
,,	45·7	0·51$_5$	0·51$_0$	+ 1·0
,,	60·9	0·27	0·28$_0$	− 3·5
,,	76·2	0·18$_7$	0·18$_0$	+ 4·0
4320	76·2	0·76$_5$	0·74$_5$	+ 3·0
,,	91·4	0·60	0·52$_0$	+ 15·5
,,	122·0	0·31$_5$	0·29$_0$	+ 7·5
,,	152·4	0·19$_8$	0·18$_6$	+ 6·5

TABLE VI

Green light distant source; white light standard.

Candle Power × 10^{-6} Green Light.*	Distance in metres (approximately).	Measured Visibility in terms of the Standard.	Visibility in terms of 10^{-6} Candle Power at 1 metre. Calculated assuming inverse square law.	Error. Percentage between columns 3 and 4.
8·1	3·0$_5$	0·91	0·8$_7$	+ 4·5
,,	6·1$_0$	0·22$_5$	0·22	+ 2·$_0$
51·$_8$	6·1$_0$	1·3$_6$	1·4$_0$	− 3·$_0$
,,	9·1$_5$	0·6$_1$	0·61$_8$	− 1·$_5$
,,	21·3	0·11$_9$	0·11$_4$	+ 4·$_5$
216	21·3	0·4$_5$	0·47$_5$	− 5·$_0$
,,	30·$_5$	0·2$_3$	0·23	0
581	30·$_5$	0·58$_5$	0·62$_5$	− 6·$_5$
,,	45·$_7$	0·28$_5$	0·28$_0$	+ 2·$_0$
,,	60·$_9$	0·16	0·15$_7$	+ 2·$_0$
1215	60·$_9$	0·36	0·33	+ 9·$_0$
,,	76·$_2$	0·20$_5$	0·21	− 2·$_5$
,,	91·$_4$	0·15$_5$	0·14$_5$	+ 6·$_5$
3725	91·$_4$	0·44$_5$	0·44$_5$	0
,,	122·$_0$	0·24$_5$	0·25	− 2·$_0$
,,	152·$_4$	0·16$_4$	0·16	+ 2·$_5$

TABLE VII

Red light distant source; white light standard.

Candle Power × 10^{-6} Red Light *	Distance in metres approximately.	Measured Visibility in terms of the Standard.	Visibility calculated in terms of 10^{-6} Candle Power at 1 metre.	Error. Percentage between columns 3 and 4.
125	9·1$_5$	1·54	1·5	+ 2·$_5$
,,	15·2$_0$	0·54$_5$	0·54	+ 1·0
,,	21·3	0·26	0·27$_5$	− 4·$_0$
,,	30·$_5$	0·13$_5$	0·13$_4$	0
519	30·$_5$	0·54	0·56	− 3·$_5$
,,	45·$_7$	0·26	0·25	+ 4·$_0$
,,	60·$_9$	0·14$_5$	0·14	+ 3·$_5$
2915	60·$_9$	0·76	0·78$_5$	− 3·$_0$
,,	76·$_2$	0·48$_5$	0·50	− 3·$_0$
,,	91·$_4$	0·33	0·35	− 5·$_5$
,,	122·$_0$	0·19$_2$	0·20	− 4·$_0$
,,	152·$_4$	0·12$_2$	0·12$_5$	− 2·$_4$

Experiment 3.—The result obtained in the last experiment has been confirmed by tests extending over a mile. Two small electric lamps, one a carbon filament and the other a tungsten, calibrated for candle-power and current, were set up at various distances and their visibility measured by means of the visibility photometer.

* The method of obtaining the percentage of light transmitted by a coloured glass is described later.

The following table gives the values obtained, using the carbon filament lamp.

Candle Power of Lamp.	Distance in metres.	Calculated Visibility on inverse square law.	Measured Visibility.	Error. Percentage.
0·18	483	$0·7_6$	$0·8_0$	+ 5
0·18	644	$0·4_3$	$0·41_5$	−4
0·69	1098	$0·5_7$	$0·5_7$	—
0·69	1610	$0·26_5$	$0·24_5$	− 7·5

The following table gives the values obtained, using a metallic filament lamp.

Candle Power of Lamp.	Distance in metres.	Calculated Visibility on inverse square law.	Measured Visibility.	Error. Percentage.
$0·18_5$	483	$0·7_8$	$0·7_4$	− 5
$0·18_5$	644	$0·4_4$	$0·4_6$	+ 4·5
$0·7_0$	1098	$0·57_5$	$0·5_7$	− 1·0
$0·7_0$	1610	0·27	$0·2_5$	− 7·5

These readings were obtained on dark, clear nights in April, with the moon behind dense clouds.

The Experiments 1 to 3 under this section confirm what would be expected from theory, that the visibility of a point source of light falls off proportionally to the inverse square of its distance from the eye of the observer. Incidentally, it should be observed that Experiment 3 shows that at least, up to ranges of one mile, atmospheric absorption is not appreciable on a night which would ordinarily be called clear and dark. Similar results were obtained on a one-mile range for green and red lights. The two different types of lamp were used in order to see if the light which had the preponderance of short wave lengths was more absorbed than the redder light. Under the conditions of the experiment, the measurements do not indicate any such difference. Several attempts have been made to work on longer ranges, but up to the present have been unsuccessful, due, among other reasons, to the prevalence of varying local mists.

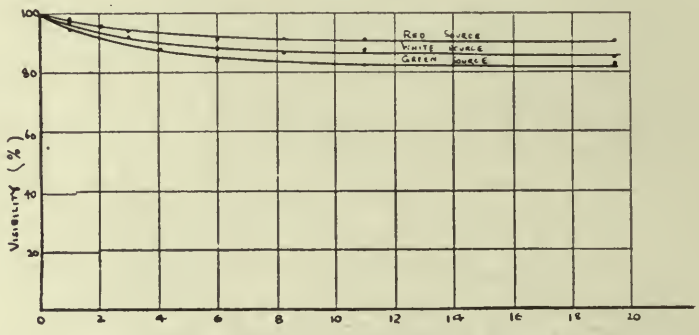
Having established the fundamental theorems that the visibility of a point source is proportional to—(1) the total candle-power of the source, and is independent of intrinsic brightness; (2) the inverse square of the distance,—it was possible to choose a suitable unit in terms of which visibility could be expressed.

The unit decided upon as being of a convenient order of magnitude was one-millionth of a candle at one metre distance. Thus the visibility of a point of one candle-power at one kilometre, neglecting absorption, would be equal to this unit, and would, by the eye, be considered equally visible. It is not suggested that a point whose visibility is equal to 2 is twice as bright to the eye as one of visibility equal to 1; but the former could have its candle-power halved or its distance increased $\sqrt{2}$ and be made equally visible.

Effect of Slight Illumination of the Field of Vision.—When observing in the open, the field of view on which the distant source is seen is rarely as black as that of the standard source, and thus in relation to the standard its visibility is diminished. In order to eliminate this disturbing factor, it was found necessary to produce equality of illumination of the two fields of vision. This was achieved by illuminating the background surrounding the standard source by means of a high efficiency electric lamp; fitted into the photometer and suitably screened. The effect of observing a point of light against a faintly illuminated background was more fully investigated by arranging a point of light in the centre of a white screen, which could be illuminated to the same degree as the field of vision in the open.

A very bright moonlight night, atmosphere clear, gave a field in the photometer whose illumination was equal to $\frac{5}{1000}$ of a metre candle. Slight mist increased the value of the illumination considerably, a value of one-hundredth of a metre candle having been recorded. Various pinholes were measured in this way for visibility, when viewed in a black field and in fields of different illuminations.

Red and green lights were also observed, the results being plotted in Fig. 7.



Illumination in $\frac{1}{1000}$ths Metre Candles

Fig. 7.—Curves showing the loss of visibility of a point source of light when observed on a slightly illuminated background.

The fact that the visibility appears to reach a constant value is rather unexpected, for undoubtedly, when working at a field illumination equal to one metre candle the point becomes nearly invisible. The above diagram, therefore, must only be considered as applying to very faint illuminations.

The dimming in each case was about ten per cent., the green appearing slightly dimmer and the red brighter than the white source, when the field had an illumination equal to $\frac{4}{1000}$ metre candle. This may be considered about an average value for practical observation of lights on clear nights.

Coloured Glasses used in Signal Lanterns.—In connection with signal lights, in which coloured-glass screens are placed before a white flame, it is often required to determine by how much the visibility of a point source is reduced by placing a coloured-glass screen in front of it. As the visibility of a point source

varies directly as the candle-power, this is equivalent to finding the percentage of total light transmitted by the coloured screen. This measurement can be very conveniently and comparatively easily made by means of the visibility photometer. The visibility of a white point source is measured; the coloured screen is then placed before the white source and the reduced visibility measured by the apparatus as previously described. Some green glasses used for signal purposes have been found to transmit only five per cent. of the light, whilst others transmit as much as fifteen per cent.

The results obtained by different observers are in close agreement so long as they have normal vision, and the figures obtained may be taken as representing the fraction of the light transmitted through the coloured glass, as appreciated by an average eye when observing a point source. Cases of defective vision are discussed later. The application of the laboratory tests to predict the visibility of lights in the open over long ranges, has been generally confirmed by such tests as the following made by different observers.

A visibility which was considered as low as could be regarded as satisfactory for practical purposes was fixed in the laboratory in terms of a white point source of known visibility. A green light was then set up in the open, and observers, moving away from the lamp, judged when it could be regarded as equally visible. The candle-power of the lamp without its green glass screen was measured simultaneously by means of a portable photometer.

Calculating the visibility of this light from the value of the transmission of the green glass determined in the manner described, it was found to agree within narrow limits with the visibility previously determined in the laboratory. The visibility thus found as the lowest which was considered desirable for practical purposes, was $0{\cdot}12 \times 10^{-6}$ candle-power at one metre distance. Using this figure, the candle-power of white light necessary for visibility at any range can be calculated, and by determining the transmission of a glass in the way described earlier, the required candle-power of the light behind the glass can be also calculated for visibility at any range.

It is interesting to note that the authors' results agree fairly closely with those calculated from the Report of the Deutsche Seewarte of 1894 (ref. cit.):—

Range.	Candle Power (British) of White Light required in clear air (from report of Deutsche Seewarte).	Candle Power of White Light (British Units) calculated from results of the author's experiments.*
1 sea mile = 1855 metres approximately	0·47	0·41
2 sea miles = 3710 metres approximately	$1{\cdot}9_0$	$1{\cdot}6_0$
5 sea miles = 9275 metres approximately	$11{\cdot}_8$	$10{\cdot}_0$

* No allowance has been made in column three for atmospheric absorption, a factor which probably comes in on the five-mile range, even in very clear weather.

Effects of using Spectacles in the Observation of Point Sources.—In the course of the work previously described, curious results were noticed when observations were made by people using spectacles. The anomalous results usually occurred when making visibility measurements of green point sources.

Experiments were undertaken with a view to determining the cause of such results, but they are not in any way exhaustive. The actual results, however, are given, together with what appears to be an explanation of the phenomena.

Experiment to ascertain the change in the visibility of a point source when viewed through lenses used as spectacles.

The visibility photometer was slightly modified for use in these experiments. The box containing the mirror and vision tubes was removed and an arrangement fitted on top of the standard tube so that a source could be viewed through spectacle lenses. The standard pinhole and wedges were used to measure the visibility of this

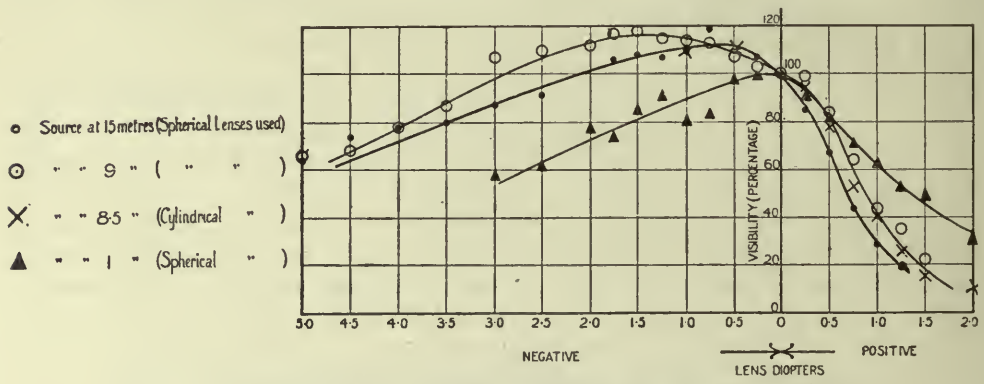


Fig. 8.—Curves showing the variation in visibility of point sources of light when viewed through spectacle lenses.

source. Plain glass was put in the path of the beam from the standard pinhole to balance the absorption and reflection losses at the surfaces of the lens.

(*a*) Two white point sources were used and the visibility of one measured when viewed through spherical spectacle lenses. The distant source was observed when fifteen metres, nine metres and one metre away. The results are plotted in Fig. 8. In all the work with lenses hereafter described, before plotting the results, the observed readings were corrected for the magnification that is produced when an object is observed through lenses. This was only appreciable on the short range of about a metre, and even then only amounted to five per cent. as a maximum correction. Too much emphasis must not be placed upon the actual shape of the curves, but their general form can always be repeated.

The negative lenses have a much less marked effect than the positive, except at the very short range when the curve is nearly symmetrical.

(*b*) In the same figure are shown plotted the results obtained when cylindrical

lenses were used instead of spherical, the range in the experiment being 8·5 metres.

It will be seen that the points fall approximately on the curve that was obtained for the visibility of a source nine metres distant, viewed through spherical lenses; and in general it was found that the cylindrical lenses produced the same dimming as the corresponding spherical. When looking at a point at a greater distance than five metres the normal eye is almost as negative as it can be, and thus on adding a positive lens, little or no equivalent negative accommodation can be made, and the focused image no longer falls on the retina. The curves show that a large dimming

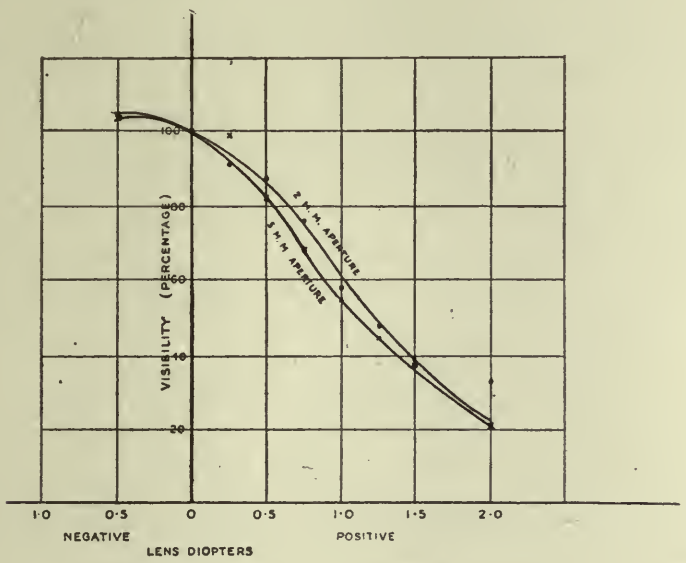


FIG. 9.—Curves showing the effect of reducing the pupilar aperture of the eye.

occurs even for so small a positive lens as 0·5 diopters. If a negative lens is added the eye can accommodate somewhat, and the visibility falls off less rapidly. When observing a point within the range of accommodation it is seen from the curves that a similar effect occurs. The eye can accommodate a little for the positive lens, and the visibility diminishes less rapidly than at longer ranges, whilst, the eye being unable to accommodate positively to such a large extent as before, the negative lenses produced a more marked effect.

The supposition that the dimming was due to the image not being focused on the fovea was not supported by the appearance of the points of light. They appeared quite well defined, even when viewed through a powerful lens. It was subsequently found, however, that the dimming *was* actually due to the spreading of the light owing to want of focus, but that with the faint sources employed, the diffused light

was not apparent. If a very bright point is observed under the same conditions the diffused light can be seen without difficulty. The above conclusion was arrived at as a result of the following experiments.

Spherical Aberration.—It was thought that the spherical aberration of the eye might produce a diffused spreading of the light, especially as the aperture of the eye becomes as large as 5 mm. after working for some time in the dark. In view of this, an experiment similar to the last was carried out, but the aperture of the eye was varied from 2 to 5 mm. by placing stops in front of the pupils of the eyes. To prevent confusion only the results for 2 mm. and 5 mm. aperture are plotted, the results for apertures 3 mm. and 4 mm. lie in the neighbourhood of the curves shown. The results are given in Fig. 9 and need no comment.

The dimming is not reduced to any appreciable extent, by using the smaller

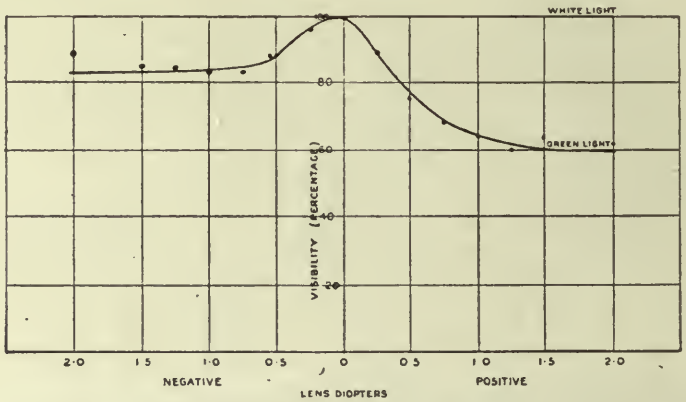


FIG. 10.—Curve showing the dimming of a green point source of light relative to a white when viewed through spectacle lenses.

aperture. From the experiments it seems conclusively shown that the dimming is due to the image on the retina being thrown out of focus, and that the diffusely spread light, around the nucleus of the image, is not appreciated, because it is so faint. The cause of it, however, does not appear to be spherical aberration. It was noted in the course of the work that differences in visibility were obtained chiefly by observers who used spectacle glasses when viewing coloured lights. It was found that an observer who used positive cylindrical glasses to correct astigmatism saw much less of a green point source than when he observed it unaided by glasses, although without them it was much less well defined. The same observer viewing the green point through a negative cylindrical lens to correct his astigmatism saw no less and sometimes more than he did without the spectacles.

Chromatic Aberration.—To investigate this effect more fully measurements of the dimming of the green and red, relative to white, were made.

The visibility photometer was used in its original form, both points being received in their respective mirrors; cylindrical lenses were placed at the top of the vision tubes and used when viewing both points. The standard point was the white one, and it was always varied by means of the wedges until it matched the other point under observation. It was found that the green point always appeared to be dimmed, relatively to the white one. The amount of this dimming is shown and plotted on Fig. 10. Here, again, the positive lenses produced the greatest dimming, the point observed being outside the range of accommodation. Fig. 11 shows the behaviour of red light relative to white for each of the authors.

It is interesting to note the brightening in the case of one of the observers. The curve for the other appears symmetrical, positive and negative lenses affecting

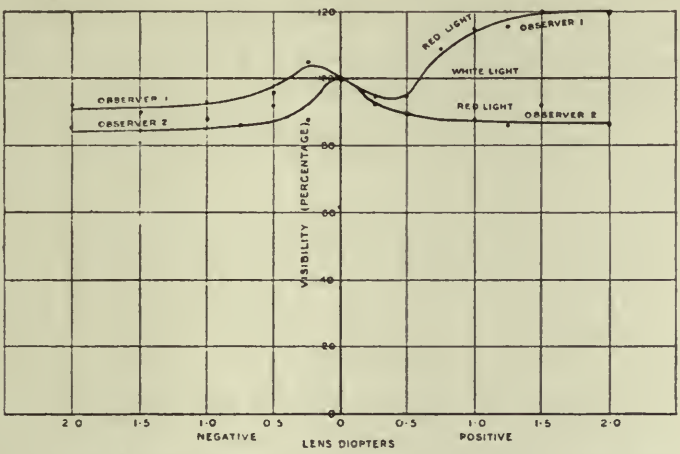


FIG. 11.—Curves showing the dimming of a red point source of light relative to a white when viewed through spectacle lenses.

the visibility of the point equally. These chromatic effects can be explained in the main on the same supposition as before.

The important point to be borne in mind is that if a person is observing a green point at such a distance that he cannot accommodate any more in the negative direction, the use of a positive lens either spherical or cylindrical will cause the point to become very much less visible, viz. for instance to the extent of fifty per cent., for a one diopter cylindrical lens. An equally powerful negative cylindrical lens would not produce serious dimming, but possibly would produce a brightening. This is of considerable importance in connection with the use of spectacle lenses by men to observe signals, and points to the necessity of testing to what extent such an observer can see a distant light, and how much light he appreciates relatively to an observer with normal eyesight. The case in which the effect would be very marked is that in which a person with long sight is corrected with positive lenses for near work, and

uses these spectacles when observing distant lights. Similarly, if a person has astigmatic sight and has positive cylindrical correcting glasses for near work, he will generally find that the use of these for the observation of distant faint lights causes considerable dimming, particularly in the case of green lights.

The authors do not suggest that the notes which they publish in the paper are in any way exhaustive. The portions of the subject which have been investigated are merely those which were immediately concerned with the visibility of steady signal lights at sea, and it has not been possible to carry the work further than was necessary for the solution of the practical problems in hand.

In conclusion, they desire, in connection with this work, to express their acknowledgments to Dr. Glazebrook, Director of the National Physical Laboratory.

Dr. SCHUSTER in opening the discussion remarked that the subject was one of considerable practical importance, and the authors seemed to have gone into the question of visibility with great thoroughness.

Dr. W. ROSENHAIN considered that the point of most interest in the paper was the fact that the authors had found that visibility depended only on candle-power, and not on intrinsic brightness. The French lighthouse engineers had always held the reverse to be the case, but there seemed no gainsaying the authors' results, and he thought that some designers would have to modify their present theories.

Mr. CHALMERS disagreed with Dr. Rosenhain in his view, and thought that there was nothing in the paper which entailed a modification of present ideas connected with lighthouse design. The paper did not deal with the use of lens and prism systems.

Dr. ROSENHAIN differed. Quite apart from the question of relation between the size of the source and the lens system in a lighthouse, the French designers had always kept the whole lens system relatively small, on the assumption that for the same candle-power the more concentrated the source, the greater would be the visibility.

Mr. RAYNER remarked that Mr. Paterson had shown what a difference there was in looking at a small hole (3 mm. in diameter at 3 M.) illuminated by a flame in a dark room, according as a green or a red glass was placed over the hole. In the case of a red glass it appeared sharp and round, but in the case of the green one the hole appeared to be surrounded by brilliant green rays, no doubt produced by some imperfection in the eye.

A further investigation of this phenomenon, in order to see what part of the spectrum gave the effect most strongly in various individuals, would be of interest; and it might be found that by using a coloured screen to reduce the intensity of the rays producing the effect most strongly, a distinct gain might result when the finest definition in a telescope or microscope was required. Such a screen might be of considerable assistance in studying lunar detail or in bacteriology.

Mr. TROTTER raised the question of what happened when an image of a point of light fell between a rod and a cone on the retina.

Dr. JOHN MILNE referred to the fatigue of different parts of the retina and to oblique vision.

Dr. S. P. THOMPSON mentioned that in their account of the paper the authors had not touched on the question of oblique vision, which greatly intensified the visibility of most faint sources of light. The question of rods and cones and that of oblique vision had been considered some years ago when the " N-rays " were discussed. These effects were probably due to indirect vision.

Dr. R. S. CLAY stated that better seeing resulted from oblique vision when the illumination was faint. He did not quite understand Mr. Paterson's remarks on astigmatism and the use of spectacles, and whether an observer actually saw less when he used his spectacles. He asked whether any results were obtained on misty nights.

Dr. C. E. K. MEES asked what precautions were taken to rest the eye; five or ten minutes of rest were not nearly sufficient, he had sometimes needed more than an hour. It depended upon the colour, and the recovery of the eye sensitiveness for red was much quicker than for green.

Mr. PATERSON, in reply to Dr. Rosenhain and Mr. Chalmers, stated that there were two questions involved. High intrinsic brilliancy might in many cases be desirable from the point of view of the design of the refracting and reflecting systems of a lighthouse, but in their view there appeared to be no object in seeking high intrinsic brilliancy for the light as a whole when viewed from a distance without lens systems. They hesitated to say whether a misty condition of the atmosphere would affect this conclusion. In reply to Mr. Trotter they thought that the way in which the rods and cones acted was very obscure, but it would be safe to say that the image of the point would hardly remain stationary during observation, but would be in a condition of rapid movement over an appreciable area of the fovea. Answering Dr. Thompson they explained that the question of oblique vision was considered in considerable detail in the paper and was a factor of great importance. The eye was more sensitive to green light in oblique than in direct vision, but was more sensitive to red light in direct vision. The question raised by Dr. Clay as to the use of spectacles was entirely a matter as to whether a spot of light was really focused properly on the fovea. Often a light appeared to the observer to be focused, but, in reality, it was surrounded by circles of diffusion too faint to be noticed, yet sufficient to cause serious diminution of visibility. Hence, if an observer's spectacles were correct for nearer vision and were slightly too positive for distant vision, he would find great diminution of visibility of the green light. The point raised by Dr. Mees was of importance. All the author's measurements were taken either after long periods in the open at night or in the dark room of the laboratory, and discrepancies were not noticed which could be attributed to a differential fatiguing effect.

SOME RECENT ADVANCES IN THE MEASUREMENT OF LIGHT AND ILLUMINATION

By J. S. Dow and V. H. Mackinney

(*Friday, June 21st,* 1912)

In presenting a paper before the Optical Convention the authors are confronted by two possibilities. They may attempt a comprehensive review of the subject, in the course of which all the recent advances are briefly mentioned; or they may select certain aspects which, for one reason or another, appear to be of particular interest at the moment, and specialize on these.

In deciding on the second course, the authors have been influenced by the consideration that the apparatus and experiments which they propose to describe seem such as to interest those connected with various branches of optics other than photometry, and to illustrate the close connection between the measurement of light and illumination and other photographic and optical problems. It may, however, be well to mention that several publications dealing comprehensively with the measurement of light and illumination have recently appeared, and may be consulted by those who desire a broad review of the subject as a whole.*

Perhaps the most interesting feature of the progress of the past ten years has been, not so much the actual advance in the construction of apparatus, as the change in the view-point from which measurements of light have been regarded. Special refinements have been introduced in the laboratory where photometry has been carried to a high stage of precision and accuracy. But at the same time there has been a growing recognition that the measurement of light and illumination is not only of considerable industrial consequence, but a fairly simple and feasible process in the hands of others besides the expert. At one time such measurements were carried out solely in the laboratory. Subsequently tests began to be made of the lamps in the street, usually by the aid of travelling photometer-carriages. Later still strictly portable instruments have been devised for the measurements of light and illumination, the first of which in this country was the apparatus devised by Sir William Preece and Mr. Trotter about 1883. With the beginning of the new century a marked interest in measurements of illumination began to be shown, and quite a number of simple instruments of this kind have since been devised. For practical purposes

* *E.g.* A.P. Trotter: *Illumination, its Measurement and Distribution*; "Photometric Units and Standards," by E. B. Rosa; "The Measurement of Light," by C. H. Sharp. (Two lectures delivered at the Johns Hopkins University, Baltimore, U.S.A., in 1910.)

the actual illumination in a room, on a table, desk or paper, is often of much greater importance for the layman than a knowledge of the amount of light given by the lamps employed.

Many instances could be mentioned of the value of such measurements. Determinations of the illumination on a table or desk, for example, enables us to judge whether the light is sufficient for reading, in a school or library, or for some industrial process in the factory. They also afford a ready means of comparing the concentrating power of different kinds of shades and reflectors, and, as will be seen later, have many other subsidiary uses in connection with the measurement of reflecting power, absorption of glasses, etc. In practice the chief requirements in such an instrument are that it should be compact and portable, and capable of being carried about with the ease of small hand camera, that it should be easy and quick to manipulate, sufficiently accurate for practical purposes, and readily set right should the reading, for any reason, vary in course of time.

In order to secure the twin qualities of simplicity and portability some concession as regards accuracy may well be made. For the great majority of practical purposes it may safely be said that an accuracy of 5 or 10 per cent. is ample, while one can imagine circumstances in which an even greater scope might be permissible. Take, for example, the determinations of illumination in schools, factories, workshops, etc. In these cases it is usually of small import whether the light is 5 or 10 per cent. too much or too little, for one cannot estimate the exact amount of light necessary for reading, etc., with this degree of nicety. On the other hand, we *do* require a ready means of ascertaining whether the value is, say 3 or 0·3 foot-candles, as the latter would be undoubtedly too little. For the moment, what is chiefly necessary is, to provide apparatus so simple and easy to use as to popularize the measurement of illumination, and enable important large variations to be observed and recorded. Again, it will be admitted that the fluctuations in the light available in practice are often considerable. For example, a variation of 4 per cent. up or down in the electrical supply voltage (which is permitted by the existing regulations, and at some times of the day and under certain conditions may even be exceeded), might lead to a variation of as much as ± 25–30 per cent. in the light from carbon-filament incandescent lamps. In the case of arc lighting there are, of course, other factors which render it difficult to approach ideal constancy, although it will be admitted that the steadiness of the light has been considerably improved in the best forms of lamps.

For these reasons it might, perhaps, be suggested that a somewhat excessive stress has sometimes been laid on accuracy, essential as this doubtless is in the laboratory. For the complexity which this extreme accuracy entails prevents the ordinary man from making use of photometry, and limits its practical applications. Since the authors became interested in this subject, they have been struck by the great variety of uses to which photometry can be put, and many most interesting and unsuspected applications have come to light.

During the last few years the Illuminating Engineering Society has been

instrumental in showing the great value of measurements of illumination in the school, library and workshop, and this latter question has recently received close attention from the Home Office. Without some means of measurement it would be difficult to specify the exact conditions of illumination required for certain practical processes. But once we have available simple instruments for the purpose the possibility of doing so becomes much less remote. One cannot expect an instrument to tell one everything about the conditions of illumination in a building. But it provides us with a *record*, and is, therefore, a valuable adjunct to personal impressions.

A Simple Apparatus for the Measurement of Illumination

In 1909 the authors had the pleasure of describing a compact and portable instrument for measuring illumination before the Optical Society. This has since been improved and remodelled, and is now known as the "Holophane Lumeter." It is hardly necessary to describe this instrument again, but a few salient points in its construction may be noted. The most novel feature is that it can be used not only to measure illumination but also surface brightness. The general appearance of the instrument (which is only $5\frac{3}{4}'' \times 4\frac{1}{2}'' \times 1\frac{3}{4}''$) is shown in Fig. 1 and the details in Fig. 2.

The small glow-lamp, B, is enclosed in a cylindrical chamber, whitened inside and uniformly illuminating the rectangular aperture, G, which is covered by a ground opal glass plate. This plate in turn acts as a source of light and illuminates the photometric screen, C. The lid, A, is attached to a cylinder fitting concentrically round the chamber, and having cut in it an aperture of special shape. As the pointer, H, is revolved this adjustable shutter covers the source, G, to a greater or less extent, and varies the illumination of the photometric screen, C, accordingly. By moving the pointer across the scale the illumination is reduced uniformly from 2 foot-candles (with the aperture fully open) to 0·2 and then to zero.

In using the instrument the observer points the instrument at the surface whose brightness he wishes to study, places his eye at E, at the same time pressing the contact at the base of the instrument and lighting the lamp. He then sees through the central aperture, D, in the illuminated surface, C, the surface to be tested, and adjusts the pointer until photometric balance is secured. The reading can then be observed on the scale in foot-candles. The construction of the screen, C, deserves a word or two of explanation. It is made by covering a glass surface with a very fine white emulsion, scraping away the centre, and then protecting by a covering glass.

In this way a screen which cannot be soiled or scratched by contact with the fingers is obtained, and, the emulsion being very thin, an exceedingly fine line of photometric division can be obtained. The process requires some care, and the authors are indebted to the skill of the makers, Messrs. R. & J. Beck, Ltd., who have now succeeded in preparing very sensitive screens of this kind.

In order to increase the range of the instrument, recourse is had to two dark

glasses (shown at M) reducing the light by $\frac{1}{10}$ and $\frac{1}{100}$ respectively. These can be placed in the path of the rays from the object studied by pulling out the knobs, N. By this means the reading can be multiplied by 10, 100 or 1000, so that values up to 2000 foot-candles can be measured. The choice of a suitable glass which is

FIG. 1.

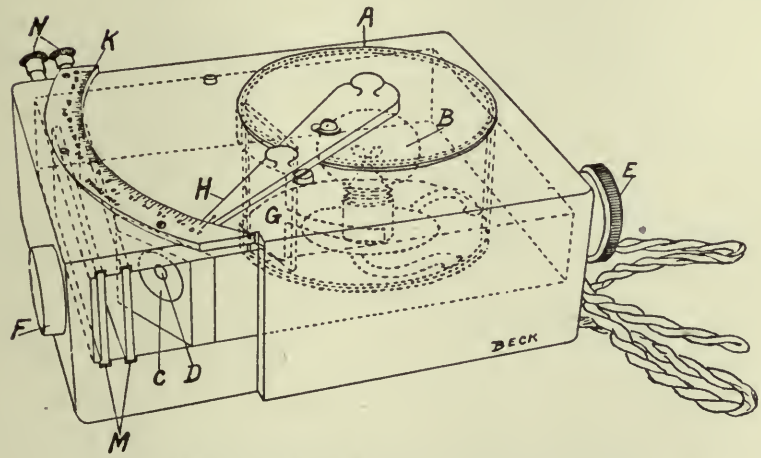


FIG. 2.—Showing working parts of Holophane Lumeter.

sufficiently opaque, uniform and neutral for the purpose naturally demands some care, but the most recent variety adopted has proved to have very good qualities in this respect.

The small lamp used in the instrument is fed from a four-volt accumulator packed in the case of the instrument, and the whole can be carried about as easily as a small hand camera. The instrument can be set to read correctly, by reference to a given illumination, by simply adjusting the position of the lamp slightly. When measure-

ments of illumination, as distinct from surface brightness, are to be made, all that is necessary is to observe the brightness of a standard mat celluloid screen supplied with the instrument, and this is also used in calibration. An interesting point which almost invariably occurs to people unfamiliar with the instrument, is the effect of the distance from which the illuminated object is observed. Within practical limits this is immaterial since the amount of light entering the eye and the area of the retinal image both diminish inversely with the square of the distance. No appreciable difference due to this effect will be observed in other measurements of illumination in a room. It should, however, be borne in mind that the area of surface included within the aperture in the field of view of the instrument is greater with increasing distance; consequently if the surface examined is not uniformly bright, one would expect this to be recorded on the instrument.

Apart from the points referred to above, there are a number of features in the design of the instrument which involve most interesting optical problems, notably the action of the diffusing glass plate (G). Those interested will find these points dealt with fully in the original paper before the Optical Society. The possibility of being able to measure both illumination and surface brightness suggests many useful applications, and it may be mentioned that this principle has been previously utilized by Dr. Clayton Sharp and Mr. P. S. Millar in the United States. Before passing on to these developments brief reference may be made to several supplementary pieces of apparatus contrived for use with the holophane lumeter, including the daylight device based on the ingenious method originally devised by Mr. Trotter and recently utilized by Mr. P. J. Waldram.* A useful addition for daylight measurements is a small cap provided with a special yellow Wratten gelatine screen, which makes the transmitted light identical to the eye with daylight, and so removes the difficulty experienced by many observers in comparing sources of light which differ in colour. For relative measurements (such as mostly occur in daylight work), the amount of light absorbed by this screen may be ignored. It is, however, quite easy to determine the approximate absorption and allow for it when absolute measurements are desired.

Some Practical Applications of Illumination Measurements

Allusion has already been made to the value of measurements of illumination in practice. Not only do such measurements constitute a valuable record, but they enable the lighting engineer to verify his calculations and to demonstrate to consumers the effect of a certain change in their installation. It is not uncommon to meet consumers whose personal impression leads them quite astray in judging an alteration of this kind, their views being based on examination of the apparent brightness of the lighting units rather than the actual working illumination provided. By the aid of a simple measuring instrument the engineer has little difficulty in showing that the recommended form of shade and reflector led to an increase in

* See *The Illuminating Engineer.* (Lond. May 1910.)

the illumination in desirable directions. In cases in which the present installation clearly gives too low an illumination a measurement tells him how much it must be increased, and what types of lamps and fixtures ought, therefore, to be introduced. A study of the transactions of the Illuminating Engineering Society during the last two years will serve to show how large a part actual measurement of illumination in schools, railways, libraries, etc., has played. But it may be observed that in many of the problems in illumination that have recently come to the front, measurements of surface-brightness are demanded to an equally great extent. For example, in considering the comparative advantages of direct and indirect lighting, the respective brightness of the walls and ceiling, and the distribution of this brightness are all-important. In preserving a record of the conditions of illumination in a room it is often desirable to map out not only the illumination on the working level, but the brightness of surrounding objects. Again another point which is now exciting much attention is the desirable intrinsic brilliancy of the sources themselves. Some authorities assert that illuminants should be so screened and shaded that their brightness should not exceed 2–3 candle-power per sq. in. of radiating area, while others would place the limit even lower. Now it is not easy to measure the actual brightness of such objects directly in the same way as the illumination on a table, owing to the fact that the values are usually far outside the customary range of portable photometric instruments as usually constructed. But it may be observed that a surface-brightness of 2000 foot-candles is equivalent to about $4\frac{1}{2}$ candle-power per sq. in., and a brilliancy not exceeding this figure could, therefore, be measured by our instrument.

In connection with industrial processes, again, it is a great advantage to be able to measure illumination in any plane, and even in places which are only accessible with difficulty. In many cases this can be done easily with an apparatus utilizing a separate test-plate. The standard screen can be readily placed at any point where it is desired to take a measurement and observed from a distance. When it is only desired to obtain relative values, and to determine surface-brightness, it is not necessary to do even this, since one has only to point the instrument at the object to be examined. Many practical cases occur in illuminating engineering in which such measurements are all that is needed, especially when it is only desired to study the distribution of illumination on a surface, *e. g.* on an illuminated placard or ceiling. An excellent illustration is provided by the holophane "Uniflux" reflector recently designed by one of the authors, which is intended for producing an even illumination over large flat surfaces such as pictures, diagrams, placards, etc. The holophane lumeter instrument proved exceedingly convenient for comparing the distribution of brightness obtainable with this reflector with those secured by other methods of lighting, and demonstrating the improved evenness of the illumination.

The facility to measure illumination and surface brightness carries with it the possibility of determining the relative reflecting power of a surface. All that is necessary is to measure the surface brightness at a given spot and then to place the

white celulloid test-card in position and measure the illumination. A simple calculation shows the relative reflecting power of the surface as compared with the test-card. This method can be readily applied to the wall-papers in any room lighted by natural or artificial light, and the following table, showing the results obtained in four typical rooms, may, perhaps, be of interest—

REFLECTING POWER OF WALL-PAPERS.

(*From Actual Results obtained in Rooms lighted by Tungsten Lamps.*)

Nature of Room.	Paper.	Surface Brightness (Foot-candles).	Illumination (Foot-candles).	Reflecting Power.
Drawing Room	Light Blue .	0·3	0·72	40 %
Hall . .	Dark Red .	0·15	0·6	25 %
Library . .	Deep Green .	0·15	1·0	15 %
Dining Room .	Very Deep Blue	0·015	0·35	4·5 %

It may be mentioned in passing, that as a result of experiments in many different schools, libraries and private houses, a surface-brightness of 0·3 foot-candle seems to be about what is usually needed to give a room a cheerful appearance. In some cases, however (*e. g.* in a library), a more subdued effect is preferred, and walls with a surface-brightness of only 0·1 foot-candle may be found.

The reflecting power of wall-papers has frequently a not inconsiderable effect on the conditions of illumination in rooms lighted by artificial means, and a series of experiments was recently carried out in a small room equipped with various wall-papers in order to ascertain in what respects calculations based on the use of holophane reflectors and tungsten lamps would have to be modified by this factor.* It was found that approximate rules could very readily be derived.

It may be observed that the reflecting power of coloured materials depends to some extent on the quality of light by which they are illuminated, and the same results will not be secured by natural and artificial light. This fact has been recently utilized by Mr. T. E. Ritchie,† of the Union Electric Co., as a means of testing the resemblance of various artificial illuminants to daylight. A series of ribbons of delicate shades of colour was prepared, and the reflecting power of the series tested with the holophane lumeter by good average daylight. The tests were subsequently repeated with a series of artificial illuminants, and it was found that the closest approximation to the daylight values occurred in the case of indirect arc lighting.

One other quality of reflecting surfaces which is not only of consequence to the lighting engineer, but plays a part in many physical problems is their "matness." An interesting illustration of the occasional advantage to be derived from

* *Illum. Eng.*, vol. iv, p. 361. (Lond. 1911.)

† "Colour Discrimination by Artificial Light," *Illum. Eng.* (Lond. Feb. 1912.)

using semi-polished surfaces is afforded by the silvered or aluminium powdered screens sometimes used for cinematograph screens. When such screens are used, the majority of the light is reflected directly back to the audience, so that the method is only applicable in fairly long and narrow rooms. From some tests which one of the authors recently carried out on materials of this kind, it would appear that the brightness of the image on a cinematographic screen (usually of the order of 1 foot-candle) might be increased as much as ten times under favourable circumstances.

It need not be said that the same appliances which serve for the measurement of reflecting power can also be used to measure the absorption of glasses, gelatines and solutions. This is a point of interest to the optical industry, for it is well known that much remains to be done in the matter of producing glass of specified absorbing power. It is conceivable that the examination of the reflecting or absorbing power of precipitates and solutions would also be of service to the chemist and physicist; an interesting case has come to our notice in which the reflecting power of the powder as measured by the holophane lumeter was utilized as an index to the quality of a certain class of coal.

Let us now turn to another branch of research which has recently interested lighting engineers, namely the study of daylight. To people who are seeking to ascertain the ideal conditions of artificial illumination the study of daylight conditions is obviously important. Moreover, it must not be forgotten that the great part of the world's work is still done by natural light. Indeed, in a great many schools artificial light is used so seldom as to be comparatively unimportant. A great many experiments have recently been made, notably by Prof. L. Weber of Kiel, with a view to testing the efficacy of the old regulations affecting window-space. Photometry is proving a most valuable tool in attacking these problems. It enables us to discriminate between good and bad natural illumination, and to recognize the distinctions from artificial light. A difficulty previously experienced in these instruments, however, was that daylight illumination was too high for convenient measurement. It was partly on this account that the authors decided to increase the range of the holophane lumeter up to 2000 foot-candles—a value which should surely answer for all practical purposes. Mr. P. J. Waldram has recently proved photometry to be of considerable value in deciding ancient-light cases and has suggested methods of obtaining a factor, independent of climatic conditions, and affected only by the nature and construction of the building, connecting the unrestricted illumination outside with the illumination in the room.

It may be suggested that photometry might often be applied by meteorologists and others interested in climatic variations. Many of the investigations of the variations in daylight from day to day, which are now tolerably complete, have been carried out by men like Weber of Kiel and Basquin of Chicago with reference to daylight illumination in schools. Such researches are also interesting to the meteorologist and the photographer. In addition to measurements of illuminations, there is no difficulty with our instrument in studying the surface-brightness of external

objects and even (when the sky is not too bright) making observations on clouds. The brightness of the sky, however, frequently exceeds our present limit of 2000 foot-candles, and the same may be said of the illumination of a white surface by direct sunlight in summer. For this class of work accessory apparatus is really needed. There is no inherent difficulty in measuring the very low orders of brightness such as are met with by night, and the authors have suggested at a recent meeting of the Royal Astronomical Association that simple photometric methods might readily be applied to obtain useful information regarding the brightness of the night sky, comets and stars, etc., with much greater precision than at present.

As an illustration of the use of the instrument in studying daylight conditions it occurred to the authors that it would be interesting to take measurements of the illumination during the recent eclipse of the sun on April 17. The measurements were made indoors in shadow, a position being selected where the initial illumination was of the order of 100 foot-candles. Photographs of the solar disc were taken simultaneously with the measurements. It is interesting to notice that the illumination at totality at about 12.10 p.m. is about 8 per cent. of the illumination when the eclipse commenced, almost exactly what the percentage area of the disc obscured (92·7 per cent.) would suggest. The upper curve shows the corresponding illumination at this time of day under normal conditions. It was taken two days alter, on Friday April 19, under a fairly cloudless sky. One would suppose that records of this kind would be of considerable service to astronomers. The process is extremely simple and could be applied in exactly the same way to an eclipse of the moon.

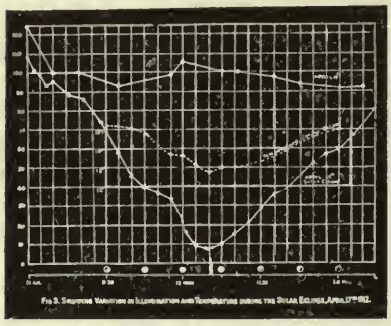


FIG. 3.

By the courtesy of Mr. A. P. Trotter we are also able to reproduce some data showing the interesting fall in temperature that occurred during the eclipse. This experiment was carried out at the Board of Trade Laboratory by Mr. P. Webberley with an ordinary thermometer placed in direct sunlight. It is curious to observe how closely the course of this curve (shown dotted in the above diagram) agrees with that obtained for the illumination. The reason for the peculiar kink in both curves which apparently occurred about 11.50 in both cases is not quite clear.

There remains one very interesting application of measurements of surface-brightness which deserves mention. In April 1911 the authors read a paper before the Royal Photographic Society in which the value of such data in connection with photography was pointed out. It is surely of interest to photographers to study the variations in the brightness of objects both by natural and artificial light, and the corresponding allowance that should be made in determining the exposure. In

the paper referred to a variety of data on this subject was presented, and it was pointed out that, by the aid of the holophane lumeter, information about the actual brightness of objects in the field could be very quickly and readily obtained in many cases in which an actinometer could only be applied with difficulty. A further advantage of the method is that one can study the brightness of the actual things to be photographed even though they may be distant and inaccessible. The fluctuations in daylight are so enormous, and the range of tone in such subjects as clouds, landscapes and the interiors of churches so extreme, that it must often require long experience and skilled judgment to get a successful result. From their experience the authors are disposed to believe that the number of failures of the inexpert photographer might be considerably diminished if he were in the habit of working on the basis of measurements of surface brightness, and in many crucial cases even professional photographers might find the method of benefit. This would be particularly true of subjects in which the tone of the various objects to be brought out differs considerably, and delicate gradations of shadow have to be revealed. In many such cases a misjudgment of exposure might not prevent the picture as a whole being presentable, but it might, nevertheless, result in the complete falsification of certain tones.

In the case of artificial light the difficulty of judging the exposure is often considerable and the actinometer cannot readily be applied. The authors' interest in this application of the holophane lumeter was aroused by the difficulty experienced in getting successful photographs of artificially lighted interiors for purposes of illuminating engineering. In many such cases it is desired not only to make a pretty picture, but to reproduce the play of light and shadow and the distribution of illumination with exactitude, and to show the actual sources of light without halation. In many cases, too, retouching is not permissible.

In work of this kind photometry and photography go hand in hand, the same instrument conveniently serving both to take measurements of illumination from an engineering standpoint and as a means of judging the exposure. The matter is of considerable importance to the various companies dealing with illumination which now make a practice of taking photographs of installations exclusively by their own light. The experience of the authors on this point has also been borne out by Mr. T. E. Ritchie, in connection with photographs taken by the light of the inverted arc.

It may be urged, however, that the value of measurements of surface brightness in photography is especially great in research, for example, in studying the compliplicated questions connected with the appearance of coloured objects to the eye and their reproduction on the photographic plate. Measurements of surface-brightness might also be a valuable adjunct to the instructor as a means of illustrating the effect of varying brilliancy and exposure. There are also many technical photographic processes in connection with which such measurements would be useful, such as lithography, the grading of plates, and in making measurements of reflecting power and opacity.

A New Apparatus for studying the Distribution of Light

During the last few years it has come to be generally recognized that it is not sufficient merely to state the intensity of a source in a certain direction, or even to give its mean spherical candle-power. It is also most important to show how the light is distributed, especially as the natural power curve of light distribution from an illuminant can now be modified very greatly by the use of suitable globes and reflectors. This tendency is illustrated by the great variety of holophane prismatic globes and reflectors which enable the distribution of light from a lamp to be entirely altered—either a " spreading " or a " concentrated " effect being obtained, as local conditions demand.

Progress has recently been made in the design of a convenient apparatus for measuring the mean spherical candle-power direct, such as the Ulbricht globe, the Matthews integrating photometer, etc. Two years ago a further simplification was suggested by Dr. Sumpner,* who found that sufficiently accurate results for practical purposes could be obtained by the use of a cubical box with a whitened interior instead of an Ulbricht sphere.

The Ulbricht sphere appears to have proved its practical utility for many commercial light sources. Yet the experience of the authors leads them to doubt whether it would give sufficiently accurate results in the case of sources equipped with a highly concentrating reflector. But in any case, as explained above, it is desirable to furnish a polar curve of an illuminant as well as its mean spherical candle-power.

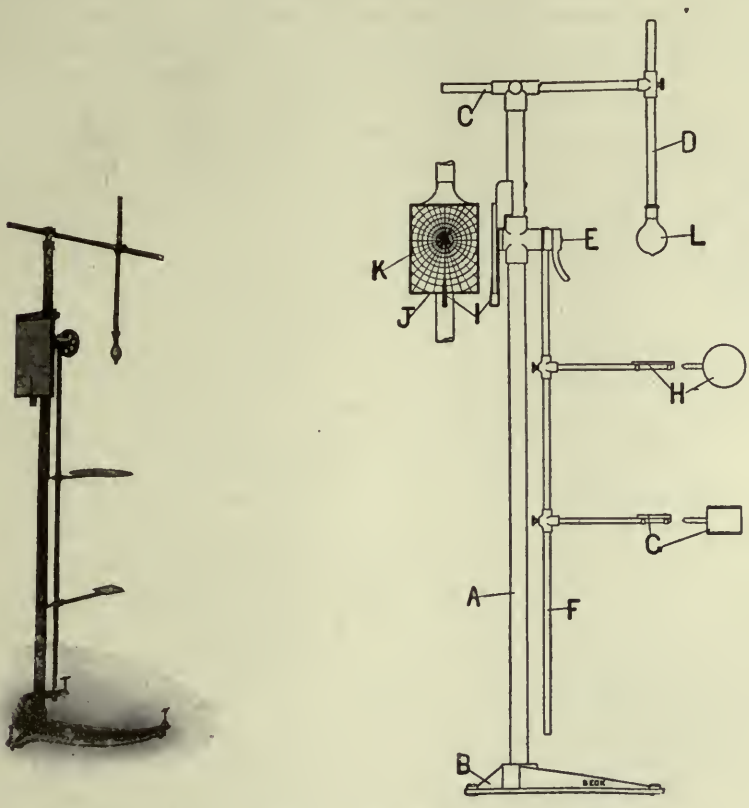
The determination of curves of this kind, and the somewhat elaborate calculations by which the mean spherical candle-power is determined, form possibly the most laborious part of commercial photometry. A great deal of ingenuity has recently been expended on the simplification of these calculations. But there seems room for more to be done in the direction of facilitating the experimental part of the work, and the authors have recently devised an apparatus which seems to have merits in this respect. It may be mentioned that a similar method has been worked out by Dr. Clayton Sharp, and was described in a lecture before the Johns Hopkins University, Baltimore, U.S.A., in 1910. The device employed by the authors was, however, worked out independently as a result of experience with the holophane lumeter. At the time of his paper, before the Illuminating Engineering Society, on " The Effect of Wall-papers on Illumination," in April 1911, Mr. Waldram, at the authors' suggestions, applied the holophane lumeter for the purpose of obtaining approximate polar curves of light distribution from a lamp actually in position in a room. The form of apparatus devised by the authors has been developed more elaborately upon somewhat similar principles.

The apparatus is shown diagrammatically in Fig. 4. Fig. 5 is from a photograph illustrating its general appearance. It is mounted on a substantial base, B; the lamp to be tested, L, being attached to an adjustable rod, D, which can be moved

* *Illum. Eng.* (Lond. May 1910.)

up and down or to and fro. When more powerful lamps are to be tested, it may be desirable to remove this arm (which can be done by means of the sliding piece, C, and to hang the lamp from the ceiling.

In any case the lamp is brought central with the point, E (corresponding with



FIGS. 4 AND 5.—New form of Polar Curve Apparatus.

A, main support; B, pedestal; C, adjustable horizontal support; D, adjustable vertical rod carrying lamp to be tested; E, pivot about which F rotates; F, rotating rod, carrying celluloid screen; G, celluloid screen; H, screen for cutting off direct light; I, pointer; J, plate on which Polar Curve Paper is placed; K, Polar Curve Paper; L, lamp to be tested.

the centre of the polar curve paper shown at K). The rod, F, rotates about E, and carries the standard celluloid surface, G, the distance of which from the lamp tested can be adjudged. As F rotates, therefore, the face of G is always presented vertically towards the lamp, L, and its brightness will be proportional to the candle-power of the lamp in that direction. There is also a pointer, I, rigidly connected with F, which rotates with it and indicates on the sheet of polar curve paper the angle with

the vertical which F takes up. It may be added that this paper is arranged to be a convenient height (about 5 ft.) from the ground.

The operation of testing the lamp is as follows : A lamp of known candle-power is first placed in position. The standard surface, G, is observed through the holophane lumeter, and its distance on the rod is adjusted until the reading of the instrument is some convenient submultiple of the known candle-power in that direction. (For example, if the candle-power were known to be 16, the reading on the instrument might conveniently be 1·6 foot-candles.) G is then clamped in position, and the lamp to be tested is constituted for the standard lamp.

The arm, F, is then rotated, the brightness of G being noted in each case, and the corresponding candle-power marked on the polar curve paper opposite the pointer. The curve is thus traced out at once, while the experiment is in progress, and there is no need for subsequent calculations.

When great accuracy is not necessary it might even be possible to use this apparatus even in rooms which have not black walls, and in which there is a certain amount of stray light. Under these conditions we make use of H, which is an adjustable black screen, and which can be inserted in any convenient position between the lamp and G. If, therefore, there is any question of the readings being affected by stray light, a reading is taken first without H in position. The brightness of G is then proportional to the illumination due to the lamp tested plus the stray light (if any). The screen, H, is then inserted, blocking out the direct light from the lamp tested. The brightness of G is now due to stray light only. Hence, by subtracting the latter reading from the former, we get the true value, *i. e.* reading without dark screen—reading with dark screen = true candle-power.

The advantages of this method may be summarized as follow—

1. The quickness and convenience with which the results are obtained.

2. The polar curve is worked out automatically while the experiment is in progress; there is no calculation necessary.

3. The apparatus is simple and portable, and can readily be moved from room to room, which an elaborate photometric bench, equipped with heavy and large mirrors, cannot.

4. Only one observer is needed for the photometric manipulations, and he can make all the observations in one position; there is no necessity to be continually crossing the room to adjust mirrors, etc.

5. By the use of the special screens provided, the apparatus can be used in an emergency in an ordinary room without darkened walls, and allowance can be made for stray light as described above. It is expected that the apparatus will be most useful for lamps of moderate candle-power (although the fact of the holophane lumeter reading up to 2000 foot-candles obviously enables very powerful sources to be tested).

Another point in connection with this apparatus deserves mention. Experience has shown that in the case of a simple source of light it is possible to repeat the curves obtained by an ordinary bench method with extreme accuracy. In the case of focusing forms of reflectors, however, one precaution is necessary, *i. e.* to

arrange for the distance between the illuminated photometrical surface and the source of light to be the same in both cases. Those who have done much work in testing reflectors of this kind are aware that the results depend to some extent on the distance at which the tests are made. In the case of most simple light sources, such as a naked glow lamp, even though they may not be, strictly speaking, " point sources," practically identical results are obtained whatever the distance of testing, because the inverse square law applies with reasonable accuracy. But in the case of concentrating the reflectors the matter becomes more complicated. We are now dealing with light which consists partly in rays which emanate according to the inverse square law, and partly with light which is concentrated into a parallel or even a convergent beam It is, therefore, desirable to adopt some standard of distance of testing such as 10 or 12 ft., this being the nearest equivalent to the distance at which the light would be required as a rule in practice. For example, if the equivalent candle-power of a lamp is given as 100, it would mean that at a distance of 12 ft. one would receive the same illumination as would be obtained from an ordinary 100 candle-power source obeying the inverse square law at the same distance.

Fig. 6.

Now, in the case of apparatus of the type shown in Figs. 4 and 5, it is difficult to make the rotating rod long enough to secure a distance of 10 ft. This difficulty can, however, be readily obviated by replacing the celluloid sheet by an inclined mirror and directing the beam on to the test screen at any required distance. This method has the approval of Dr. Sharp, and his experience is substantially confirmed by that of the authors When testing a highly focusing reflector, and sometimes also in dealing with very powerful illuminants, this mirror method is preferable. Fig. 6 shows a photograph of the apparatus with the mirror in position.

The peculiar advantages of the combination of this form of polar curve apparatus with the holophane lumeter have been described above. The saving in time is often remarkable, owing to the facts that a reading with the holophane lumeter is obtained more quickly than one on a photometrical bench, and that the results are expressed direct in candle-power, without any complicated calculations. Not infrequently a complete curve has been obtained in from five to ten minutes.

The Holophane Portable Standard of Light

Some experiments have also been made on a new form of Portable Standard of Light, and Fig. 7 is a photograph of the first rough model constructed by Messrs. Beck. It consists essentially in a low-voltage tungsten lamp which is fed by a portable accumulator packed away in a convenient form of case. A mounted celluloid screen attached to a graduated metal tape is also provided in the case, and the screen can thus be set up any desired distance from the small tungsten lamp, which acts as a secondary standard, and any desired illumination produced. It is, of course, also possible to illuminate any other photometric surface by this lamp, and thus use the apparatus for checking comparison lamps used in photometry in the usual way.

The most novel part of the apparatus is the means employed of maintaining the voltage across the standard tungsten lamp constant. This is accomplished as follows. A small carbon filament lamp is run in parallel with the tungsten lamp and in series with both is placed a suitable adjustable rheostat. (This can be seen on the right at the back of the box in the illustration.) Between the two lamps is placed a specially designed form of photometric screen by the aid of which their intensities can be compared. The two photometric surfaces are inspected through an aperature on the side of the box furthest from the observer. The exact nature of this screen need not be described in detail. It is sufficient to say that the observer sees three slots. When the centre one is in balance that on the left hand is too bright and that on the right hand too dark; the side slots thus act as " gauge glasses." In order to facilitate accurate judgment a yellow glass may be placed in front of the aperture so as to make the colour of the light from the two lamps appear the same.

The action of this part of the apparatus depends on the fact that any change in the P.D. supplied by the cell affects the two lamps unequally, and therefore puts the screen out of balance. Suppose, for example, that the voltage of the cell falls by 1 per cent. The candle-power of the carbon-filament lamp will be reduced by 6 to 7 per cent. and that of the tungsten lamp by 3 to 4 per cent. This difference is immediately revealed on the photometric screen, and it will be observed that the resulting change in brightness is almost the same as that of the standard—so that the device is a very sensitive means of detecting any change in P.D.

When using the apparatus the operator first regulates the resistance in such a way that the metallic-filament lamp gives some convenient light, say one candle-power. (This is ascertained by comparison with a suitable standard in the laboratory.) He then alters the position of the comparison carbon-filament lamp until the photometric screen appears to be in balance and clamps the lamp in this position. The standard is now ready for use, and the position of the comparison lamp should not be altered until it eventually becomes necessary to set the standard once more.

FIG 7.

[*To face page* 84.

Should the voltage of the cell fall at any time this will affect the lamps unequally, and the changed conditions are made evident. By readjusting the resistance he can secure photometric balance once more and bring the standard back to its original candle-power. A similar adjustment can be made should the voltage of the cell be abnormally high owing to its having been recently charged. The standard will, therefore, always give the same light provided the lamps themselves do not alter. As, however, it is intended that this should be used as a " reference standard " only, so that it should be unnecessary to run the lamps for more than a minute or two at a time, such a change should only take place very gradually. Should the lamp prove, as a result of testing in a laboratory, to have diminished in candle-power the standard can readily be rearranged to give the same light as before by repeating the original adjustment. As an additional precaution it is proposed to provide two sets of lamps, each of which can be alternately brought into circuit, and which should give concordant results.

It will be observed that this portable standard acts as a temporary custodian of the standard of light in a somewhat similar way to the holophane lumeter, but over longer periods of time and with greater accuracy. It is first set from a suitable laboratory standard, and is always capable of readjustment to the correct value should it vary. Having once been set, however, it should be suitable for use for a long time, and owing to its portable nature can be readily carried about from place to place or sent away to be rechecked very easily. It should, therefore, be very useful to those who require a convenient and easily operated standard, and wish to avoid the necessity for installing a large number of standards at different places.

It is not, of course, intended that the apparatus can take the place of a carefully adjusted laboratory standard for very accurate work; but the accuracy should suffice for commercial purposes and for checking illumination photometers. This last requirement is one which will doubtless be met with more in the near future as measurements of illumination in schools, factories, etc., become more common. Experiments carried out with the rough model over several months, the instrument being checked every few days with special holophane lumeters, gave consistent results to within 6 to 7 per cent., and it is expected that, with the improvements introduced as result of this experience, the accuracy will be substantially improved.

In devising the forms of apparatus described in this paper the aim has been to secure convenience and simplification in ordinary practical work where extreme accuracy is not really necessary. At the present moment it seems better to make a small concession in this direction, so as to have apparatus available for the urgent demands of present illumination, rather than to postpone the popularization of measurements of illumination until laboratory conditions can be reproduced in practice. At the same time, bearing in mind the conditions under which a large amount of commercial testing is at present carried on, the authors are disposed to believe that the ordinary degree of accuracy even in laboratories frequently does not exceed that obtainable with the portable apparatus described. With the three

appliances, the lumeter, the polar curve apparatus and the portable standard of light, one should be able to carry out practically all the operations undertaken in the ordinary laboratory with the additional advantages that the work can be done quicker, that the apparatus is portable and can readily be removed from one room to another, and that illumination measurements can be made outside the laboratory.

Work in Technical Institutions and Colleges

These qualifications should be specially appreciated in technical institutions and colleges in which space is often limited, and it is often desirable to transfer experiments from one room to another. It may also be suggested that the changes which have come over the science of illumination during the last few years really call for a modification of the laboratory practice in photometry at technical institutions.

It may safely be said that the experiments carried on to-day are often somewhat out of touch with practice and are needlessly laborious to the student. Most of them are designed to prove the various laws of photometry; but so much time is often spent on the experiments, owing to the fact that the apparatus is old-fashioned and inconvenient and the methods of proof needlessly laborious, that the student rarely gets much beyond this stage. As an illustration we may mention the determination of polar curves and calculation of mean spherical candle-power. Not infrequently every value is calculated out from the bench reading on the inverse square law, and the number of readings taken far exceeds that in practice. In a modern laboratory labour-saving devices avoid much of this routine work, and there is no reason why the student, once he has mastered the fundamental principles, should not make use of these simplifications. Moreover the difficulty which students who are novices in this work find in reading the photometer is often augmented by the fact that primitive and insensitive screens, which would trouble even an expert, are employed. The psychological effect of this is unfortunate, because the student receives an unjustified impression of the difficulties and uncertainties of photometry.

So much time is often expended on the study of his tools that he does not arrive at the stage of using them. Yet, as a matter of fact, there are numberless important applications of photometry which are both useful and interesting as laboratory experiments. The ordinary tests on lamps should now be supplemented by experiments on the latest forms of shades and reflectors, data should be ascertained as to the comparative reflecting power of various surfaces, the effect of wall-papers in rooms on the illumination, the absorption of glasses, etc. The student should also be familiarized with practical measurements of illumination, and should make experiments in rooms lighted by various forms of fittings. The newest types of semi-indirect and indirect fixtures, for example, afford endless op ortunities for

intelligent study. A number of instances might doubtless be found of cases in which the measurement of light and illumination might prove a valuable adjunct in physical and chemical studies, while on the physiological side such questions as the effect of " glare," excessive contrast, and the formation of shadows would well repay investigation.

In conclusion, the author would like to emphasize again the fact that the promotion of these students demands the simplification and popularization of methods of measuring light and illumination; and in this paper the results of some experiments in this direction have been described. But much still remains to be done, and we may hope to see remarkable progress in this direction during the next few years.

Mr. A. P. Trotter said that several kinds of photometers had been invented for measuring surface brightness, but none had come into practical use until the authors had produced this excellent apparatus. They had not only produced a very portable instrument, but had done a large amount of practical work with it. The result was that it seemed likely that some branches of industrial photometry would be revolutionized.

The primary measurement in Photometry was that of candle-power. The next was that of illumination, with the unit of one foot-candle which, for all practical purposes, was equal to one candle-metre. The third was surface-brightness—not a good term—the unit of which was the brightness of a perfectly white surface illuminated with one foot-candle. The authors used a test-plate of white celluloid. This could be used without correction for measurement of illumination, but in expressing surface-brightness, the albedo of the test-plate should be allowed for. It was probably about 0·85. There was no advantage in trying to get nearer to unity; but measurements of surface brightness had no meaning unless they were referred to an albedo of unity, as well as to an arbitrary unit of illumination.

It was not surprising that others were adopting photometers for the measurement of surface brightness. Mr. Edgcumbe was showing one in the Exhibition, and he (Mr. Trotter) had been so impressed with the importance of measurements of surface brightness that he had designed an attachment to his portable photometer to enable such measurements to be made.

The new development in practical photometry which was now possible was the measurement of illumination in inaccessible places or on a plane, such, for example, as the front of a telephone switchboard, where an illumination photometer could not be placed.

Mr. R. S. Whipple referred to the point concerning the constancy of the light intensity of the portable standard of light. Waidner and Burgess had dealt with this problem in their examination of Holborn pyrometers. If the lamps were run for eight hours at an overload, and then always at an underload, the candle-power would not vary for years. That might be advisable in this case.

Mr. C. C. Paterson thought the standard exceedingly convenient, the second lamp taking the place of the moving-coil amperemeter. The Holophane Lumeter had successfully been tried on his suggestion in coal-mines.

Mr. Chalmers had also used the Holophane Lumeter for measurement in connexion with the obtaining of even illumination on vertical surfaces and had found it possible to make measurements very rapidly and sufficiently accurate for his purpose, an accuracy of from five to ten per cent. being obtained.

Prof. S. P. Thompson dwelt upon the importance of the instruments for legal disputes as to the amount of light; surveyors had no longer any excuse for being inaccurate now they were supplied with numerical data.

Dr. Schuster added that architects should likewise study the problem of the correct illumination for rooms; so far all they could tell the inquirer was that so many square feet of flooring required so many square feet of windows; but they could not give any explanation as to the distribution of the windows.

SOME ALLOYS SUITABLE FOR INSTRUMENT WORK

By W. Rosenhain, B.A., D.Sc., of the National Physical Laboratory

(*Monday, June 24th*, 1912.)

The metals used in the construction of instruments do not appear to have received the amount of careful attention which they deserve. The majority of instrument-makers content themselves with using any miscellaneous brass tubing or casting that may be available, while here and there an effort may be made to lighten instruments by the use of aluminium or some little-understood aluminium alloy. There seems to be no doubt that a study of the materials available and of their properties, with a careful mutual adaptation of the materials and design, would lead to considerable improvement in our instruments. The purposes served by different instruments, and even by different parts of the same instrument, vary so widely that the materials to be employed for them should be varied in a corresponding manner.

The wide variations of purpose which have just been referred to, make it impossible to lay down any general specification of the requirements which should be met by metals or alloys intended for instrument work, but there are certain properties which are obviously desirable, and these may briefly be referred to. We have first of all strength and stiffness. It is true that in a good many cases the stresses which come upon the parts of an instrument are so slight that the thinnest sections which it is possible to employ are of ample strength, even when the metals themselves are comparatively weak, but in other cases the requirements, as regards strength and rigidity, are much more serious, and in these cases it becomes desirable to study the tensile strength and elastic modulus of the materials to be employed. As regards elastic modulus, which governs the stiffness of a given section of material, it must be remembered that steel stands far higher than any non-ferrous alloy yet discovered; in fact, the strongest of the aluminium-bronzes, whose strength exceeds that of the softer kinds of steel, have an elastic modulus rather less than one-half that of steel, while the light alloys of aluminium have a modulus not much more than one-third that of steel. On the other hand, it is to be remembered that for *equal weight* the stiffness of sections made of certain light alloys may be greater than that of steel.

The second universal requirement for the materials for instrument-making is that it should be possible to work them either hot or cold, and that they should be capable of being machined and, in some cases, of being polished. Where alloys are required only in the form of castings it may be permissible to use a material which would be too hard and brittle to allow of rolling into rods or drawing into tubes, while for polishing purposes, again, the harder metals are preferable. The property of being easily machined, particularly at high speeds and with good finish, is, however, a peculiar one and must be ascertained specially for each alloy.

The durability of the materials is also of considerable importance. In this connection we have to consider first of all what might be termed "intrinsic durability," *i. e.* whether the metal is free from internal sources of disruption. Such internal sources of disruption may be of several kinds; they may be due to the composition and constitution of the alloy. Some alloys are known, although they are never intentionally used, which undergo spontaneous disintegration in some cases in a few hours and in other cases after a period of years. Apart from experiments, however, on materials which have been insufficiently studied the risk of encountering this class of phenomenon is negligible. On the other hand, the treatment which is applied to some alloys is such as to leave within them the seeds of subsequent disintegration. Thus excessively hard-drawn wire or rod, and particularly thin stampings which have undergone very severe local deformation without adequate annealing are liable to undergo what is known as "season cracking." This phenomenon appears to be due to the fact that the severely distorted crystals of such a metal tend to rearrange themselves in more stable forms. This is what normally occurs when metal is softened by heating it (annealing), but when the cold work has been

very severe a gradual rearrangement may occur even at the ordinary temperature, and as such rearrangement is generally accompanied by a small change in volume, splitting or cracking may result. This is one reason why it is in itself undesirable to use metals whose strength has been artificially raised by such processes. To a certain extent such treatment is, of course, permissible—and in some cases even desirable; but it is often carried too far, and this practice may be ascribed to the fact that the material itself is inherently too weak properly to serve the purpose required. It would be preferable to use an inherently stronger alloy rather than to rely upon mechanical hardening.

Apart from these internal causes of possible disruption the materials of instruments depend for their durability also upon the power of resisting external causes of deterioration. Such external causes are principally mechanical wear and corrosion. Mechanical wear is, of course, confined to those parts of the instrument where movement occurs, but where these movements are of a delicate kind, and a prolonged maintenance of adjustment is necessary, resistance to wear becomes an extremely important property. There can be no doubt that for this purpose many of the alloys ordinarily used are decidedly unsuitable. A soft brass, chosen because of the ease with which it can be machined and polished, is far too soft for making the slides and the rack and pinion gears of a microscope, and similar considerations apply in many other cases.

As regards corrosion, in the majority of instruments, and particularly those which are used indoors or in otherwise protected situations, very little difficulty need arise. The majority of alloys can be adequately protected by a good lacquer, and where lacquer is insufficient, paint, enamel, or other protective coatings can be used. On the other hand, where instruments are exposed to severe conditions, such as the spray of sea-water, wind and weather—and, still worse, the fumes of many chemical and industrial operations—deterioration by corrosion becomes a very serious matter. It must, however, be admitted at once that there are no known metals other than the noble metals which can successfully resist the severest of these influences. As regards sea-water, the use of some of the alloys mentioned below if conducted in a rational manner would probably prove distinctly beneficial. In such cases, however, it is necessary so to design the instrument that no electrolytic action can be set up between different parts, and for that purpose it is necessary that all metallic materials should consist of the same alloy. Even the juxtaposition of a drawn tube and a casting of the same material will give rise to slight galvanic effects, although these are much smaller than the electric currents which are generated by the juxtaposition in contact with sea-water of two alloys of different composition. It should be remarked that in this respect exposure to sea air and spray is rather more severe than actual exposure to sea-water.

In the more severe cases which occur with instruments intended for use in works and in chemical laboratories the avoidance of corrosion could only be effected by rigorously covering all metallic parts with carefully applied and adequately thick

layers of protective material. The proper choice of these coatings is a matter requiring considerable care, but it lies outside the scope of the present paper. It must be emphasized, however, that in such instruments if real utility is aimed at the whole idea of " prettiness " must be abandoned, there must be no display of bright metal at all, and in many cases it will be eminently desirable if all the metallic parts of the instrument are enclosed in an air-tight box with wood or ebonite handles protruding, while the indicating dial of the instrument is observed through a glass window. The prejudice against instruments which is still widespread in industrial circles is largely based on the idea that they are " delicate," and that they will not stand the usage of industrial life. As a matter of fact the cause of failure or derangement is in most cases due to corrosion rather than to mechanical breakage.

A further property of metals which is important from many points of view in instrument-making is their weight. The demand for lightness is in certain cases imperative, while in many cases a reduction in weight would be very desirable. In view of this fact, it is rather surprising to find that the use of aluminium and its alloys is not far more widespread among instrument-makers than it appears to be. Some of the causes which may have retarded the use of light metals will be discussed below.

It is not the purpose of the present paper to discuss in detail the properties of the materials most commonly used in instrument-making, namely, brass and gun-metal. Anything like a full discussion of these two groups of alloys would in itself form subject-matter for several papers longer than the present one. In the present instance, however, the author merely desires to draw the attention of instrument-makers to the fact that certain groups of alloys not ordinarily employed by them have recently received very careful and thorough investigation, and that some of them appear to possess properties which would render them of special value and interest for certain purposes in instrument-making. For a full account of these alloys and for the investigations which have been carried out upon them, reference must be made to the Eighth, Ninth, and Tenth Reports of the Alloys Research Committee of the Institution of Mechanical Engineers.* It will be found, however, that these Reports, written chiefly from the point of view of the metallurgist and engineer, and covering in all some 785 pages of the *Proceedings* of the Institution of Mechanical Engineers, present a rather formidable front to the instrument-maker in search of information, and for that purpose the author has thought it desirable to abstract

* Eighth Report to the Alloys Research Committee of the Institution of Mechanical Engineers on the Properties of the Alloys of Aluminium and Copper, by H. C. H. Carpenter, M.A., Ph.D., and C. A. Edwards. (1907.)

Ninth Report to the Alloys Research Committee of the Institution of Mechanical Engineers on the Properties of some Alloys of Copper, Aluminium and Manganese, by W. Rosenhain, B.A., D.Sc., and F. C. A. H. Lantsberry. (1910.)

Tenth Report to the Alloys Research Committee of the Institution of Mechanical Engineers on the Alloys of Aluminium and Zinc, by W. Rosenhain, B.A., D.Sc., and S. L. Archbutt. (1912.)

from those Reports data concerning a few materials which appear to be of interest in the present connection.

The first group of alloys to be considered may be described as belonging to the class of bronze: that is to say, they are heavy alloys of considerable strength. Their properties so far as strength and ductility are concerned are summarized in Table I. In the first column of this Table are given the reference numbers of the alloys. The Roman numerals refer to the Alloys Research Report in which they are described, the other number being that by which each alloy is identified in the Report which deals with it. By means of these numbers those interested can readily refer back to the original Reports for fuller details concerning any of these alloys. The second column gives the percentage composition of the alloys, impurities being neglected.

The first six alloys of the Table consist of copper and aluminium alone, while the last three also contain from 1 to 3 per cent. of manganese. In the next column of the Table is stated the condition in which the alloy has been tested. The term "Rolled Bar" refers to material which has been hot-rolled to a diameter of $\frac{13}{16}$ in. or $\frac{7}{8}$ in., starting from a billet 3 ins. in diameter, while the material described as "Cold-drawn Bar" has been drawn cold from $1\frac{1}{4}$ in. diameter down to $\frac{13}{16}$ in. or $\frac{7}{8}$ in. diameter.

On examining the figures in the Table it will be seen that as regards Sand Castings alloy No. IX. 2 attains a strength of over 35 tons per sq. in., while in the Chill Castings the same alloy attains 39·9 tons, both these figures being accompanied by an elongation of well over 20 per cent. If greater ductility is required alloy No. VIII. 12 provides a sand casting with a tensile strength of just over 30 tons, and 36 per cent. elongation, while such an alloy as No. VIII. 9 will furnish a sand casting with a tensile strength well over 20 tons per sq. in. and an elongation exceeding 70 per cent. With these figures we may compare the data for ordinary brass in which the tensile strength of sand castings never exceeds 19 tons per sq. in., and this figure is very rarely attained in commercial brass. In the form of castings, therefore, the alloys shown in Table I present possibilities of strength far in excess of anything that can be obtained in brass, but it must be admitted that for this very purpose they present some special difficulties. If the attempt were made to melt and cast them, by the methods usually employed in brass-founding, failure would result, owing to the fact that, when molten, these alloys are considerably more sluggish than brass, and their contraction in freezing is decidedly larger. Owing to these difficulties it is sometimes stated that it is impossible to obtain satisfactory castings of these alloys, but this statement is incorrect, as the author has found that a little care and some experience of the peculiarities of these metals makes it quite possible to obtain perfectly sound results—at all events in castings of moderate size; although the cost is somewhat increased by the necessity of using comparatively large "gates."

In the form of rolled bar it will be seen that these alloys attain a tensile strength of over 40 tons, coupled in the case of alloy No. IX. 6 with an elongation of 39 per cent.

TABLE I

Mechanical Properties of Alloys of Copper with Aluminium (containing up to 10 % *of Aluminium)*

Reference Number.	Composition, per cent. Copper.	Aluminium.	Manganese.	Condition.	Strength. Yield Point. (Tons per square inch.)	Breaking Stress. (Tons per square inch.)	Ductility. Elongation per cent. on 2 inches.
VIII. 7.	94·24%	5·76%	—	Sand Casting	4·8	17·8	67·0
,,	,,	,,	—	Chill Casting	6·0	18·8	61·0
,,	,,	,,	—	Rolled Bar	11·8	28·4	74·2
VIII. 8.	93·27%	6·73%	—	Sand Casting	4·8	18·65	—
,,	,,	,,	—	Chill Casting	—	19·96	69·0
,,	,,	,,	—	Rolled Bar	10·4	28·85	71·0
VIII. 9.	92·65%	7·35%	—	Sand Casting	6·6	21·3	71·0
,,	,,	,,	—	Chill Casting	—	21·58	84·0
,,	,,	,,	—	Rolled Bar	10·6	29·68	72·5
,,	,,	,,	—	Cold-drawn Bar	24·6	31·42	48·0
VIII. 11.	91·33%	8·67%	—	Sand Casting	9·8	28·1	48·0
,,	,,	,,	—	Chill Casting	—	30·8	55·0
,,	,,	,,	—	Rolled Bar	11·1	36·7	38·0
VIII. 12.	90·62%	9·38%	—	Sand Casting	9·7	30·38	36·25
,,	,,	,,	—	Chill Casting	10·5	33·98	43·50
,,	,,	,,	—	Rolled Bar	17·7	38·00	34·00
,,	,,	,,	—	Cold-drawn Bar	29·0	38·83	32·00
VIII. 13.	90·10%	9·90%	—	Sand Casting	11·3	31·70	21·7
,,	,,	,,	—	Chill Casting	12·4	36·93	30·5
,,	,,	,,	—	Rolled Bar	14·8	38·10	28·8
,,	,,	,,	—	Cold-drawn Bar	40·4	43·94	13·0
IX. 2.	89·06%	10·02%	0·92%	Sand Casting	14·00	35·76	22·5
,,	,,	,,	—	Chill Casting	16·00	39·90	25·0
,,	,,	,,	—	Rolled Bar	23·04	42·84	22·5
,,	,,	,,	—	Cold-drawn Bar	42·00	50·04	16·0
IX. 3.	88·00%	9·99%	2·01%	Sand Casting	13·20	34·44	24·0
,,	,,	,,	,,	Chill Casting	16·80	37·00	25·0
,,	,,	,,	,,	Rolled Bar	21·40	40·72	29·0
,,	,,	,,	,,	Cold-drawn Bar	40·88	52·08	10·0
IX. 6.	88·11%	8·91%	2·98%	Sand Casting	10 80	31·60	24·0
,,	,,	,,	,,	Chill Casting	14·80	34·40	26·0
,,	,,	,,	,,	Rolled Bar	20·00	40·00	39·0
,,	,,	,,	,,	Cold-drawn Bar	38·92	44·00	22·0

In this form also the strength attained is far greater than anything reached by brass, and there is no difficulty in hot-rolling any of these alloys.

In the form of cold-drawn bars the alloy No. IX. 3 attains the astonishing strength of 52 tons per sq. in., while still showing an elongation of 10 per cent. It will, of course, be understood that a material of these properties cannot be drawn and handled in the same manner as the weaker and softer brasses, but the author's experience with these alloys suggests that there is no insurmountable difficulty in producing from them tubes and other sections that may be required.

Apart from the mere question of tensile strength and ductility these alloys present certain interesting features. In the first place it is important to note that those containing manganese, *i. e.* the last three in the Table, are very readily machined, working as easily and yielding as good a finish as ordinary brass to which lead has been added for the purpose of facilitating machining. An exception in this respect must, however, be made for the cold-drawn material which does not machine so readily.

Perhaps the most important feature of these alloys from the point of view of the instrument-maker is their very considerable resistance to abrasion. Abrasion tests on the last three alloys described in Table I have been made by subjecting discs of the alloys 1 in. in diameter, and $\frac{1}{4}$ in. thick to rolling abrasion in contact with a hardened steel roller $2\frac{3}{4}$ ins. in diameter under a load of 789 lb. After 107,000 revolutions the weight of material removed was found to be :—

Alloy IX. 2, 11·6 milligrammes.
Alloy IX. 3, 13·0 ,,
Alloy IX. 6, 10·0 ,,

For comparison similar tests were made on discs of rail and tool steels, and the results obtained were :—

Rail steel, 64·5 milligrammes.
Tool steel, 27·4 ,,

Taken in conjunction with the ductility of the alloys these results certainly indicate remarkable and valuable properties. Advantage has, indeed, already been taken of these properties for instrumental purposes. In a measuring machine, designed by Col. R. E. Crompton, C.B., two small steel balls were at first employed for the purpose of making contacts, but it was found that owing to the magnetic property of the steel the balls acquired small " whiskers " of magnetic particles, and these interfered with the accuracy of the measurements. The steel balls have since been replaced by small spheres of the alloy No. IX. 3, and the difficulty in question has disappeared entirely.

The alloys described in Table I are also remarkable for their power of resisting

corrosion, particularly the action of sea-water. Thus the alloys Nos. VIII. 9 and VIII. 13 respectively lost 0·00087 and 0·00081 lb. per sq. in. per month of 30 days, when completely immersed in the sea, while the loss in Naval Brass similarly exposed was 0·00124 and in Muntz Metal 0·00123 lb. respectively. The behaviour of the alloys IX. 2, 3 and 6 is still more favourable. Specimens of these alloys which have been immersed in the sea, and then exposed between wind and water for several years, show little more than a dulling of the surface.

The most serious objection to the use of these alloys is undoubtedly the difficulty of obtaining them commercially, as they are not manufactured so extensively as the older alloys of copper with zinc and tin, but once the value of these alloys is fully recognized there is little doubt that the enterprise of metallurgical manufacturers will meet any demands that may arise. This difficulty is one which applies to the introduction of every new material, and it is to be hoped that both instrument-makers and metallurgical manufacturers will rise above any inherent conservatism in such a matter.

The second group of alloys described in the Alloys Research Reports mentioned above which are of interest to instrument-makers, are the light alloys consisting principally of aluminium. The use of light metals in instrument-making appears at the present time to be unduly restricted. There is no doubt that here, as elsewhere, the excessive claims made for aluminium in the early days of that industry are much to blame in creating disappointment and subsequent prejudice. Not only were the claims advanced far beyond the truth, but the aluminium which was available in the early stages of the industry was far less pure, and in many ways far less satisfactory, than that which is now obtainable, while the recent very marked drop in the price of aluminium has also materially changed the situation. The time has, therefore, arrived, when the attention of instrument-makers should be seriously directed towards the materials now available, and particularly to the fact that many of the alloys possess properties far superior to those of aluminium itself. As regards the pure metal itself it undoubtedly possesses the fundamental advantage of lightness coupled with very great ductility and very high conductivity, both thermal and electric. On the other hand it is weak, and can only be rendered stiff enough for most practical purposes by cold work, and reasons have already been given why it is not desirable to rely upon cold work to provide the necessary strength in our materials. Further, pure aluminium is in some respects difficult to machine, particularly at high speeds, but it possesses the curious property that its qualities in this respect are improved by age. It is found that freshly cast aluminium is much more difficult to machine than the same material which has been allowed to rest for several months. Finally, aluminium itself possesses the unpleasant property of being very easily abraded; even when severely rubbed with the fingers it is apt to produce a slight metallic stain. Most of these disadvantages are absent from the best of the light alloys, but these again have certain difficulties to contend with.

TABLE II

Light Alloys Consisting Principally of Aluminium

Reference Number.	Composition. Al.	Cu.	Mn.	Zn.	Condition.	Strength. Yield Point. (Tons per sq. inch.)	Breaking Stress.	Ductility. Elongation per cent. on 2 inches.	Density.	Specific Tenacity.
VIII. 33	Pure Aluminium				Sand Casting	2·5	4·9	24·0	2·69	50·5
,,	,,				Chill Casting	2·3	5·22	37·0	2·71	53·4
,,	,,				Rolled Bar	6·5	7·20	30·5	2·71	73·6
,,	,,				Cold-drawn Bar	8·5	8·70	19·5	2·71	89·9
VIII. 37	96·24	3·76	—	—	Sand Casting	4·9	7·49	5·0	2·77	74·92
,,	,,	,,	—	—	Chill Casting	5·4	9·60	10·5	2·79	95·3
,,	,,	,,	—	—	Rolled Bar	11·6	17·00	21·0	2·79	168·8
,,	,,	,,	—	—	Cold-drawn Bar	18·5	20·00	7·5	2·79	198·6
IX. 11	96·17	2·89	0·94	—	Sand Casting	6·0	7·48	5·0	2·67	77·61
,,	,,	,,	,,	—	Chill Casting	7·27	12·05	13·5	2·74	121·7
,,	,,	,,	,,	—	Rolled Bar	12 80	16·50	15·0	2·79	163·9
X. 20	79·85	—	—	20·15	Sand Casting	10·00	13·07	0·7	3·06	118·2
,,	,,	—	—	,,	Chill Casting	7·80	13·82	4·0	3·08	123·6
,,	,,	—	—	,,	Rolled Bar	17·30	22·64	20·5	3·09	202·3
,,	,,	—	—	,,	Cold-drawn Bar	19·90	22·30	12·7	3·09	199·9
X. 25.3	72	3	—	25	Sand Casting	5·71	18·25	2·0	3·26	155·1
,,	,,	,,	—	,,	Chill Casting	5·63	20·22	3·5	3·27	171·3
,,	,,	,,	—	,,	Rolled Bar	21·76	30·91	16·75	3·29	260·3
,,	,,	,,	—	,,	Cold-drawn Bar	31·50	33·30	13·00	3·29	279

In the first place, a very considerable number of light alloys have from time to time been placed on the market with little more than a fanciful name to recommend them. While some of the commercially advertised light alloys are no doubt very useful, others are very much the reverse, being put together without any scientific study, while claims are advanced for them with little regard for the truth. As regards all such alloys it is desirable to ascertain whether they really possess the strength claimed for them, how they behave as regards corrosion, and whether they are subject to spontaneous disintegration or warping. Generally speaking the author would advise instrument-makers to confine themselves either to alloys already well known to them from practical experience or to such new alloys as have had full and careful scientific investigation. It is not suggested that the alloys described below are in any sense the only ones to which this description applies; they are merely quoted here as examples with which the author is familiar from his own investigations

The composition and properties of the light alloys, described in the three Reports of the Alloys Research Committee referred to above, are given in Table II, this Table being arranged in the same manner as Table I.

For the sake of comparison, under the reference number VIII. 33 the data for pure aluminium have been included. It will be seen that even in the cold-drawn condition it falls short of a tensile strength of 9 tons per sq. in. Alloy No. VIII. 37 represents the most favourable member of the series of alloys of aluminium with copper alone. In the form of rolled bar it attains a strength of 17 tons per sq. in. with an elongation of 21 per cent. In the cast state also this alloy is far superior to pure aluminium. An alloy of very similar properties is No. IX. 11, which contains approximately 3 per cent. of copper and 1 per cent. of manganese. In itself this alloy is not very much superior, as regards mechanical properties, to No. VIII. 37, but it possesses two points of special interest. In the first place the presence of manganese appears to reduce the tendency of these alloys to undergo corrosion in sea-water; at all events there is good reason to believe that IX. 11 is superior to pure aluminium in this respect. Machined specimens of this alloy on exposure to sea-water become coated with a dark-coloured patina, which appears to act to some extent as a protective covering. A further point of interest about this alloy is that it forms the basis from which has been derived the light alloy commercially known as Duralumin. This remarkable substance owes its superiority to IX. 11 to the presence in it of 0·5 per cent. of magnesium. The presence of this small proportion of magnesium appears to confer upon the alloy the property of hardening gradually after a particular kind of heat treatment, and as a result of this hardening process, while the ductility is slightly reduced the tensile strength is very nearly doubled.

Alloy No. X. 20 may be regarded as probably the best alloy of the aluminium-zinc series. Castings of this material attain a tensile strength of over 13 tons per sq. in., but show very little ductility. The rolled bar, on the other hand, attains a tensile strength of 26·6 tons combined with an elongation of over 20 per cent. It will be noticed that both this and the next following alloy, X. 25. 3, show specific gravities materially higher than the first three metals in the Table, and in the latter case particularly it might be urged that lightness has been too much sacrificed by the addition of such considerable proportions of zinc as 20 and 25 per cent. The figures in the last column of the Table, headed "Specific Tenacity," have been added in order to show the relation of strength and weight in a correct light. The specific tenacity as there tabulated is defined as the breaking stress in tons per square inch divided by the weight of a cubic inch in pounds. The figure may also be regarded as presenting the breaking load of a bar of such cross section that every inch-length of it weighs 1 lb. So far as strength is concerned this quantity may be regarded as a Figure of Merit for the alloy, and it will be seen that this is by far the lowest in pure aluminium, although this is the lightest material in the Table, and the maximum value of the specific tenacity is found in the alloy X. 25. 3. For the sake of comparison it may be stated that the specific tenacity of mild steel on the same

definition is 105. It should also be noted that when materials of different specific tenacities are compared, as regards their strength when used as beams, the comparison in favour of the material of higher specific tenacity is still more marked, since for a given weight of beam not only the areas of the flanges, but also the depth of the beam, is proportionately increased. The value of such an alloy as X. 25. 3 in the cast form where no particular ductility is necessary is obvious from the figures given in the Table, and it may be stated that no difficulty attaches to the casting of this alloy. The process of rolling it into bars, however, entails a certain amount of difficulty which has yet to be overcome on the commercial scale, but in view of the high ductility which the alloy develops when rolled hot it appears probable that means could be found for producing in this metal both tubes and other desired sections.

As regards corrosion, there is no doubt that both aluminium and its alloys are not as resistant as might be desired. Certain of the alloys, as has already been indicated, are slightly superior to the pure metal, but the great majority of them are slightly more corrodible. In most cases, however, and certainly in all those described in Table II, this corrosion is not very marked so far as exposure to the air is concerned. To obtain this result, however, it is essential that any zinc which may be used in the manufacture of the alloys shall be of the purest kind obtainable. The presence of impurities, and particularly of lead, tends to increase the rate of corrosion in these alloys very materially. On the whole it may be taken that aluminium and its alloys are not more corrodible than iron or steel; and particularly where their surfaces are not subject to friction, and the protective coating is allowed to form itself, the corrodibility is very much smaller than that of iron. None the less, in the case of instruments exposed to adverse conditions, the protection of light alloy surfaces against corrosion is eminently desirable. For this purpose most of the ordinary protective coverings may be used, but it is an essential condition to the successful use of these materials that any source of electrolytic action should be carefully avoided. This implies that light alloys must not be brought into contact with other metals, particularly with copper and brass, in circumstances where the presence of moisture will facilitate electrolytic action. Where such action is set up extremely rapid corrosion of the aluminium alloy will result; but a little care on the part of the designer or constructer of the instrument will suffice to avoid this source of deterioration.

The above brief account of some of the properties of a few special alloys, which have been investigated at the National Physical Laboratory, has been given in the hope of arousing in instrument-makers an interest in the fresh developments which are taking place in this country and abroad in the production of new and valuable materials on the basis of our increasing knowledge of alloys and their properties. It is hoped that if such interest be aroused important results may follow, for not only may many existing instruments be improved by the use of superior materials, but other and new problems in instrument-making may prove to be capable of solution by their aid.

Dr. Benedicks (of Stockholm) remarked that too little attention was often paid to strength in instrument-making. Microscopes, which were practically built of one material only, should have the strength of lathes. He would ask Dr. Rosenhain which of his alloys he considered the most suitable for hydrographic instruments to be used in sea-water.

Dr. Rosenhain replied that the IX_3 alloy might answer, provided it could be drawn into suitable forms.

A NEW SPECTRAL COLOUR-MIXING APPARATUS

By F. W. Edridge-Green, M.D., F.R.C.S., Beit Memorial Research Fellow

(*Monday, June 24th,* 1912.)

In this apparatus there are two systems, each consisting of three slits (movable), a common collimating lens and a train of three prisms.

The last prisms of each system touch by one edge. There are, therefore, two parts, each of which may be occupied by a pure or mixed colour. These two parts are viewed by means of a single telescope, which has a division from the junction of the last two prisms to the objective, and from the objective to a gate (or square stop) at its focal point. This gate is also at the focal point of a modified Ramsden Eye-piece, and its outer edges are adjustable. By this means the field is divided into two completely isolated parts by the thin division plate. Each part may be illuminated by a pure colour, or any mixture of three colours.

In order to prevent stray light on the slit side of the prisms, the sources are enclosed in tubes having the slit at one extremity. One tube is directed towards the dispersion prisms, while the others lie at right angles, small total reflecting prisms directing the light to the dispersion prisms. By this means it is possible to have all three slits quite close together in one spectral region. The light sources also slide along their tubes, so that the intensity of the light of any one slit may be adjusted.

I am indebted for many of the details to Mr. H. S. Ryland, designer for the makers, Messrs. Reiner & Keeler, 9 Vere Street, W.

Dr. Edridge-Green remarked that there was scarcely any phenomenon of colour vision which the Young-Helmholtz theory explained satisfactorily, and no one had even attempted to answer his objections. Even the facts of colour-mixing

were not satisfactorily explained by this theory, as a considerable amount of one colour could be added to another without changing the colour of the latter. Yellow was a simple sensation, not compounded of red and green sensations, though red and green mixed made yellow. The proof of this was that the eye could be fatigued for red without changing yellow to green, or be fatigued to green without changing yellow to red.

Dr. Edridge-Green in referring to Mr. Blakesley's paper, agreed that the spectroscope should undoubtedly be used for testing colour-blindness, but not by colour-mixing. He had quite recently examined a man who had passed the ordinary Board of Trade tests, and had avoided all confusion of colours; yet in his (E.-G.) spectrometer the man had seen only purple and red and not distinguished any colour between them.

Dr. C. E. K. Mees thought that colour filters might be used in testing the eye as to the power of distinguishing faint signal lights; he would not recommend this for testing engine-drivers, however.

ON MIRROR SPECTROSCOPES AND SPECTROGRAPHS

By R. A. Houstoun, M.A., Ph.D., D.Sc.

(*Monday, June 24th,* 1912.)

All spectroscopes listed by makers and employed for ordinary laboratory use have lenses in their telescopes and collimators. Spectroscopes with concave mirrors have been employed so far only in research work in the infra-red. They are, however, also suitable for the visible and ultra-violet, and, with the purpose of calling attention to their advantages, I describe here two instruments that have been made and used * in the University of Glasgow.

The first of these instruments employs the Wadsworth mirror-prism combination, which is little known in this country, although frequently used in research work in America. The Wadsworth mirror-prism combination consists of a prism and mirror mounted together on the prism table, with the plane of the mirror and the plane that bisects the refracting angle of the prism both meeting in the axis of rotation of the prism table. The diagram represents a special case of this arrangement; ED and CA, the traces of the planes in question, meet in A, through which the axis of rotation of the prism table passes.

* "The Absorption of Light by Inorganic Salts," *Proc. Roy. Edin.* Nos. i–vi. (1911.)

Consider any ray, FGHJK, which passes through the prism at minimum deviation and is reflected by the mirror at J. Then JK is parallel to FG. The path of the ray through the prism GH is also parallel to the base of the prism, BD. From A draw AP, AN and AM respectively perpendicular to FG, JH and KJ. Then AN = AM by equal triangles and AN = AP by symmetry; consequently AP = AM. Suppose that the ray FG is white light; the colour in this ray that suffers minimum deviation emerges along JK after passing through the system. Now let the mirror-prism system be rotated through an angle about A, but let the ray FG remain fixed. Then AP and, consequently, AM remains fixed and the position of JK is unaltered by the rotation. But the colour of the light that emerges along JK is now different.

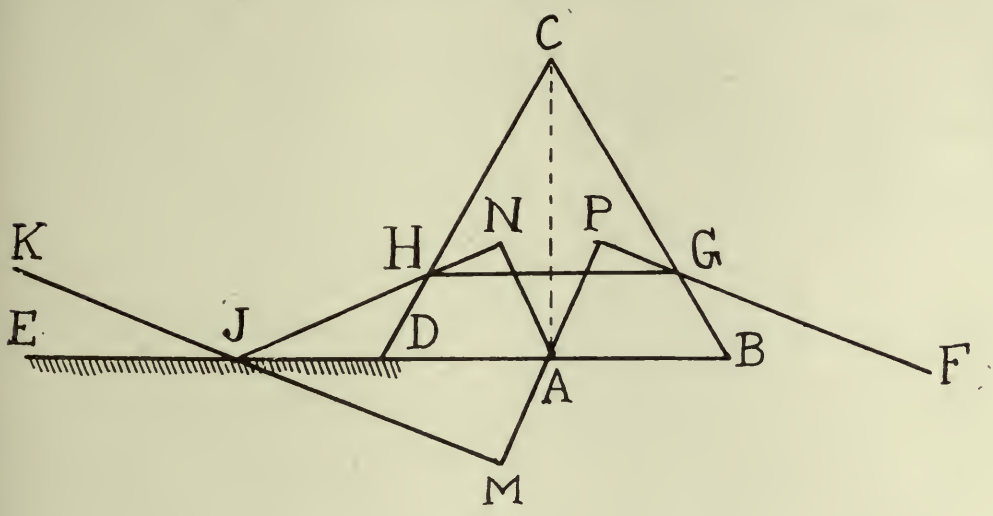


Fig. 1.

Suppose now that the single ray FG is replaced by a beam of parallel rays; each colour in turn as it suffers minimum deviation is undeviated, and at the same time suffers the same constant parallel displacement.

The next diagram shows how these properties are taken advantage of. The mirror-prism combination is shown inside a rectangular box into which we are looking vertically down, the lid having been removed. The mirror is denoted by the letter S. The slit is attached to a piece of brass tubing which slides in a short piece of tube fixed in the side of the box. The light from the slit is rendered parallel by a concave mirror, M, then passes through the mirror-prism combination, falls on the other concave mirror, M′, and is brought to a focus on the photographic plate at P. Owing to the obliquity of the incidence, the distances of the slit and P from their respective mirrors are given not by $\frac{r}{2}$, but $\frac{(r \cos \phi)}{2}$ where ϕ is the angle between the incident

beam of light and the normal to the mirror. If the ends of the spectrum are to come out sharp the photographic plate should be inclined at 79° to the incident rays, and not at right angles, as is shown in Fig. 2.

The second instrument is represented in Fig. 3. The figure represents only the essentials. The slit, S, is attached to a brass tube sliding in a wooden block and clamped in position by a screw. A thermopile and photographic plate carrier, with two brass grooves for holding the slide, are shown at the side. They slide between the guides shown, the screw passing through the slot. They are fixed in position by tightening a nut on the screw. The light from the slit is rendered parallel by the concave mirror, M, passes through the fixed prism, which is set so that the middle of the spectrum suffers minimum deviation, falls on the mirror, N, and is brought to a focus on the photographic plate. The mirror, N, is rotated about a vertical axis

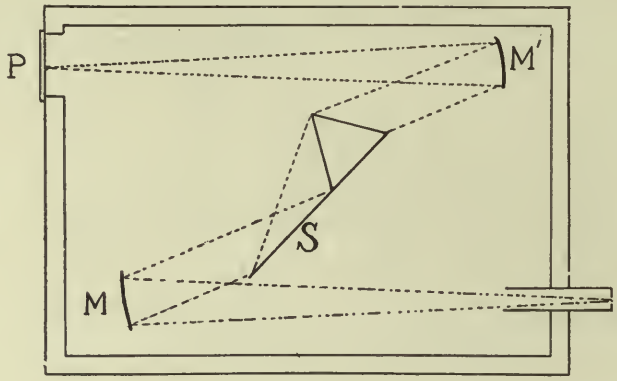


Fig. 2.

through its centre by means of a micrometer screw; hence all the colours of the spectrum can be made to pass in succession across the centre of the photographic plate. The curve along which the spectrum lies can be calculated from the formula for the oblique incidence of a thin pencil on a concave mirror. It does not deviate appreciably from a straight line.

Both instruments have automatic focusing. If the distance and inclination of the photographic plate are right for the visible spectrum, then, when the ultra-violet or infra-red is rotated on to the plate, it is in focus too. The first instrument has also automatic minimum deviation. If the N*a* line, when at minimum deviation, comes into focus on the plate at a certain point, P, all other lines in the infra-red or ultra-violet come into minimum deviation as they pass across P. The advantage of the second instrument over the first is that the light is reflected only on two metal surfaces as against three. Automatic minimum deviation is, of course, very useful when a Cornu double prism is employed; theoretically, the latter gives a single image only when at minimum deviation.

Both instruments are very rigid, the moving parts being small. In the first the

glass prism can be replaced by a quartz prism or rock-salt prism, and the instrument will work quite as well without any further alterations. Diffuse light caused by reflection at grazing incidence from the inside of blackened tubes is entirely eliminated. Both instruments are of the constant deviation type, and can be used as monochromatic illuminators. There is no chromatic aberration.

For the mirrors I have used concave lenses silvered over, or mirrors made of solid nickel of great purity. Silver reflects better than nickel in the visible spectrum, slightly better in the infra-red, but much worse in the ultra-violet. In the most

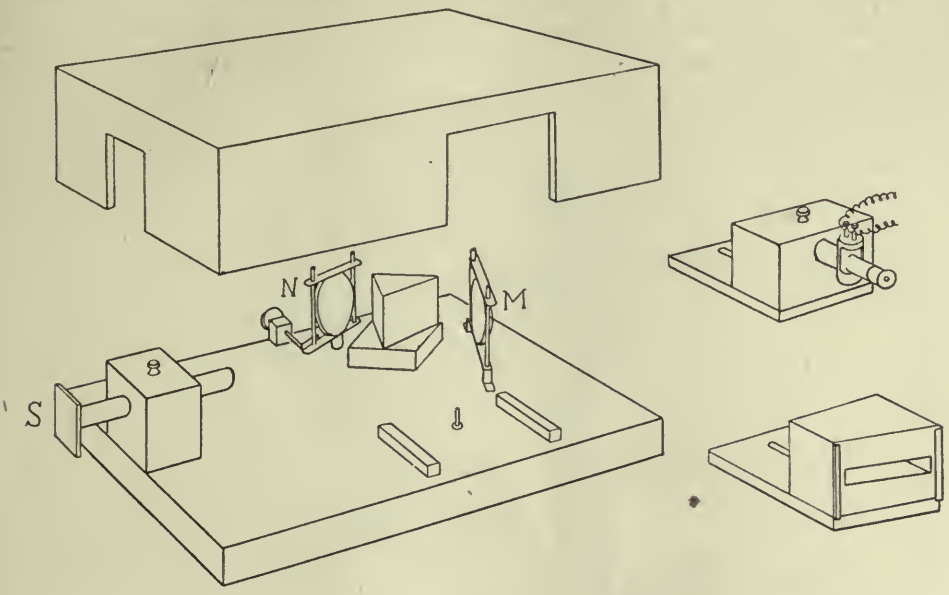


Fig. 3.

unfavourable part of the ultra-violet nickel mirrors reflect only 37·8 per cent. of the incident light. Hence, in the second instrument, only 14·3 per cent. of the light gets past the mirrors as against the 75 per cent. that gets through the lenses of a quartz spectrograph. The photographic plate is, however, not nearly so oblique to the rays as in a quartz spectrograph, and, when this is considered, the ratio of the intensities of the light on the plate in the two cases is about two to one in favour of the quartz spectrograph for the same aperture at the point in the spectrum most unfavourable to nickel. But if the new Kahlbaum Spiegel magnalium is used for the mirrors the advantage should lie the other way.

I have used the second instrument both as a spectrograph in the ultra-violet and with a thermopile in the infra-red.

The Chairman regretted that there was no time for discussing this interesting communication

THE SCOTOMAGRAPH

By J. H. Tomlinson, M.R.C.S.

(*Monday, June 24th*, 1912.)

The examination of the sensitiveness of the retina is necessarily subjective, and presents many difficulties. The ordinary perimeters which are used in daylight are convenient, but have certain drawbacks. The light is variable, according to the time of the day and the time of the year, and according to the state of the weather,

Fig. 1.

especially in cities like London. Moreover, the quadrant is unevenly illuminated. When it is in the vertical plane there is less light than when it is in the horizontal plane. The pigment colours used in testing colour-fields are not standardized and they change their hue from exposure, deposit of dust, etc. Perimeters with electric lamps travelling on quadrants have been proposed, so that examination can be made in a darkened room. These do not seem to have come into general use; probably the inconvenience of working in a dark room, the difficulty of keeping a watch on the eye of the patient, together with the uncertainty of a small electric lamp, have

militated against the adoption of this form of instrument. But for accurate work I am convinced that it is essential that there should be nothing in the field of view except the test-object. In the case of daylight perimeters, light is coming into the eye from the walls of the room, from the furniture, and from the clothes of the surgeon who is superintending the examination. Bjerrum of Copenhagen has recommended the use of a large black cloth screen, 4 metres square, opposite to which the patient is seated, and the use of a small ivory bead on a black rod as the test-object. But while this accentuates the need of a proper background, the inconvenience of the method is manifest. The instrument which I have designed, and to which I have given the name of "Scotomagraph," provides a perfectly black background, for the patient looks into a closed box or casing; at the same time it can be used in an ordinary room, a small velvet screen being placed between it and the light; thirdly, the tendency of the eye to wander is largely neutralized by having stereoscopic fixation; fourthly, there is provided a ready means of registration; and fifthly, the light is parallelized, so that the patient does not need to use his accommodation to see the test-object, an important consideration with old people.

It must be remembered that the eye is in a different condition physiologically from that of an eye being examined by a daylight perimeter, for in the latter case it is light-adapted, whereas in my instrument it is in a state of dark-adaptation. Our knowledge of the colour-fields in the dark-adapted eye, is mainly derived from Abney's beautiful researches in 1897. My object has been to produce a clinical instrument something of the type he employed for laboratory investigation. He used two forms of perimeter: one consisted of a hollow white hemisphere with a window at the pole, which could be illuminated by monochromatic light. The head, or at any rate the eye under examination, was directed towards the peripheral part of the hemisphere, a bead of luminous paint being used as the fixing-point. In the second form of instrument he employed an ordinary perimeter quadrant having a white surface instead of a black one. The eye was fixed as usual, and a patch of monochromatic light was projected on to the quadrant from a mirror mounted near the eye on a universal joint, the light being received on the mirror through an opening at the fixing-point. My instrument is somewhat of the type of the latter apparatus. I employ a mirror which gives a second image of the spot used as the fixing-point, but this image is received directly by the eye; it is not projected. The mirror is hinged to give a radial displacement of the image, and the whole instrument is rotatable about its axis to bring the second image on every part of the retina. The diagram shows two positions of the mirror. When the mirror is at *a* the second image appears

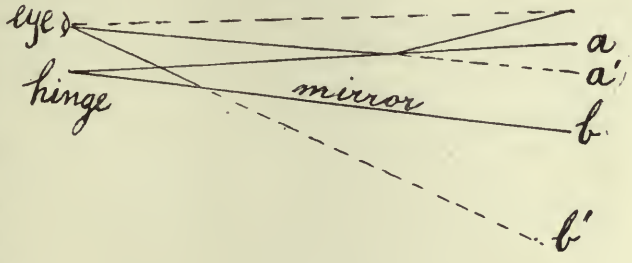


FIG. 2.

to come from the point a', and when it is at b from the point b'. By moving the mirror on its hinge from one extreme position to the other the second image traces a radial path on the retina from the centre to about 60°. A series of radial paths may be traced by rotating the instrument into successive positions, and rocking the mirror on its hinge. A series of concentric paths may be traced by successively fixing the mirror at different angles and rotating the instrument. The eye-tube opens into a wide chamber or casing, having sufficient diameter to allow the mirror freedom to make an excursion of 30°. This chamber is closed below by a plate which is hinged for opening and carries a diaphragm having a series of apertures, any one of which can be brought into the axis of the instrument to serve as the fixing-point of the eye. A chart, having a central aperture, is placed on the plate and the latter is then closed up and clamped.

The eye to be examined looks into a tube standing axially some distance above the aforesaid casing, which rests by trunnions on a horseshoe-shaped stand. The inclination of the instrument can be adjusted by a telescopic stay provided with a fixing-screw. The casing contains the hinged mirror and its actuating cam. The mirror carries a barrel in which a spring needle is housed. The needle can be projected to mark the chart through the medium of a flexible tube and pneumatic ball. The top of the casing is graduated and the flange which operates the mirror cam is provided with an indicating pointer, so that the position of the mirror at any moment can be read off from the outside.

The free eye looks down a cranked tube provided with two totally reflecting prisms, so that an aperture placed beyond the limits of the casing is visible to the eye. This tube is carried by a movable ring embracing the casing, so that it can be brought either to the right or the left of the axial tube. Note that the eye-piece is at a lower level than that of the axial tube, the object being that when the eye under examination is pressed against its eye-piece, there is an interval between the free eye and *its* eye-piece, thus enabling the surgeon to watch the latter eye to see that it does not wander. In practice, however, it is found that a patient has very little difficulty in fixing, and not much temptation to follow the reflected image as it moves over his retina, if he is warned against doing so from time to time. In using the instrument, it is essential that the patient should be thoroughly at ease, preferably with his elbows on the table. A sheet of white blotting-paper (or in some cases grey paper) should be spread in front of the instrument and be illuminated by an electric or other lamp, a screen preventing the direct light from entering the patient's eye.

A chart having been placed on pins mounted on the hinged plate, the latter is closed up and fixed by a turn-lever. The mirror should be in its extreme outer position so that the second image is practically out of sight. The patient's interpupillary distance should be measured and the eye-lenses of the instrument adjusted a corresponding distance apart. The two lower adjusting screws should now be manipulated until the image seen by the free eye is a little distance above that seen by the eye to be examined. The patient having been seated at the instrument is asked if he sees the two images one above the other. He may remark that the images are streaky.

This is because he is not looking quite centrally down the eye-pieces. A little movement of the eyes will put this right. He may then say one image is above the other, but to the right or left. If so, he should move the appropriate adjusting-screw until the images are exactly one above the other, then with the other screw he brings down the upper image until it coalesces with its fellow. When this is done, the two images " click togther " and it is somewhat difficult to separate them.*

[This is the best method of fusing the images, for the eyes have very little vertical adjustment; by bringing the images one over the other the internal and external recti are in repose, so that their action does not complicate the fusion.]

In dealing with an ametrope, his free eye may be corrected by inserting a lens in the eye-piece. He then sees one image clearly defined, although the other image is ill-defined; this does not, in general, prevent stereoscopic fixation.

Having now got the object-aperture clearly fixed, the flange controlling the mirror is rapidly turned, bringing the second image into sight as a falling star. The patient is told to take hold of the flange and move it backwards and forwards so as to understand the method to be employed and to practice seeing the moving object without following it with his eye. The blind spot of the eye under examination should next be mapped out. Its centre is normally between 14° and 17°. The mapping out of the blind spot interests the patient and gives him a little practice in the use of the instrument, and, in addition, it may reveal anomalies. It is well for the surgeon to control the movement of the casing about its axis and retain the pneumatic ball in his hand, the patient taking hold of the mirror-moving flange with both his hands, one at each side.

When taking ordinary fields of vision a series of radial excursions of the second image are taken and the limits marked off. If glaucoma is suspected, the mirror is set at successive angles, say, 15°, 20°, 25° and 30°, and the whole instrument turned slowly, so that the second image traces a series of concentric circles. The smallest aperture should be employed and the light be reduced. To test the colour-fields, small plugs fitted with coloured gelatines are inserted in the light aperture of the chart plate. The gelatines are selected from Wratten & Wainwright's lists and are practically monochromatic. It is necessary when using them to replace the white paper by a lamp with frosted glass screen. The free eye looks in the ordinary way at white paper. The fusion is not so complete, but the white light overpowers the coloured light and after a while the fixing-point is considered white. The judgment as to the colour of the second image is thus unbiased.

* The idea of giving the free eye a fixing-point was suggested to me by my friend, Mr. Frederick Cheshire, F.R.M.S.

ON ASPHERICAL SURFACES FOR SPECTACLE LENSES (GULLSTRAND'S CATARACT LENSES)

By M. von Rohr, Ph.D.

(*Monday, June 24th*, 1912.)

Before treating the real object of my lecture, I think it advisable to give a rough sketch of the direction taken at the Carl Zeiss works with regard to spectacle lenses.

As far as I know the literature, the theory of spectacle lenses has been confined to paraxial objects and images, and under this restriction the object has been fully discussed by means of simple and even elegant formulæ. But it will be granted, I

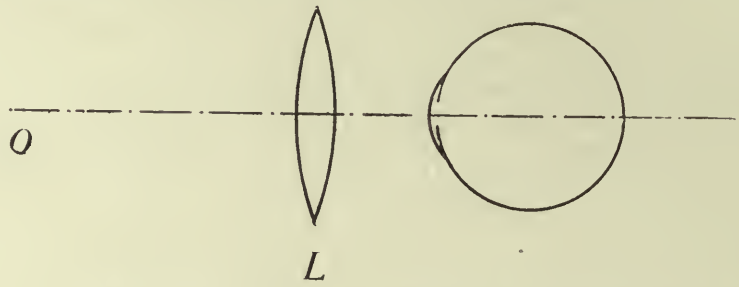


Fig. 1.—Eye behind the spectacle lens and looking in the direction of the axis.

think, that this restriction is too narrow in so far as the theory only applies to directions of the line of vision which are in close proximity to the axis of the spectacle lens, in other words to paraxial directions. All the formulæ before mentioned, therefore, will hold good only in the case that the axis of the spectacle lens is directed towards that particular object point that is the principal object of contemplation. In order to realize these conditions we should have to move our head continually in looking around. We all know that this supposition is not fulfilled in general, and that we look around not with moving head but with moving eye realizing what is called " direct vision."

It is a remarkable fact, that optical instruments for visual use have not been constructed with the clear intention of helping direct vision, and although the nature of the eye's movement in direct vision has been defined as early as 1826, as a rotation on a point (the centre of rotation of the eye) situated in the middle of the eye, the design of optical instruments has very seldom taken any notice of this very important fact. I shall not deny that some efforts have been made to construct visual instruments of a better efficiency in direct vision—and my late lamented friend, Mr. R. H.

Bow of Edinburgh, takes a prominent place in the roll of these inventors—but any success, however restricted, in these endeavours cannot be spoken of before the exertions of some ophthalmologists. I may mention here the names of Dr. Ostwalt (1898), Prof. Tscherning (1899), and not much later (1901) of Dr. Percival.

In Jena we began our calculations and constructions at a much later date—1908—certainly without knowledge of these predecessors, but under the influence of Prof. Gullstrand of Upsala, whose authority in these matters will not be doubted. I may as well state the problem that we have put before ourselves for solution. Given a spectacle lens of a certain power (or powers in astigmatic lenses), to find the form of the surfaces that—for an eye rotating under the action of the muscles on its centre of rotation—the image of an object element may be clearly

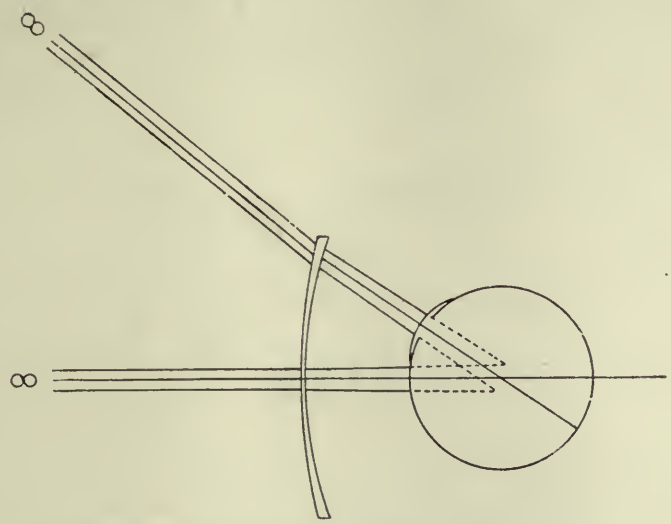


Fig. 2.—Incident parallel pencils pass after refraction through the centre of rotation.

depicted on the middle of the macula lutea. In other words it is the problem of widening the field of view of the spectacle lens and developing a field of direct vision. Such lenses we shall subsequently call after Gullstrand's expression, *point image lenses.*

It would be a very great privilege for me to develop the theory of these lenses to an audience like the present, and there would be no want of matter even for a lengthy discourse, as the different lenses, viz., ordinary spherical, astigmatic and prismatic forms have all been treated by us. To-day time will not allow me to give anything like the whole theory, and we shall restrict our attention to a small, but interesting group of the first class, *i. e.* to ordinary lenses having surfaces of revolution in general, spherical surfaces.

Some words are necessary about the optical problem, when due regard is taken

of the eye's rotation in direct vision. Systems of a similar kind are extremely rare in technical optics; we may cite as an instance some (widest-angle) panoramic cameras with rotating lenses; but, as every photographer knows, these systems are not used now-a-days. The correct treatment of the eye's rotatory movement was, therefore, not immediately understood, as precedent cases were wanting. As in direct vision with steady head one point belongs to all possible positions of the line of vision, *i. e.* the centre of rotation, we shall calculate all point image spectacle glasses for such a back stop that the eye's centre of rotation may coincide with the middle of this diaphragm. We see that the centre of the back stop is common to all possible secondary axes of oblique pencils, or in our mode of expression to all image-side *directions* of the *principal rays* of the spectacle lens. I may mention here that we have fixed the distance between the eye's centre of rotation and the nearest lens vertex to nearly one inch (25 mm.), and that is equivalent to a distance of 12 mm. between the vertices of the cornea and the nearest lens surface.

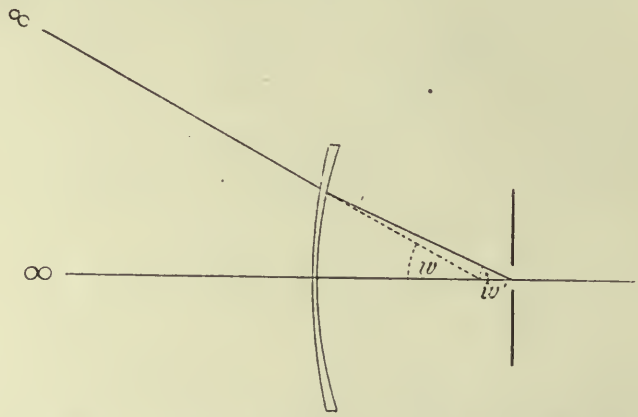


Fig. 3.—Centre of rotation serving as back stop for the spectacle glass.

Our problem for spectacle glasses having been fixed exactly, we may state at this stage that the spectacle glass is to be treated in quite a different manner compared with the elegant rules for paraxial rays referred to above. Whereas in this case any axial object point will give existence to one image point and one only, the question is here much more complicated, and we do not know at the outset whether object points lying on principal rays of finite obliquity are at all rendered by the spectacle lens. To answer this question thoroughly would imply the development of the theory of oblique pencils, but we shall—as that is impossible here—only give the results: In general lenses do not render object points lying on oblique rays, because the thin pencils after their passage through the lens are affected with *astigmatism of oblique pencils*. In consequence thereof, no single image point corresponds to the object point, but two short bright lines lying on the principal ray

at a finite distance from each other. If it be our object to construct point image lenses we must try to correct the spectacle glasses with their back stop at our chosen distance of 25 mm. for astigmatism of oblique pencils, within a certain angle of vision, say of $2 \times 30°$.

As the power of the lens and the distance of the stop are given, there is only one variation left, viz. the form or (as the opticians say), the *bending* of the lens. The question arises, therefore, whether it is possible or not simply by bending to do away with the astigmatism of oblique pencils. I cannot here develop the theory which might be given in a very elegant manner by means of Prof. Tscherning's approximate equations for infinitesimal obliquities, which, however, hold good remarkably for definite obliquities of the line of vision as well. You will be interested to hear, that if there be any real solution at all, always *two* solutions are forthcoming. As you see from diagrams the forms of the spectacle lenses therefore differ with

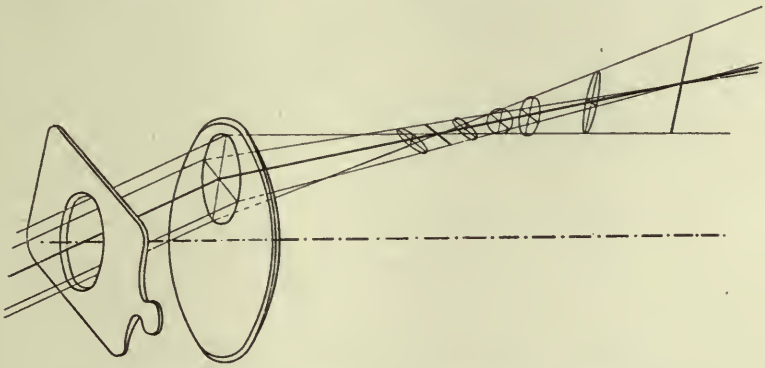


Fig. 4.—Incident parallel pencil and the astigmatism of oblique pencils.

regard to the bending, and we have to distinguish between a more and a less bent form, or, to use my mode of expression, between a Wollaston and an Ostwalt meniscus. I think you will agree with me that the names of the two scientists which have first attacked this problem should be connected with these forms.

But there are limits for these double solutions, and they only occur for lenses of the power D_1, if D_1 lies between

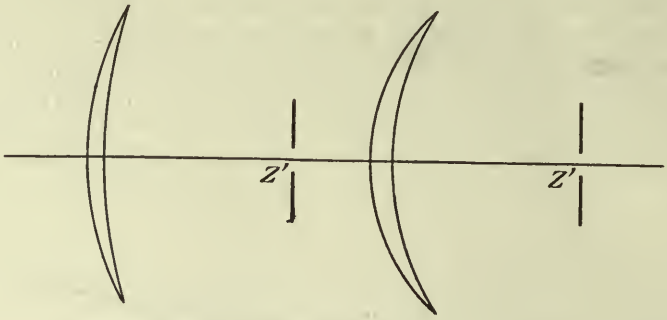
$$-21 \text{ dptr.} \leqq D_1 \leqq 7\tfrac{1}{4} \text{ dptr}$$

For any other power the approximate equation has two imaginary roots which need not trouble us here.

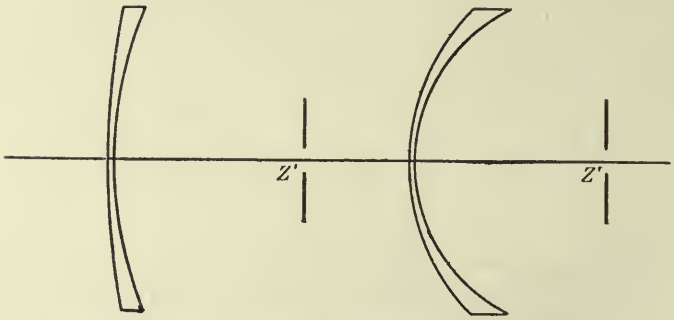
You will see at once that for dispersive lenses the limit is not of any consequence, but it is different with collective glasses, for nearly all cataract lenses are outside the above limits, and consequently cannot be constructed as point image lenses, if they are planned as single lenses with spherical surfaces. There might be a way out of this difficulty by employing, after H. Parent's idea, two consecutive single

lenses with spherical surfaces, and in 1908 I made some computations in this direction. The astigmatism of oblique pencils was, indeed, considerably diminished, but the mounting of the two lenses was too delicate for practical wearing.

The problem can, however, be put in another form as proposed for the first time by Prof. Gullstrand: is it possible to correct the astigmatism of oblique pencils by giving an aspherical form to either of the two surfaces of the cataract lens?



Ostwalt's and Wollaston's form for $D_1 = + 5$ dptr.



Ostwalt's and Wollaston's form for $D_1 = - 5$ dptr.

FIG. 5.

What is an aspherical surface? It is a surface of revolution where the meridian curve is different from the arc of a circle. We shall, therefore, count under this head elliptic, parabolic and hyperbolic surfaces of revolution, because the meridian curves are respectively arcs of an ellipsis, a parabola or a hyperbola. Although such surfaces of revolution realized by means of conic sections have been used in the construction of spectacles, they do not seem to have proved of decided advantage. In our case the problem is to form the meridian curve in such a way that the astigmatism of the oblique pencils may totally disappear for a given angle of obliquity. The solution is possible, and conducts to an arc of a transcendental curve. The

corresponding surface of revolution nearly coincides in the centre with a sphere (the *osculating* sphere), whereas it shows towards the margin a difference increasing with the distance from the centre. As we shall soon see, this distance is small even at the margin, and in all cases hitherto realized aspherical lenses have more substance at the margin than they would have if the aspherical surface was replaced by the osculating sphere, and we may use the figurative expression of addition of substance. With regard to the difference some figures will possibly interest you. Let us suppose, for instance, a greatly (795 times) magnified aspherical lens with a giant diameter of 104 ft., the radius of the osculating sphere will amount to 365 ft., *i. e.*, the height of the cross on St. Paul's Cathedral. If we draw a line from the centre of the osculating sphere towards the lens margin, its length would, of course, be exactly 365 ft., but the aspherical surface would have a somewhat shorter distance, viz., exactly by five inches shorter and amount only to 364 ft. 7 ins. This supposition

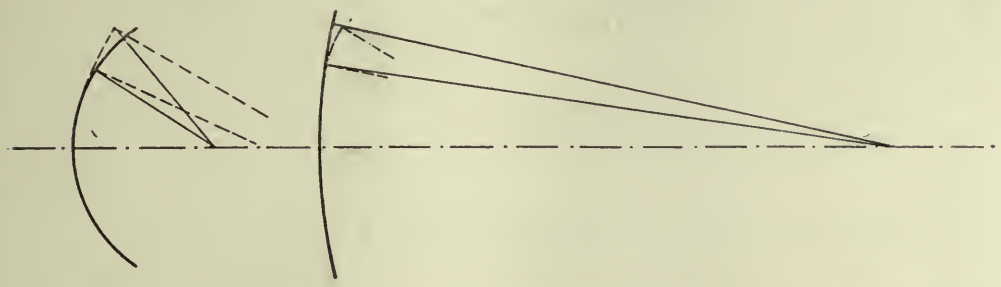


Fig. 6.—Deviation of the normals in connection with the addition of material.

will give you some idea of the necessary accuracy as all the figures are to be reduced in the proportion of 1 : 795. Therefore, for the real lens of 12 dptr. the radius of the actual osculating sphere turns out to be 140 mm. and the difference of the aspherical surface 0·16 mm.

If you ask me, how is it possible that such an apparently insignificant deviation from the ordinary spherical form may correct the astigmatism of oblique pencils, I must give the following answer. The correction of this aberration not only depends on this distance itself, but also on its first and second derivative. And this is shown in the figure, as you will notice a certain deviation of the normals (increasing towards the margin), connected with the addition of material. As such complex functions also enter into the very complex formulæ of the astigmatism of oblique pencils it follows immediately that not only the amount of the final distance is important, but also the law after which this distance reaches its amount at the margin. In order to make the nature of the transcendental curve as clear as possible I have given—in great magnification—the diagram showing the addition of material as dependent on the arc of the osculating sphere; it is evident that towards the margin the

addition increases out of all proportion. It goes without saying that the working of such surfaces must be done with the utmost precision from the centre up to the margin, otherwise the astigmatism of oblique pencils will not be corrected.

From the rendering of the test types it will be evident that the correction of the astigmatism of oblique pencils is everything that can be desired, and this correction can be obtained even in other cases.

We are all aware that the above supposition, of an eye operated on for cataract with such a result that the cornea is quite symmetrical, is not the rule. After the operation generally a certain astigmatism is found as the cornea shows two planes perpendicular to each other, in which the curvature has the greatest and least values. The correction of the operated eye has, therefore, to be effected with an astigmatic lens, and it goes without saying that the calculation of a corresponding astigmatic lens is much more difficult. These difficulties, too, are overcome by means of an aspherical surface, and it is possible to construct an asphero-toric lens which will show throughout the entire field of direct vision an astigmatism of constant amount, and with such an orientation of the axes that it is annihilated by the astigmatism of

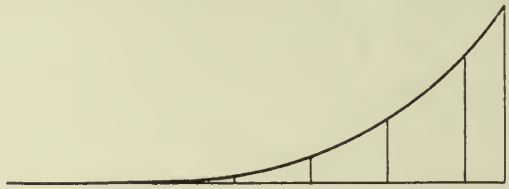


Fig. 7.—Increase of addition towards the margin.

the rotating eye; in consequence thereof a point image is formed on the macula lutea of the rotating eye whatever be the obliquity of the line of vision.

From the test types of the diapositive you will see that the quality of the images is very satisfactory, whether the line of vision be lying in the first or in the second meridian, or in a plane forming an angle of 45° with either.

In all cases it is of great importance that these spectacle glasses are properly adjusted, so that the centre of rotation lies at 25 mm. distance, or that the distance between the vertex of the cornea and the nearest lens surface amounts to 12 mm.

At the same time the wearers of the new glasses must learn to use their eyes in the natural way—*i. e.* in direct vision. If a considerable time has elapsed since the operation the patients may have become—in consequence of the astigmatism of oblique pencils of the ordinary glasses—so used to turn their heads in looking around, that they continue this custom also with the new glasses and so deprive themselves of the full benefit of their principal qualities.

Mr. T. Smith asked what were the actual corrections aimed at, and to which of the two periscopic types did the Gullstrand lenses approximate.

Prof. Silvanus P. Thompson asked whether it was not better to consider the nodal point of the eye rather than the centre of rotation.

Dr. von Rohr replied that the lenses were corrected for astigmatism of oblique pencils, the object being assumed to be at infinity. As no accommodation is possible in aphakic eyes it is not necessary to consider near points; for reading the lenses are corrected for the distance for which the reading addition has been chosen.

The general form of the lenses is such that the concave surface is less curved than in the Ostwalt type. The aspherical surface would permit of the orthoscopic correction being made, but the general form of the lens would then be inconvenient; the actual lenses are, therefore, made so that the shape of the lens is not unusual while the orthoscopic condition is nearly satisfied.

In answer to Prof. Thompson he (Dr. von Rohr) pointed out that in direct vision the eye is rotated, and that the new position of the line of vision (cutting the former position in the centre of rotation) must be taken into account in the construction of the spectacle lens. The nodal point N′ of the eye has no great value for direct vision, because for *finite* obliquities the marginal parts of the retina are of so little use for the perception of details of form, and, therefore, to distinguish these we rotate the eye so as to bring the image on to the *maculed lutea.*

With reference to the practical manufacture of these lenses he referred inquirers to the English Patent, 8932/1900.

He concluded by expressing his appreciation of Dr. Percival's work in connexion with the astigmatism of spectacle lenses, though he could not accept all Dr. Percival's conclusions.

IMPROVED INDIRECT OPHTHALMOSCOPY

By Lionel Laurance and H. Oscar Wood

(*Monday, June 24th,* 1912.)

In the usual method of indirect ophthalmoscopy, a large concave mirror, of about 10″ focus, is held some 20 ins. or 30 ins. from the observed eye. The light diverging from a source some 40 ins. from the mirror is reflected therefrom to form a real image between the observing and observed eye, and some 12 ins. or so from a strong condenser, of about 3 ins. focal length, held close to the observed eye. The light passing through this condenser is strongly converged to about 4 ins., and so enters the eye to be brought to a focus in the vitreous, whence it diverges and illuminates a certain fundus area.

The light diverging from each point of the illuminated fundus emerges from the

eye as a parallel beam in emmetropia, as a divergent pencil in hypermetropia, and as a convergent pencil in myopia; and is brought to a real focus by the condenser at its focal distance in emmetropia, beyond it in hypermetropia, and within it in myopia. This real image of the fundus, formed in the air between the condenser and the observing eye, is what is seen; it is projected by the observer on to the condenser itself, the image being larger in hypermetropia and smaller in myopia than in emmetropia, but the apparent size, except in the last case, varies with the distance of the lens from the observed eye.

The magnification in all states of refraction is the ratio between the size of the real image, formed by the condenser, and the fundus itself. In emmetropia, the latter subtending a certain angle at the nodal point about 15 mm. distant, the image is formed under the same angle at the focal distance of the lens—some 75 mm.—and therefore the magnification is $75 \div 15 = 5$ times (approx.).

The magnification in emmetropia remains the same for any distance of the lens from the eye. In hypermetropia the magnification is greater, but becomes less on withdrawing the lens, and in myopia the magnification is less, but becomes greater on withdrawing the lens. In all conditions, however, the magnification is the same when the lens has its focal point coincident with the anterior focus of the eye.

The area of fundus that can be seen varies directly with the size of the pupil, inversely with the length of the eye observed, and also inversely with the distance between the observed and observing eye.

If the light enters the eye along the visual axis the pupil contracts vigorously, and any observation is difficult to make, so that in practice it is better to project the light along the optic axis, and, to effect this, the observed eye should turn slightly towards the nasal side.

The greatest field of view is obtained when the lens is distant from the eye such that the greatest possible virtual image of the pupil, *i.e.* equal in size to that of the condenser, is obtained; but this distance is inconveniently great with a large + 13D lens, since the latter must be held in the air beyond a possible stretch of the hand when any of the observer's fingers rest against the brow of the subject. If, to obviate this difficulty, the condenser be increased in power, the aberration, already very prominent in the weaker lens, is considerably increased.

By moving the condenser to and fro, or varying the distance of the mirror, the fundus area illuminated becomes larger or smaller. In the latter case the condensation of light is increased, and the fundus is better illuminated, while when the area seen is increased, the illumination is less intense. The maximum illumination and minimum field is obtained when the image of the source is formed sharply on the retina.

Contraction of the pupil, difficulty in holding the condenser sufficiently far from the eye while at the same time keeping in view the aerial image, together with the reflections from the surfaces of the eye,and still more those from the surfaces of the lens, are among the difficulties experienced in the practice of indirect ophthalmoscopy. In addition the strong aberration of a single thick condenser renders the apparent dilatation of the pupil exceedingly hard to maintain.

The two prime conditions to be simultaneously fulfilled in order to secure the maximum field of view in indirect ophthalmoscopy are: (*a*) the apparent dilatation of the pupil should be as great as possible; and (*b*) the area of fundus so disclosed should be uniformly and brightly illuminated.

The first condition is realized, for any given size of condenser, when the real conjugate focus of the observed pupil is projected in the neighbourhood of the nodal point of the observing eye. In Fig. 1 let *ab* be the observed pupil, L the condenser,

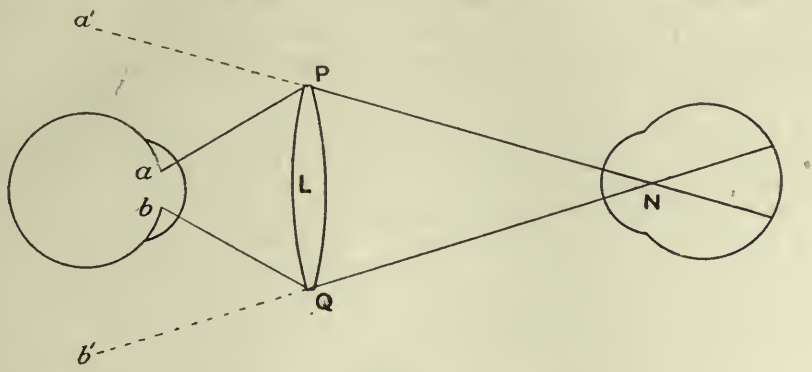


Fig. 1.

and N the nodal point of observer. Then when L is so placed with respect to *ab* that the extreme rays *a*P, *b*Q are converged to N, the apparent size of the pupil, of which *a' b'* is the projection, is equal to the diameter of the condenser. It therefore follows that the larger is L, the greater will be the field of view, but the spherical aberration

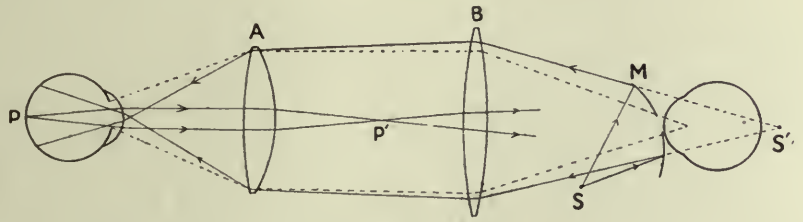


Fig. 2.

of very large single condensers limits the practicable diameter. To a very great extent, however, this difficulty can be overcome by employing a system of two lenses such that, by sharing the refraction between them, spherical aberration may be reduced to a minimum. Referring to Fig. 2, this is secured by placing the component lenses at distances from the observed and observing eyes equal to their respective focal lengths. The light diverging from the observed iris then emerges approximately

parallel from the first lens, to be converged to the observer's nodal point by the second; the resulting virtual image of the observed pupil, therefore, suffers from little aberration.

To obtain the best possible results we need fixity of the aerial image, whereby its view is rendered easy, together with maximum definition, field, illumination and magnification, with an absence of lenticular reflections.

The suggested improvements in indirect ophthalmoscopy would arise from—

(*a*) The employment of an apparatus such that the observed eye is kept in a fixed position by means of a head-rest and a convenient fixation point, alteration of the latter being within the manipulation of the observer, so that the direction of the visual axis may be controlled.

(*b*) The employment, in connection with the above, of a more or less aplanatic condensing system, of convenient focal length and optical arrangement.

(*c*) An optical arrangement such that the virtual image of the pupil is as large as possible—filling the whole field of view.

(*d*) An optical arrangement which at the same time causes maximum condensation of light in the observed eye, it being so arranged that the real image of the aperture of the mirror lies at the cornea of the observed eye, so that the greatest possible amount of light enters the pupil.

(*e*) An optical arrangement which secures an increased magnification, firstly by the employment of a weaker condenser than is usual, and secondly by using an auxiliary lens, whereby additional virtual magnification may be obtained, and accommodative effort on the part of the observer relieved.

(*f*) A method of arranging the lenses of the optical system so that, by tilting them, all the annoying and confusing images of the source of light, caused by reflection from the surfaces, are shifted from the line of view; such other aberrations introduced by the obliquity of the lenses being neutralized by the introduction of a compensating cylindrical element.

These improvements in indirect ophthalmoscopy are embodied in the Orthoscope manufactured by Messrs. Gowlland of Croydon. The condensing system consists of a sphero-cylindrical element, A, so tilted in the plane of its cylindrical axis that sphericity of effect is practically re-established, but reflections from its surfaces thrown out of the line of vision; the effective power of this lens is approximately 9D. The second or auxiliary lens, B, is considerably weaker, being about 4D, also slightly tilted to eliminate reflections, the small degree of radial astigmatism thus introduced being compensated by a slight extra tilt of the first lens. Thus the power of the system as a whole can be rendered, for practical purposes, equal in all meridians. The concave mirror, M, is about 4 ins. in focal length, and the small source, S, is so placed that its virtual image, S′, is approximately at the focal point of the auxiliary lens, B. Thus the projected light emerges parallel from the latter, and after traversing the interval between it and A, converges towards the focal distance, slightly beyond which is placed the observed eye, E. A real image of the mirror, M, may then be seen

in the pupillary plane of the latter, and this fact serves as a guide in adjusting the instrument for observation.

The relative powers of the component lenses, the curvature of the mirror, and the various separations, are so calculated that, when the maximum illumination of the observed fundus is secured, *i.e.* when the real image of the mirror is projected on to the pupilliary plane of the observed eye, the apparent size of the pupil is a maximum, no portion of the iris being visible.

Now, light diverging from any point, P, of the illuminated fundus emerges, in emmetropia, parallel from the eye to meet at P′ in the focal plane of the first lens. Taking the focal length of the latter as being 110 mm. (9D), we have that the magnification of the real image, P, with respect to the fundus, is $\frac{F}{f}$ where F is the focal length of the first lens and *f* the anterior focal length of the eye. The latter, as usual, being taken to be 15 mm., the magnification of the image P′ is $110 \div 15 = 7{\cdot}5$ (approx.), and the final image, as seen by the observer through the second lens, receives an additional magnification of about one-and-a-half. The total magnification of the fundus is thus approximately ten as compared with the five obtained by the ordinary indirect method, but no definite figure can be given, as the refraction of the eye under observation must be taken into account. In normal circumstances, however, a portion of the fundus, say the optic disc, which is about 1·5 mm. in diameter, is seen as an image some 15 mm. in diameter.

As previously stated, the area of fundus visible depends entirely upon the diameter of the condensers employed, and in this present instrument, the effective diameter of the first component is 3 ins., or 75 mm., so that the extent of fundus visible for any one position of the observed eye is $75 \div 15 = 5$ discs widths of uniformly illuminated fundus. The latter, however, can be easily explored by means of movable fixation objects, at which the subject can gaze.

When the instrument is to be used as a stationary retinoscope the condensing system is swung clear to one side of the body, and a small circular diaphragm placed before the illuminant. The mirror itself is controlled by a weak spring, which causes the former, when released, to assume its normal position, so that the beam is projected along the axis of the tube. The observed eye is viewed in a vertical plane containing the visual axis, instead of to one side of it, as is usual; this is obtained by causing the eye to be raised slightly to fix, in a small mirror on a universal joint at the top of the body, any convenient object at infinity behind the observer, such as a set of test types, faintly illuminated. This latter has the effect of relaxing accommodation, especially under the influence of the fogging lenses required to bring the point of reversal to about 66 cms., where the observer's eye is situated. Thus, when all movement of the shadow and reflex ceases, the observed eye is either naturally or artificially myopic to the extent of 1·5D, the distance between the operator and subject being 66 cms.

The reflex, in these circumstances, is very brilliant and well defined, the band appearance sought for in astigmatism being obtained much more easily than in the

ordinary method. The apparent size of the pupil is also larger, and the movements are easily controlled by means of the automatic mirror. The near source of illumination is advantageous in that the actual source, or virtual image formed by the mirror, is practically at the focal distance of the meridian corrected; therefore, the definition of the band is a maximum in all circumstances. It has not been thought advisable to add the usual battery of lenses, as, besides being unnecessary and expensive, it is very difficult to place them sufficiently close to the observed eye to secure accurate results. A simple adjustable lens-holder, attached to the head-rest, is therefore employed in conjunction with the trial case.

NOTES ON SPECTROGRAPHS

By W. H. Eagle, B.Sc.

(*Tuesday, June 25th*, 1912.)

As I have for the last few months been largely occupied with calculations concerning quartz and glass spectrographs, I thought that a brief notice of some of the results obtained might not be without interest, particularly as the conditions to be satisfied in the design of spectrographic apparatus are quite different from those to be taken into account in designing ordinary photographic objectives, and so forth.

Although the design of each part of a spectrograph will hereafter be considered separately, we may specify at this stage the general conditions to be satisfied. There must be sharp definition over the whole spectrum, on a flat plate if possible, which plate, however, need not necessarily be perpendicular to the axis of the instrument; hence achromatism is not essential, the difference of focus of the different wave lengths being corrected by tilting the plate. Moreover, in the great majority of spectrographs, where we are dealing with a slit, considerable astigmatism is permissible in the image; so that, although a flat image is very desirable, it is not absolutely necessary that this image should be homocentric, and herein lies a difference from ordinary photographic systems. The factors at one's disposal are—

(*a*) The length of the collimator, the design of its lens and the wave length (or wave lengths) for which it is focused.

(*b*) The orientation of the prism system.

(*c*) The design of the camera objective.

Spectrographs may be divided into three classes—

1. Those used chiefly for the visible spectrum, and which are not expected to be used beyond about λ 3500.

2. Those intended for use over the ultra-violet spectrum down to about λ 2000.

3. Those for use beyond about λ 1850, which require to be used in exhausted chambers (owing to the absorbing effect of the air for light of very short wave length) and are used only for special work; they will not be further considered in this paper.

Most spectrographs used for astronomical purposes belong to class (1). Since the more remote portions of the ultra-violet spectrum are not required in these instruments, their optical parts may be constructed of glass; but the dense flint glasses, such as those of $n_D = 1{\cdot}75$, must not be employed for the prisms, as these glasses absorb strongly in the more refrangible part of the spectrum.

For spectrographs of class (2) glass cannot be used. Any of the following materials may be selected for the prisms and lenses—

Fluorite.	Vitreous Silica.	Sylvine.
Quartz.	Rock-salt.	Calcite.

Of these, fluorite in the colourless form transmits down to λ 1250*; but it is now very rare and expensive, and is only used for instruments of class (3).

Quartz is the material by far the most commonly used in spectrographs of this description; in thicknesses of 1 to 2 mm. it is transparent to λ 1500, but a thickness of 2 cm. absorbs at about λ 1600. The direction of rotation and of the optic axis makes very little difference to the transparency.

Vitreous silica is slightly less transparent than crystalline quartz.

Rock-salt transmits to λ 1800, but it requires protection from the moisture of the atmosphere. It is sometimes used for combining with quartz to make achromatic objectives (which must be in the triplet form) for use in the ultra-violet. Its refractive index for D is almost exactly the same as that of quartz, but it is slightly less dispersive than quartz in the red, and very much more dispersive in the ultra-violet, as may be seen from the appended table.

Sylvine more or less resembles rock-salt, and is but little used in optics except for the infra-red.

Calcite is transparent only to λ 2140, and its strong birefringence makes it objectionable in spectroscopy. It is, however, often used in making quartz-calcite objectives. Needless to say, these objectives cannot be cemented, since the balsam absorbs ultra-violet light beyond about λ 2950. Castor oil, which is transparent to the ultra-violet, may be put between the lenses.

The following table exhibits the refractive indices and ν-values $\left(= \frac{n_{2749} - 1}{n_{2144} - n_D}\right)$ for these substances—

* Lyman: *Astrophysical Journal*, xxv. 45. (1907.)

	n_D	n_{2749}	n_{2144}	ν
Fluorite	1·43385	1·45966	1·48457	9·06
* Quartz (ordinary) . .	1·54426	1·58753	1·63047	6·82
Quartz (extraordinary) .	1·55337	1·59813	1·64270	6·70
Vitreous silica . . .	1·45848	1·49613	1·53390	6·58
Rock-salt . . .	1·54413	1·62687	1·73221	3·33
Sylvine	1·49038	1·56380	1·66182	3·28
Calcite (ordinary) . .	1·65836	1·74150	1·84582	3·95
Calcite (extraordinary) .	1·48639	1·52266	1·55992	7·11

The optical parts will now be considered separately.

I. *The Collimator.*—Since the prism system gives no astigmatism when the pencil of light incident upon it consists of parallel rays, it is an advantage to have the rays parallel for as many points as possible in the spectrum. Hence an achromatic objective is used whenever possible. It requires to be corrected for spherical aberration for about the centre of the region operated upon, but correction for coma is relatively unimportant, since the rays are nearly parallel to the axis. For glass spectrographs an ordinary telescope objective may be employed; the chromatic correction, however, should not be made for C and F, but rather for D and G′, or even for F and H_δ; the ratio of the powers of the two components (in a doublet), however, must not be so large as to make it impossible to correct the spherical aberration other than by figuring.

Where glass cannot be employed, quartz may be combined with either rock-salt or fluorite. The first of these combinations gives a considerable secondary spectrum, but the second gives a remarkably achromatic combination, there being practically no secondary spectrum. Quartz-calcite combinations may also be used, but, as pointed out above, calcite is not sufficiently transparent for many purposes.

Although it is well to keep the secondary spectrum as small as possible, the outstanding differences are sometimes made use of (viz. by changing the glasses of the collimator objective) to give a final correction to the curvature of the spectrum. (See, for instance, the four-prism spectrograph described on p. 127.)

Except in the comparatively rare cases where achromatic combinations of quartz with rock-salt, fluorite or calcite are employed, the collimator lens of spectrographs, when not of glass, is usually a simple quartz lens. Although it can then collimate the light for only one point in the spectrum, and its spherical aberration can be completely corrected only by figuring, it nevertheless possesses the advantage of offering less absorbing material to the light. It should certainly be figured to remove residual spherical aberration (the curves being already chosen in the proper ratio to give minimum spherical aberration for the centre of the spectrum—about λ 2980). It is not advisable to carry a large amount of aberration through the prisms, with the intention of correcting it in the camera system.

* Right Quartz. For left quartz $n_D = \begin{matrix} 1{\cdot}54424 \text{ (ordinary).} \\ 1{\cdot}55335 \text{ (extraordinary).} \end{matrix}$

In three- or four- prism instruments, where the spectrum is photographed in sections, the collimator is usually focused for the central ray of the portion being photographed. With single-prism quartz spectrographs, however, where the whole spectrum is photographed at once, the collimator is now focused for a point near the end of the ultra-violet portion (about λ 2300 in a quartz spectrograph), when the definition in the red, though not good, is passable; whereas if the collimator were focused for the less refrangible rays, the definition in the far ultra-violet would not be sufficiently good.

The influence of the focusing of the collimator upon the curvature of the spectrum will be discussed further on.

II. *The Prism System.*—Although compound prisms are used in some forms of spectroscope (*e. g.* the Evershed Solar Spectroscope) where compactness combined with a large dispersion is desired, they are seldom or never used in spectrographs, because the dense flint glasses used in their construction absorb strongly in the violet and ultra-violet regions of the spectrum.

If i_1, i_1', i_2, i_2' are the angles of incidence and refraction at the first and second surfaces of the prism respectively, and α the refracting angle, we have by differentiation of the equations

$$\sin i_1 = n \sin i_1'$$
$$i_2 = i_1' - \alpha$$
$$\sin i_2' = n \sin i_2$$

the following expression for the dispersion of a single prism in air—

$$\delta i_2' = - \frac{\sin \alpha}{\cos i_1' \cos i_2'} \delta n.$$

The dispersion thus depends both upon the refracting angle α of the prism and the angle of incidence i_1 of the mean ray. The dispersion will be a maximum (∞) at grazing emergence ($\cos i_2' = 0$); but on account of the great loss of light by reflection it is not practicable to use the prism in this position; further, the image of the slit would appear very broad. The dispersion will be a minimum for some particular value of i, determined by the product

$$\cos i_1' \cos i_2'$$

being a maximum. Thollon * has shown that this position is given approximately by the equation

$$i_1' = - n^2 i_2.$$

For instance, with a 60° prism of refractive index 1·5, minimum dispersion would be reached for the position $i_1 = 84° 15'$; for a prism of refractive index 1·6, this position would not be reached at all, dispersion decreasing continuously until $i_1 = 90°$. A prism of 40° and refractive index 1·77, would exhibit minimum dispersion at $i_1 = 62° 44'$ and minimum deviation at $i_1 = 37° 15'$. Thus, the higher

* *Comptes Rendus*, lxxxix, 93. (1879.)

the refractive index of a prism, the smaller must be its refracting angle if minimum dispersion is to be obtained.

Hence, if the prism is set at minimum deviation for a ray which has a refractive index of, for example, 1·6, the ray which has a refractive index of 1·5 will traverse the prism nearer the position of minimum dispersion, and the less refrangible end of the spectrum will be compressed while the more refrangible end will be relatively expanded; so that if the prism is adjusted at minimum deviation for the more refrangible end, the spectrum will be relatively short.

In single-prism quartz spectrographs, where the whole spectrum is photographed at once, it has been found by several observers, and independently by instrument makers themselves, that the prism (of the Cornu type) should be set at minimum deviation for the more refrangible portion of the ultra-violet (about λ 2200). This is discussed in connection with the results obtained with single-prism quartz spectrographs.

When we are dealing with instruments with more than one prism, the prisms are set, like the collimator, for the middle of that portion of the spectrum which is being photographed; if this were not done, the outer rays would traverse the further prisms too far from the position of minimum deviation.

As was pointed out by Pickering * in 1868, those prisms are the most favourable which completely polarize the incident light on reflection at a surface, since the loss of light by reflection cannot then exceed one-half. We have then—

$$\tan i, = n$$
$$\sin i,' = \frac{1}{n} \sin i,$$
$$a = 2\, i'_1.$$

With $n = 1{\cdot}5$ we find $a = 67° 22{\cdot}8'$
,, $n = 1{\cdot}65$,, $= 62° 26{\cdot}2'$
,, $n = 1{\cdot}7$,, $= 60° 55{\cdot}8.$

Thus the most favourable prism-angles for glasses used in spectroscopy lie between 60° and 67°, and we may consider the usual 60° prism as good in this respect.

The important subject of the loss of light in prisms was dealt with by Pickering (*loc. cit.*) and also by Hartmann.† From Hartmann's results it appears that for the same dispersion it is better to employ a small number of prisms of large angle than a large number of small angle, unless the dispersion is very large in comparison with the number of prisms. For instance, with prisms of flint glass 0·102 ($n_D = 1{\cdot}6489$), one prism of 60° gives more light than two of 45°, although both systems give about the same dispersion; two prisms of 64° give the same illumination and the same dispersion as three of 57°, but less light than four prisms of 49°; four prisms of 60° give more light than five of 55°, but five prisms of 62° give more light than four prisms of 65°, for the same dispersion. Hartmann further shows that

* *American Journal of Science*, (2) xlv. 301.
† *Zeitschrift für Instrumentenkunde*, xx. 20. (1900.)

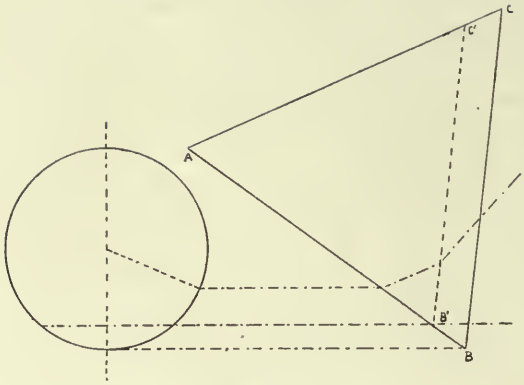


FIG. 1.

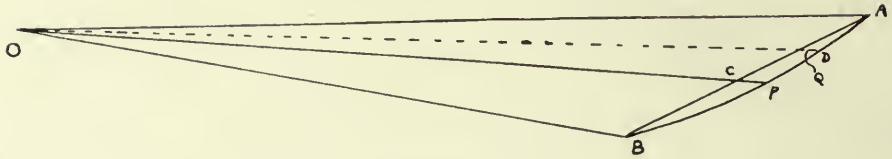


FIG. 2.

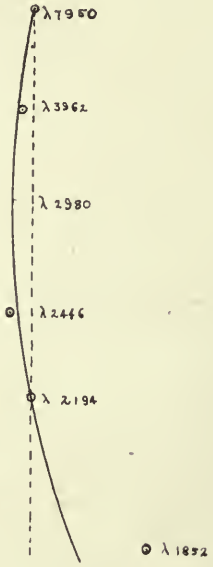


FIG. 3.

[*To face p.* 125.

compound prisms are nearly always inferior to simple prisms in illumination, apart from their opacity to the ultra-violet, and the great influence which errors of the surfaces have at high angles of incidence on the flint components. He further shows that a small prism, which does not include the whole of the beam from the collimator, may be more efficient in transmitting light than a prism which completely fills the beam. This apparently remarkable result is due to the loss by absorption in the larger prism being greater than the loss of the light which passes outside the apex of the smaller prism. Thus the prism AB′C′ is more efficient than the larger prism ABC. (Fig. 1).

As the beam is spread out by dispersion, the more remote prisms require to be increased in size to take in the whole of it. When a fairly long stretch of spectrum is required at one exposure, the requisite increase in size of for example, say, the fourth prism, is very marked.

III. *The Camera Objective.*—Considerable attention has recently been given to the design of special camera objectives for spectrographs, and their necessary requirements are now well specified. They are as follows—

1. The system should consist of thin lenses of non-absorbing material, and the design should be such as not to give rise to a flare spot.

2. The objective need not be achromatic, since the plate may be tilted to take up the difference in focus of the different parts of the spectrum. In single-prism quartz spectrographs the plate is tilted more than 60° from the plane normal to the axis; but very high tilts are objectionable, since the sizes of the blur-circles (or rather ellipses) rapidly increase, and the coarseness of the grain of the plate becomes apparent; so that it is preferable to use achromatic systems if the more important conditions mentioned below are not thereby sacrificed.

3. The system must be free from spherical aberration, its aperture being determined by the amount of illumination desired. The spherical aberration should, if possible, be corrected for two or more parts of the spectrum; at any rate, the chromatic variation of spherical aberration must not be large.

4. The system must be free from coma; if possible, for two or more parts of the spectrum. Any considerable variation of coma with refractive index is highly objectionable, and must be avoided in the design. It is to be noted that not only does the refractive index vary at different points along the spectrum, but the convergence of the light incident upon the camera objective changes also.

5. If the instrument is to be used only with a slit, the correction of astigmatism is not essential; but it is extremely desirable that the vertical images of the slit should be brought to a focus on a flat plate; hence we need only consider the correction of the curvature of the spectrum-image formed by the tangential rays.

The most important consequence of these conditions is that the objective may be made of one material only, which is usually the same as that of the prisms. Since the image need not be homocentric, the Petzval condition may be largely ignored. It will be noted that the problem of rendering a spectrum flat, in which the images

of a single slit, due to light of different wave lengths, are spread over a photographic plate, is not quite the same as that of flattening the image of an extended object produced by white light, where, moreover, the plate is practically perpendicular to the axis; in the case of a spectrum the prism produces what are virtually a number of slits in contiguous positions, and the camera objective produces images of these, not usually at a constant focal distance.

In instruments to be used for sources other than narrow slits, however, astigmatism must be corrected, and the problem is then a different one; in this case the curvature due to the change of refractive index with wave length should be compensated by the Petzval curvature due to the obliquity; both the Petzval curvature and the obliquity varying with the wave length.

I have found the Seidel expressions for the aberrations convenient in the design of these camera objectives. When, however, the angles of incidence are large, trigonometrical methods should also be employed.

A simple thin lens is often used as camera objective in quartz spectrographs, but is, of course, far from perfect. If the corrections are made for about λ 2980 ($n = 1{\cdot}5771$) we have $r_2 : r_1 = 9{\cdot}38$ for correction of coma, and $r_2 : r_1 = 6{\cdot}18$ for minimum spherical aberration. In practice the ratio $r_2 : r_1$ is taken about midway between these, and spherical aberration is removed as far as possible by figuring the lens. Such lenses impart considerable curvature to the spectrum, making it concave towards the lens. In small, single-prism quartz spectrographs a converging meniscus is often used, with its concave side towards the prism; a stop is interposed between the lens and the prism. This design enables a flat field to be obtained with these instruments, but the system cannot be corrected for spherical aberration (except by figuring), and is necessarily affected by coma.

Systems which consist of a combination of positive lenses are sometimes employed; for instance, the camera objective may consist of a system of two convergent meniscus lenses, symmetrically disposed with respect to a central stop. Such a system has considerable under-corrected spherical aberration, and therefore the diameter of the stop must be kept small, which increases the time of exposure. This system can, however, be made free from coma, and with a small stop $\left(\frac{F}{20}\right)$ the definition is excellent; the curvature, moreover, is small, for the system may be so constructed that the mean radius of curvature of the spectrum is 5·4 times the equivalent focal length for λ 2980.

A *résumé* of various types of camera objectives for one- and three-prism instruments has been given by Plaskett.* His experiments show very clearly how the ordinary achromatic telescope objective, or even the triple objective, always render the spectrum strongly concave towards the lens. For three-prism spectrographs he considers the Zeiss "Chromat" to be the best. This is a non-achromatic objective designed by Hartmann,† of the same material as the prisms, and consisting

* *Astrophysical Journal*, xxix, 290. (1909.)
† *Zeitschrift für Instrumentenkunde*, xxiv, 257. (1904.)

of a thin positive and a thin negative lens with a small air-lens between them; the plate has, of course, to be tilted (14°), but the definition is sharp over the whole of a flat plate (field of 14°).

In a large four-prism stellar spectrograph which I recently had occasion to design (p. 122), the deviation was 180° for Hγ (λ 4341). The most important part of the spectrum for the purposes of this instrument was that comprised between λλ 4730 and 4102, corresponding to an angular dispersion of 2° 2½′ and 2° 12½′ respectively, from the axial ray λ 4341. The four prisms of light flint glass ($n_D = 1{\cdot}5751$; ${}_c\nu_F = 41{\cdot}7$) were adjusted at minimum deviation for this ray. The collimator lens was a cemented doublet of borosilicate crown ($n_D = 1{\cdot}5108$, ${}_c\nu_F = 63{\cdot}0$), and light flint (the same as was used for the prisms), slightly chromatically over-corrected with respect to the rays 4730 and 4102 (the difference of power for these rays being 0·00019 times the total power for Hγ); it was carefully freed from spherical aberration for Hγ, the work being done trigonometrically; its focal length for Hγ was 24·5 ins., and the slit was placed at the focus for this wave length; the clear aperture was 1·5 in.

The camera lens had a focal length of 18 ins. for Hγ and an aperture of 2·375 ins., of which, however, the marginal portions were occupied only by the outer deviated rays of the spectrum. It was finally designed to consist of two meniscus lenses, the powers of which were in the ratio $-2 : +3$; there was a small air space enclosed between the lenses, of which the negative one was towards the incident parallel light. Both lenses were of the same glass as the prisms (light flint), so that a tilt had to be given to the photographic plate. The correction of the objective for spherical aberration and coma was made with respect to the ray Hγ. With this system the spectrum (λλ 4730 – 4102) was 1·38 ins. long, and the curvature over this range, as measured by the sag of the plate (concave towards the lens) was only 0·004 in., which corresponds to a radius of curvature of 61·20 ins., or 3·4 times the focal length of the camera objective; over the whole of a quarter-plate (corresponding to a total field of 13° 30′), the sag would be just 1 mm. The length of the comatic flare at the edges of the spectrum was only 0·000169 in. for λ 4730 and 0·000096 in. for λ 4102.

This result may now be compared with that given by an ordinary cemented achromatic object-glass similar to that of the collimator. Such a system was originally calculated for the instrument in question, both the collimator (focal length 26·25 ins. for λ 4341) and the camera objectives being made, however, of borosilicate crown (as above) and ordinary flint ($n_D = 1{\cdot}6184$, ${}_c\nu_F = 36{\cdot}4$). With this system, which gave a spectrum 1·322 ins. long, the tilt was only 0° 29′, but the sag of the plate was now 0·0282 in., corresponding to a radius of curvature of 7·745 ins., or 0·43 times the focal length of the camera objective; the sag for the whole length of a quarter-plate, corresponding to a total field of 13° 28′, would be no less than 0·2917 in. There was also considerable inward coma at the edges of the field. Hence the advantage of using a non-achromatic combination is at once apparent.

The above figures are the result of trigonometrical calculation of the ray-paths. When the curvature is small, it may be further controlled by means of the

secondary chromatic errors of the collimator objective. For example, in the above case of the non-achromatic camera system, it was found that with a collimator objective of borosilicate crown and ordinary flint there was still a slight amount of curvature. When light flint was substituted for ordinary flint the amount of secondary spectrum was slightly increased, but the increase was almost sufficient to cause the curvature to vanish. When ordinary silicate crown and ordinary flint were tried, a calculation showed that the increase of secondary spectrum was so great that it caused the image to become convex towards the camera lens.

In this connection a few details of a spectrograph specially intended for the examination of the ultra-violet regions of corona- and chromospheric spectra may be of interest. This instrument was made by Messrs. Hilger two years ago, and a very interesting series of photographs was obtained with it during the recent solar eclipse of April 17.

In this instrument there is no slit, the light from the corona of the sun, after reflection from a heliostat, being incident directly upon the prism system, which consists of two 60° prisms of quartz adjusted in the position of minimum deviation for λ 3611. The region of the spectrum operated upon extended from the visible spectrum down to about λ 2700, and λ 3611 was chosen as the centre of this region.

The camera objective is a cemented triple of quartz and rock-salt with a focal length of 53·4 ins. and an aperture of 2·5 ins. The rock-salt component is a divergent meniscus with one side very nearly plane. This is enclosed by two convergent lenses of quartz, one of which does not deviate far from the biconvex form, the other being a weak convergent meniscus. This system was made achromatic for λλ 7682 and 2749. and the corrections for spherical aberration and coma (by the Seidel formulæ) were made for λ 3611. It will be noted that the presence of the third, though very weak lens enables coma to be corrected in the cemented system. Castor oil was used as the cement between the lenses, and the whole combination was luted into its cell to prevent access of atmospheric moisture to the rock-salt.

The following figures indicate the amount by which both the tangential and the sagittal images deviate from the plane through the image-point of λ 3611, perpendicular to the axis. In the table, θ is the angle of inclination of the ray to the axis and X is the oblique distance from the lens to the image-position, so that X cos θ

Wave Length	θ	X cos θ		Astigmatic Difference	Curvature	
		Tangential Rays	Sagittal Rays		Tangential Rays	Sagittal Rays
		inches	inches			
7682	4° 16′	53·35	53·68	+ 0·33	+ 0·05	— 0·28
4861	2° 28	53·33	53·45	+ 0·12	+ 0·07	— 0·05
3611	0	53·40	53·40	—	—	—
3034	2° 41′	53·40	53·50	+ 0·10	+ 0·0	— 0·10
2749	4° 51′	53·26	53·60	+ 0·34	+ 0·14	— 0·20

represents the distance parallel to the axis from the image to the second principal plane of the lens.

It will be observed that the tangential image is concave towards the object glass while the sagittal image is convex.

A small observing telescope at the side of the instrument, fitted with a reflecting prism, serves to ascertain whether the sun's rays are incident upon the first prism at the proper angle.

Single-Prism Quartz Spectrographs

I will now give a few of the results which I have recently obtained during the course of some calculations with regard to these instruments. The object was to ascertain the effect of various factors upon the image, with a view to the possible improvement of these spectrographs.

The instrument assumed for most of the calculations was as follows—

Collimator Lens.—A simple double convex lens of quartz, $r_1 : r_2 = 6{\cdot}18$, this being the required ratio for minimum spherical aberration for the centre of the spectrum (λ 2980). The focal lengths were as follows—

Wave Length.	*Focal Length.*
7950	24·131 ins.
3962	·23·278 ,,
2980	22·517 ,,
2446	21·492 ,,
2194	20·792 ,,
1852	19·226 ,,

Prism.—Length of face, 2·56 ins.; Cornu type; centre of face, 2·16 ins. from collimator lens.

Camera Lens.—A simple quartz lens assumed to be close to the prism. If this assumption is made, the calculation of the curvature is much simplified, since the curves of the lens then have no effect upon it, the addition to the Gauss-value of the convergence of the rays being simply $\left(3+\frac{1}{n}\right)\frac{\tan^2\theta}{2f}$ where θ is the inclination of the chief rays to the axis, and f the focal length of the lens. As no diaphragm is assumed to be interposed, the virtual position of the stop for the camera lens will be where the rays of different wave length cross the axis (λ 2980) after emergence from the prism; this position is not strictly constant throughout the spectrum, but its average position is about 0·23 in. inside the prism. The total angular field is only about 8°, so that the assumption that the stop is on the surface of the lens is very nearly justified (in fact, it was shown by calculation in one case that the error introduced by this assumption into the estimation of the curvature was only 0·4 per cent.).

The focal lengths of the camera lens were—

Wave Length.	*Focal Length.*
7950	22·816 ins.
3962	22·010 ,,
2980	21·291 ,,
2446	20·322 ,,
2194	19·659 ,,
1852	18 179 ,,

The factors which were varied were—

(1) The position of the prism with respect to the incident light.

(2) The point of the spectrum for which the collimator was focused.

(3) The length of the collimator.

In each case the three wave lengths, 7950, 2980, 2194 were traced through the apparatus, the astigmatic effect of the prism being taken into account, and a straight line was assumed drawn from the image-line of λ 7950 to that of λ 2194.

Thus (Fig. 2) let O be the assumed common stop and A, P and B the points where the tangential image-lines of λλ 7950, 2980 and 2194 respectively meet the horizontal plane. Let AB meet OP in C. Then CP represents what is here called the " curvature " (much exaggerated in the figure); it is seen to be approximately equal to the sag of the plate divided by the cosine of the angle of tilt (*i. e.* 90°— ∠ ACP), assuming the plate adjusted so that λλ 7950 and 2194 focus simultaneously upon it; it is the difference in focus in the direction of the axis, between the centre and the edges of the spectrum. The length CP (= OP − OC) is easily calculated by successive solution of the triangles AOB and AOC.

Let a circle be drawn through the points APB. Then its polar equation may be written

$$r^2 + A\,r\cos\theta + Br\sin\theta + C = 0.$$

Let O be taken as the pole and OP as the initial line. Then from the known lengths OA, OP, OB, and the known angles POA, POB, the constants A, B and C in the above equation may be determined. The radius (R) of the circle APB is then given by

$$R = \sqrt{\tfrac{1}{4}(A^2 + B^2) - C}.$$

This may be taken as giving an indication of the amount of curvature of the spectrum.

In some cases the oblique focal distances for λλ 3962, 2446 and 1852 were also computed, and then the radii vectores of the circle APB corresponding to the angles θ for these wave lengths were calculated from the above equation. The difference between these quantities for a given ray gives the amount by which the spectrum at that point deviates from the circular form APB. Thus, if OQD represents the path of the ray λ 3962, Q the position of its tangential image-line,

and D the point where OD meets the circle APB, QD is the amount of deviation from the circle APB.

I regret that, in the time allowed for preparing this paper, I have not been able to find the radius of curvature at several points along the spectrum. I considered, however, that the above expedient of tracing through λλ 3962, 2446 and 1852 would give some idea of the shape of the spectrum image in absence of a knowledge of the radius of curvature at different points.

The following is the notation used in the tables below—

i_1, the angle of incidence at the first surface of the prism.
i_1', ,, refraction ,, ,, ,, ,,
i_2, ,, incidence ,, second ,, ,,
i_2', ,, refraction ,, ,, ,, ,,
θ, the angle made by the rays with the axis on leaving the prism. (For λ 2980, which traverses the axis of the camera, $\theta = 0$.)
u', distance from the collimator lens of the image of the slit formed by it.
s', distance of the focal line of the tangential rays from the second surface of the prism, after refraction through the prism.
x, distance (oblique to the axis) of the image from the pole O. In the case of λ 2980 x represents the (axial) distance of the centre of the plate from the origin.
r, θ, the polar co-ordinates of points on the circle through the image-points of λλ 7905, 2980 and 2194.
R, the radius of this circle.
$x - r$, the deviation of the image from this circle.

The direction of the incident light is taken as the positive direction.

All dimensions are in inches.

The expression " tilt of plate " here signifies the angle through which the plate has to be turned from perpendicularity to the axis, in order that the rays of λλ 7950 and 2194 may focus simultaneously upon it.

The expression

$$\delta i_2' = \frac{\cos i_1 \cos i_2}{\cos i_1' \cos i_2'} \delta i_1 = m \, \delta i_1$$

is clearly a measure of the apparent breadth of the slit as seen from the point where the light leaves the second surface of the prism, and hence is directly related to the width of the spectrum line as seen on the plate. The factor m is therefore included among these results. In the position of minimum deviation $m = 1$.

To determine the effect of the above factors (1) and (2), five calculations were made—

(*a*) Prism adjusted at minimum deviation for the centre of the spectrum (λ 2980) and collimator focused for the same position.

(*b*) Prism adjusted at minimum deviation for λ 2194 and collimator focused for λ 7950.

(*c*) Prism adjusted at minimum deviation for λ 2194 and collimator focused for the same position.

(*d*) Prism adjusted at minimum deviation for λ 7950 and collimator focused for λ 2194.

(*e*) Prism adjusted at minimum deviation for λ 7950 and collimator focused for the same position.

These cases will be considered separately.

(*a*) *Prism and collimator both adjusted for* λ 2980.

This is probably the adjustment which one would naturally make for a spectrograph. The results are given below.

	λ 7950	λ 3962	λ 2980	λ 2446	λ 2194	λ 1852
i	52° 3′ 0″	52° 3′ 0″	52° 3′ 0″	52° 3′ 0″	52° 3′ 0″	52° 3′ 0″
i'	48° 34′ 24″	50° 19′ 21″	52° 3′ 0″	54° 41′ 9″	56° 44′ 32″	62° 25′ 26″
θ	+ 3° 28′ 36″	+ 1° 43′ 39″	∝	− 2° 38′ 9″	− 4° 41′ 32″	− 10° 22′ 26″
u'	− 336·810	− 689·047	∝	+ 472·094	+ 271·301	+ 131·523
s'	− 379·905	− 733·513	∝	+ 424·429	+ 222·242	+ 79·0100
x	24·1423	22·6616	21·2910	19·3386	17 9067	14·2276
r	,,	22·6739	,,	19·3205	,,	14 5587
$x - r$	—	− 0·0123	—	+ 0·0181	—	− 0·3311
m	0·9447	0·9709	1·000	1·052	1·100	1·277

Total dispersion (λλ 7950 − 2194) = 8° 10′ 8″.
Length of spectrum λλ 7950 − 2194 = 6·90325.
Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 1{\cdot}0006.$
Tilt of plate = 64° 53′ 45″.
Curvature = 0·31783.
R = 43·9491.

(*b*) *Prism adjusted for* λ 2194; *collimator for* λ 7950.

	λ 7950	λ 3962	λ 2980	λ 2446	λ 2194	λ 1852
i	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″
i'	46° 29′ 5″	48° 10′ 52″	49° 50′ 58″	52° 22′ 49″	54° 20′ 26″	59° 40′ 16″
θ	3° 21′ 53″	1° 40′ 6″	—	2° 31′ 51″	4° 29′ 28″	9° 49′ 18″
u'	∝	+ 658·870	+ 336·810	+ 196·570	+ 150·264	+ 94·5870
s'	∝	+ 810·578	+ 396·754	+ 209·160	+ 151·249	+ 71·9227
x	22·7125	21·4039	20·2025	18·4666	17·2544	14·0176
r	,,	21·4161	,,	18·4830	,,	14·3646
$x - r$	—	− 0·0122	—	− 0·0164	—	− 0·3470
m	0·8790	0 8940	0·9225	− 0·9625	1·000	1·132

Total dispersion (λλ 7950 − 2194) = 7° 51′ 21″.
Length of spectrum (λλ 7950 − 2194) = 6·09479.

Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 1{\cdot}013.$

Tilt of plate = 63° 52′ 15″.

Curvature = 0·24672.

R = 42·5220.

(*c*) *Prism and collimator both adjusted for* λ 2194.

	λ 7950	λ 3962	λ 2980	λ 2446	λ 2194	λ 1852
i	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″
i′	46° 29′ 15″	48° 10′ 52″	49° 50′ 58″	52° 22′ 49″	54° 20′ 26″	59° 40′ 16″
θ	3° 21′ 53″	1° 40′ 6″	—	2° 31′ 51″	4° 29′ 28″	9° 49′ 18″
u′	− 150·264	− 194·658	− 271·301	− 637·869	∝	+ 255·278
s′	− 197·643	− 243·558	− 321·650	− 690·320	∝	+ 197·446
x	25·6344	24·1515	22·7922	20·8816	19·5024	16·0695
r	,,	24·1617	,,	20·8673	,,	16·3229
x − *r*	—	− 0·0102	—	+ 0·0143	—	− 0·2534
m	0·8790	0·8940	0·9125	0·9625	1·000	1·132

Total dispersion (λλ 7950 − 2194) = 7° 51′ 21″.

Length of spectrum (λλ 7950 − 2194) = 6·85454.

Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 1{\cdot}015.$

Tilt of plate = 63° 45′ 1·8″.

Curvature = 0·25341.

R = 52·2566.

(*d*) *Prism adjusted for* 7950; *collimator for* λ 2194.

	λ 7950	λ 3962	λ 2980	λ 2446	λ 2194	λ 1852
i	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″
i′	50° 17′ 14″	52° 5′ 11″	53° 52′ 12″	56° 36′ 25″	58° 45′ 28″	64° 47′ 52″
θ	3° 34′ 58″	1° 47′ 1″	—	2° 44′ 13″	4° 53′ 16″	10° 55′ 40″
u′	− 150·264	− 194·658	− 271·301	− 637·869	∝	+ 255·278
s′	− 152·877	− 185·544	− 240·948	− 501·091	∝	+ 125·039
x	26·6084	24·9077	23·3398	21·1120	19·4737	15·2130
r	,,	24·9233	,,	21·0879	,,	15·6609
x − *r*	—	− 0·0156	—	+ 0·0241	—	− 0·4479
m	1·000	1·030	1·067	1·130	1·190	1·420

Total dispersion (λλ 7950 − 2194) = 8° 28′ 14″.

Length of spectrum (λλ 7950 − 2194) = 7·88726.

Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 0{\cdot}9980.$

Tilt of plate = 65° 5′ 25″.

Curvature = 0·36411.

R = 50·4421.

(*e*) *Prism and collimator both adjusted for* λ 7950.

	λ 7950	λ 3962	λ 2980	λ 2446	λ 2194	λ 1852
i	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″	50° 17′ 14″	50° 17′ 14′
i'	50° 17′ 14″	52° 5′ 11″	53° 52′ 12″	56° 36′ 25″	58° 45′ 28″	64° 47′ 52″
θ	3° 34′ 58″	1° 47′ 1″	—	2° 44′ 13″	4° 53′ 16″	10° 55′ 40
u'	∝	+ 658·870	+ 336·810	+ 196·570	+ 150·264	+ 94·5870
s'	∝	+ 617·276	+ 294·005	+ 152·007	+ 104·409	+ 46·5994
x	22·6987	21·2243	19·8459	17·8559	16·3730	12·5234
r	,,	21·2374	,,	17·8359	,,	12·8577
$x - r$	—	− 0·0131	—	+ 0·0200	—	− 0·3343
m	1·000	1·030	1·067	1·130	1·190	1·420

Total dispersion (λλ 7950 − 2194) = 8° 28′ 14″.
Length of spectrum (λλ 7950 − 2194) = 6·93704.
Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 0{\cdot}9840.$
Tilt of plate = 66° 4′ 22″.
Curvature = 0·38858.
R = 37·9126.

From these results several interesting facts are deducible.

Orientation of the Prism.—Comparing positions (*c*) and (*d*) and also (*b*) and (*e*), we may draw the following inferences as to the effect of adjusting the prism for a point in the far ultra-violet—

1. Reduction of total angular dispersion. (The reason for this is set forth on p. 124.)
2. Reduction of the length of the less refrangible part of the spectrum relatively to that of the more refrangible.
3. Considerable lessening of the tilt.
4. Shortening of the spectrum.
5. Diminution of curvature, as measured by that motion of the inclined plate in the direction of the axis, which is necessary in passing from the common focal plane of λλ 7950 and 2194 to that of the mean ray of the spectrum.
6. Slight increase in what may here be termed the " radius of curvature " of the spectrum, as measured by the radius of the circle through the image points of λλ 7950, 2980 and 2194.
7. Over the greater part of the spectrum the angular magnification of the breadth of the slit-image does not exceed unity.

Adjustment of the Collimator.—Comparing positions (*b*) and (*c*) and (*d*) and (*e*), we notice the following effects of focusing the collimator for a point in the far ultra-violet—

1. Diminution of the tilt (considerable when the prism is set for λ 7950.)

2. Lengthening of the spectrum.
3. Curvature practically unaltered, but—
4. "Radius of curvature" largely increased, owing to the lengthening of the spectrum.

The values of $x - r$ give some indication as to the shape of the spectrum-image. This matter will be made clearer on referring to Fig. 3 (not drawn to scale), which gives a diagrammatic representation of the effect. The photographic plate is shown perpendicular to the axis of the camera in order that the effect may be more apparent. It is seen that in no case is the spectrum a circle. In every case the image-line for λ 1852 lies considerably inside the circle through the image-lines of λλ 7950, 2980 and 2194. The quantity $x - r$ is numerically least in position (*c*) and greatest in position (*d*). In every case, with the exception of (*b*), the image-line for λ 2446 lies outside the circle in question, the quantity $x - r$ being numerically least in position (*c*) and greatest in position (*d*). In all cases the image-line for λ 3962 lies inside the circle in question, $x - r$ being again least in (*c*) and greatest in (*d*).

Hence when the prism is set for λ 2194 the rapid falling-off of the focus in the extreme ultra-violet is not so intense as when the prism is set for the red (λ 7950). In fact, the setting of the prism for λ 7950 tends to flatten the visible end of the spectrum at the expense, so to speak, of the far ultra-violet.

The inference from the above results is that the best setting for both prism and collimator is that the prism should be set in the position of minimum deviation for a point far into the ultra-violet, and that the collimator should also be focused for a point in this region — in the present case, λ 2194.

No advantages are apparent in using the prism and collimator in their mean positions (λ 2980). Tilt and curvature are both increased, but the spectrum is scarcely increased in length.

I will now give the results of some calculations made to ascertain the effect of altering the length of the collimator in a single-prism quartz spectrograph. The instrument assumed was the same as for the above calculations, with the exception of the collimator. Both prism and collimator were adjusted for λ 2194.

(*a*) *Collimator made twice its previous length* (*i. e.* focal length of collimator lens for λ 2980, 45·034 ins., of camera lens, 21·291 ins.).

Length of spectrum (λλ 7950 − 2194) = 5·47647.

Ratio $\dfrac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 1{\cdot}080.$

Tilt of plate = 57° 30′ 33″.

Curvature = 0·1809.

R = 38·4198.

(*b*) *Infinitely long collimator* (*i. e.* a collimator simultaneously in focus for all the rays of the spectrum: a perfectly achromatic objective would fulfil this condition).

Length of spectrum (λλ 7950 − 2194) = 4·31463.

Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 1{\cdot}145.$

Tilt of plate = 48° 28′ 46″.

Curvature = 0·123.

R = 28·2630.

From these figures the following effects of lengthening the collimator are observable—

1. Reduction of the tilt; and—
2. Diminution of the length of the spectrum.
3. Although the sag of the plate is reduced, the radius R is greatly reduced, so that the amount of bending required to be given to the plate is really greater than before.
4. Of course, the effects of astigmatism in the prism will be reduced as the collimator is lengthened.

Use of Vitreous Silica in Spectrographs

The advances now being made in the manufacture of vitreous silica for optical purposes made it interesting to try the effect of substituting it for quartz. Some calculations were therefore made with a single-prism spectrograph, the collimator and camera lenses of which were of equal focus (21·2910 ins. for λ 2980), the prism and collimator being assumed adjusted for λ 2194; the assumptions made in the calculations were the same as in the above cases.

1. *Prism and lenses of vitreous silica.*

	λ 7950	λ 2980	λ 2194
i	49° 52′ 2″	49° 52′ 2″	49° 52′ 2″
i'	43° 29′ 10″	46° 21′ 22″	49° 52′ 2″
θ	2° 52′ 12″	—	3° 30′ 40″
u'	− 137·36	− 255·36	∝
s'	− 165·5	− 285·2	∝
x	26·47	23·00	19·55
m	0·9200	0·9510	1·000

Total dispersion (λλ 7950 − 2194) = 6° 22′ 52″.

Length of spectrum (λλ 7950 − 2194) = 7·370.

Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 0{\cdot}9030.$

Tilt of plate = 69° 59′.

Curvature = 0·20.

R = 99·23.

2. *Prism of quartz, lenses of vitreous silica.*

	λ7950	λ 2980	λ 2194
i	54° 20′ 26″	54° 20′ 26″	54° 20′ 26″
i'	46° 29′ 5″	49° 50′ 58″	54° 20′ 26″
θ	3° 21′ 53″	—	4° 29′ 28″
u'	− 137·359	− 255·356	∝
s'	− 180·953	− 302·921	∝
x	26·0898	22·8915	19·4933
m	0·8790	0·9225	1·000

Total dispersion (λλ 7950 − 2194) = 7° 51′ 21″.
Length of spectrum (λλ 7950 − 2194) = 7·28427.
Ratio $\frac{\text{length of spectrum } \lambda\lambda\ 2980 - 2194}{\text{length of spectrum } \lambda\lambda\ 7950 - 2980} = 0{\cdot}9960.$
Tilt of plate = 65° 10′ 50″.
Curvature = 0·15765.
R = 100·118.

Comparing these results with case (*c*) above, it is seen that the use of vitreous silica throughout produces the following effects—

1. Reduction of total dispersion.
2. Increase in the length of the less refrangible part of the spectrum, relatively to that of the more refrangible.
3. Large increase in the tilt, and—
4. Increase in the length of the spectrum (λλ 7950 − 2194).
5. Diminution of the curvature, and great increase in the mean radius of curvature, so that the photographic plate would require much less bending than in case (*c*) above.

If, however, we use a quartz prism with lenses of vitreous silica, the increase in tilt is not nearly so great, and the radius of curvature is still further increased, while the length of the spectrum is scarcely altered. This would appear to be a very favourable combination for use in spectrographs.

I much regret that the time allowed me for the preparation of this paper has not permitted me to furnish more complete results, or to discuss fully the problems met with in connection with spectrographs of two or more prisms. I am hoping, however, to extend the above methods to these instruments, and a slight sketch of the general problem may not be out of place.

In the first place we may remark that only when a system of m prisms is adjusted in the position of minimum deviation for a certain ray is the dispersion in that region m times the dispersion of a single prism. In spectroscopy, however, the prisms are usually so adjusted, and the angular dispersion with two prisms is about twice as

great as with one. The tilt will, however, be reduced by the increased dispersion (assuming the focus unaltered) and, therefore, the length of the spectrum will not be increased in proportion. Secondly, it will be seen that it is more important to have the light well collimated than it is with one prism—thus, an achromatic objective should be used in the collimator where possible; unless the incident light is parallel, the astigmatic effect of the prisms upon rays that will depart considerably from the position of minimum deviation in the further prisms will be serious. Thirdly, the increased angles (θ) at which the outer rays of the spectrum are inclined to the axis of the camera will cause the mean radius of curvature of the spectrum to be less for the same focus of the camera.

In conclusion, I wish to express my thanks to Messrs. A. Hilger, Ltd., for the use of their eight-figure calculating machine.

Mr. A. S. Eddington said that the problem was of great interest to them at Greenwich, where they had to adjust a four-prism spectrograph, the plate had to be at great tilt and they were going to use a curved film. The complications were too great for calculations, and they would try an experimental method; they were not in a position to design a new instrument, but would have to use the quartz spectrograph which they possessed.

The Astronomer Royal remarked that Mr. Eagle had rather cut the knot for them. His paper would show them what wave-length to use in setting the prism and collimator.

Mr. S. D. Chalmers pointed out that experimental methods were not so well adapted to the problem of selecting the best focal length for the collimator or the most favourable type for the objective.

THE VARIATIONS OF A BINOCULAR TESTING BENCH

By William Shackleton, F.R.A.S., A.R.C.Sc.

(*Tuesday, June 25th*, 1912.)

In the *Proceedings of the Optical Convention*, No. 1, 1905, p. 143, Dr. Drysdale described a method of testing the parallelism of the axes of binoculars. This method is now used by many manufacturers and others for testing the accuracy of the finished article. Briefly the method described in the first volume of the *Proceedings* consisted in allowing two parallel beams of light to fall on the objectives of a binocular, the emergent beams from the eye-pieces being focused on a screen by means of a convex lens, Fig. 1; if the images coincided on the screen the binocular was in

adjustment, whilst want of coincidence indicated that the axes of the binocular were not parallel.

The above answered well, the objections to its use being that two collimators were needed, also a lens of rather short focal length and large diameter to form the images on the screen. This apparatus was modified so that only one collimator and a projection lens of less diameter were required.

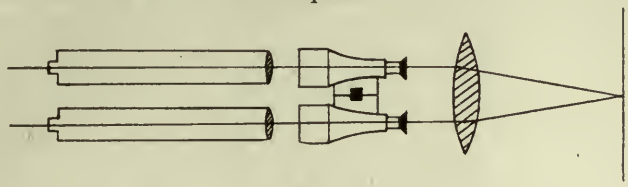


Fig. 1.

In using this modification, one barrel of a binocular is interposed between the collimator and the convex lens; the binocular is then traversed by means of a suitable slide, transversely to the line of collimation until the second barrel is in the position formerly occupied by the first barrel; the amount of displacement of the image on the screen gives a measure of the inclination of the axes of the binocular (Fig. 2).

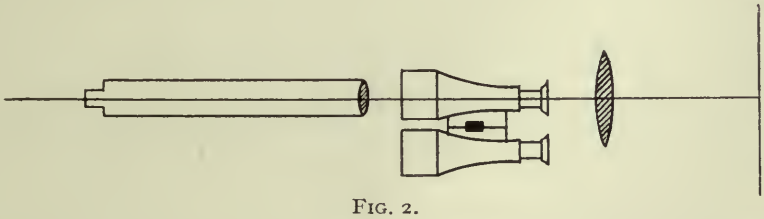


Fig. 2.

This method is satisfactory for the usual Galilean binocular with magnifying powers of not more than six, the only objection—which also applies to the preceding arrangement—is that a somewhat darkened room is required, and also if the projection lens is of the same focal length as the collimating lens the image on the screen

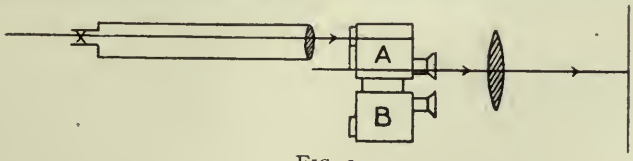


Fig. 3.

is somewhat coarse, since its width is the width of the collimator slit × magnifying power of binocular. Such a magnified image is not suitable for reference.

When we come to test prismatic binoculars of large stereoscopic power and increased magnifying power, the arrangement described in Fig. 2 is quite unsuitable, for if we arrange the apparatus for axial transmission as in Fig. 3, we get on inter-

posing barrel " B " the emergent beam striking the projection lens on the margin or missing it altogether if the lens is smaller than the inter-objective width of the binocular (Fig. 4).

If the projection lens were affected with spherical aberration, considerable displacement might be observed of the screen image, if the emergent rays from the binocular did not traverse the lens under similar conditions. We can arrange the projection lens, if its diameter allows, in such a way that the emergent pencil from barrel " A " traverses near the margin of the convex lens, and on interposing barrel

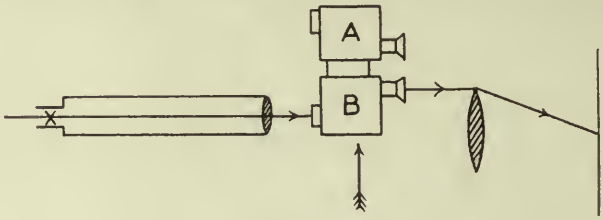


FIG. 4.

" B " the emergent beam falls on the other margin but on the same zone, thus eliminating displacement due to spherical aberration of the projection lens.

When tests at numerous interocular separations are being made, it becomes laborious to adjust the binocular each time in such a position as to minimize errors due to spherical aberration. Now in modern prismatic binoculars the inter-objective width often exceeds five inches, thus a large projection lens is necessary, also the magnifying power of prismatic binoculars is usually greater than that of the Galilean type, hence the screen image is correspondingly coarser and more difficult to accurately observe.

It is obvious, however, that such an optical system as above described is capable of other variations, but before examining these, we will see what angles we wish

FIG. 5.

to measure and select the optical system best suited for the purpose we have in view. Suppose that when barrel " A " of the binocular is placed before the collimator, the emergent beam proceeds in the direction SX. If barrel " B " is now interposed, having an inclination κ to the axis of barrel " A," the emergent beam will diverge at an angle δ from the original direction SX, as shown in Fig. 5.

Now— $\phi = \delta + \kappa$

and— $\frac{\phi}{\kappa} = M$ (magnification)

$\therefore \delta = \kappa (M - 1)$ . . . . . (I)

This formula applies to all optical combinations giving erect images, whether of the prismatic binocular type, Galilean or erecting binocular telescope.

It is to be noted that κ is of the contrary sign to δ, thus if one rotates the binocular clockwise, the emergent beam moves counter clockwise, hence the formula becomes—

$$\delta = \kappa (1 - M) \quad \ldots\ldots\ldots\ldots \quad (II)$$

M is positive; for the magnification of a telescope adjusted for telescopic vision is—

$$M = -\frac{F}{f}$$

but for the Galilean, F is negative and f positive

$\therefore$ M is positive.

Hence the equation—

$$\delta = \kappa (1 - M) \quad \ldots\ldots\ldots\ldots \quad (III)$$

holds for the Galilean and erecting combinations when the usual conventions as to sign are employed.

It is rarely that inverting binoculars come under examination, but the conditions in this case are as below—

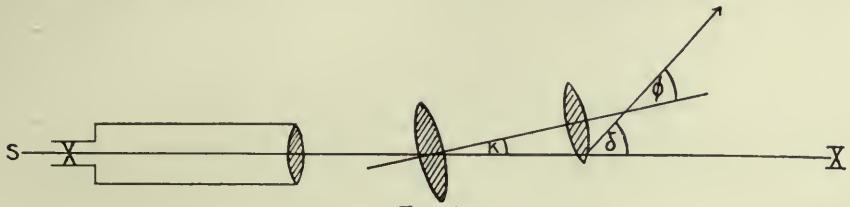


Fig. 6.

$$\delta = \phi + \kappa$$

$$\frac{\phi}{\kappa} = M$$

$$\therefore \delta = \kappa (M + 1) \quad \ldots\ldots\ldots\ldots \quad (IV)$$

but M is negative in the Kepler telescope—

$$\therefore \delta = \kappa (1 - M) \quad \ldots\ldots\ldots\ldots \quad (V)$$

which is the same equation as (II).

It will be noted that in the Kepler telescope δ is of the same sign as κ, thus if one inclines the telescope clockwise to its original position the emergent beam moves in the same direction.

If then we have a binocular the axes of which are inclined at an angle κ to each other, the emergent beams will be inclined at an angle of δ where—

$$\delta = \kappa (1 - M) \quad \ldots\ldots\ldots\ldots \quad (VI)$$

If δ is greater than the binocular accommodation of the eyes two distinct images are

seen, and even if the binocular accommodation is not exceeded, but δ is large, good seeing is difficult on account of the eyes straining to superpose the two images.

The angle δ being a function of the eyes alone, it is the same for all binoculars, and it is necessary to select a suitable value which average eyes will tolerate without strain.

In practice I have found that if an allowance of 1° in the horizontal plane and 2° in the vertical plane is made, average eyes superpose the images without strain.

On this assumption the tolerance in angle between the axes of a binocular is tabulated below—

TABLE I.

Magnifying Power.	Allowance in angle between the two axes of a binocular.	
	Horizontal Plane.	Vertical Plane.
3	30′	10′
4	20′	$6\frac{2}{3}$′
5	15′	5′
6	12′	4′
7	10′	$3\frac{1}{3}$′
8	$8\frac{4}{7}$′	$2\frac{6}{7}$′
9	$7\frac{1}{2}$′	$2\frac{1}{2}$′
10	$6\frac{2}{3}$′	$2\frac{2}{3}$′
11	6′	2′
12	$5\frac{5}{11}$′	$1\frac{9}{11}$′

Let us consider some other variations of the apparatus. In Figs. 1 and 2 we can turn the binoculars round, so that the incident rays fall on the eye-pieces and emerge at the objectives.

Then the deviation—

$$\delta' = \kappa - \phi$$

$$\frac{\kappa}{\phi} = M \quad \therefore \delta' = \kappa\left(1 - \frac{1}{M}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VII)}$$

Combining I and VII we get—

$$\delta = \delta' M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VIII)}$$

Obviously the disadvantages of this arrangement are loss of light due to the smallness of the effective aperture of the eye-pieces, and a much smaller displacement of the emergent beam for a corresponding error in binocularity, thus for projection on a screen it is unsuitable, except that some gain is experienced by the slit images being thin, since the optical train minimizes.

We can vary the arrangement of Figs. 3 and 4 by traversing the collimator transversely to the line of collimation instead of moving the binocular, and by this means considerable gain is experienced since the binoculars can easily be set so that

the emergent pencils traverse the projecting lens on the same zone; further the diameter of the lens need not be much greater than the pupillary distance.

In place of the projecting lens and screen, we may observe with a telescope when a semi-darkened room becomes unnecessary; but here a difficulty arises, for with the objectives turned to the collimator and observing the emergent beam, the slit or fiducial mark is twice magnified and becomes unsuitable for reference.

If, however, we adopt the arrangement of Fig. 7, and observe the emergent beam with a telescope of the same magnifying power as the binocular, the slit or

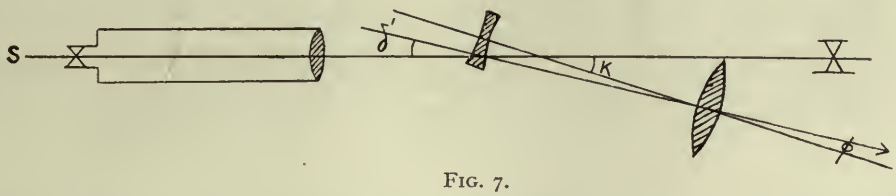


Fig. 7.

fiducial mark appears of the same size as seen through the collimator with the naked eye, and is suitable for reference, moreover, the disadvantage of a faint screen image due to the small effective aperture of the eye-pieces is no longer material since the telescope image is bright enough.

Since the observing telescope magnifies M′ times the apparent deviation $\delta M'$ (see equation VIII) can be made equal to δ.

This, then, appears to be the most suitable arrangement for testing the parallelism of the axes of binoculars. The complete arrangement is shown in Fig. 8, where the collimator and binocular remain at rest whilst the observing telescope is capable of moving parallel to itself. The advantages of this arrangement are—

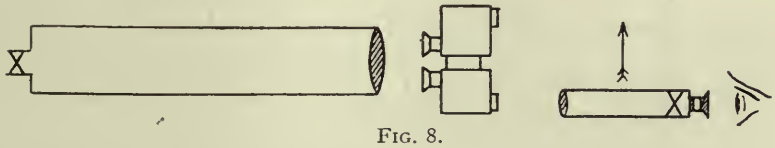


Fig. 8.

No darkening of the room required.

One collimator only needed of not more than 3 in. aperture, since the pupillary distance does not exceed $2\frac{5}{8}$ ins.

Any inter-objective width may be observed.

A small observing telescope only needed, the movement of which is close to the observer and under control.

The fiducial mark appears of magnification unity if the observing telescope has the power same as the instrument under examination. There is one other important advantage, viz., a diagram may be used as the fiducial collimator mark, to indicate the tolerance limits. Such a diagram will apply to all binoculars of

whatever power, thus the testing of an assortment of binoculars would be fairly rapid. We saw from Table I that whilst the allowance for axial error varies for every power of binocular, yet the tolerance in binocular accommodation is the same for all.

If we optically reverse Fig. 5, we see that the angle δ can be obtained by placing a reference mark at a suitable distance from the axis in the focal plane of the objective.

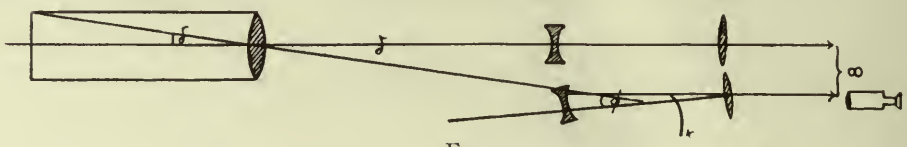


Fig. 9.

$$\phi = \delta + \kappa, \text{ but } \phi = \delta + \kappa$$
$$\therefore \delta = \kappa\,(M - 1). \quad \text{(See I.)}$$

Hence, by making a diagram of the requisite size to subtend the angular allowance in horizontal and vertical binocular accommodation the whole apparatus becomes self-contained.

A suitable shape for the diagram is that shown in Fig. 10.

To make a test, the binocular is adjusted for solar focus, then interposed between the collimator and the observing telescope, and aligned so that the centre

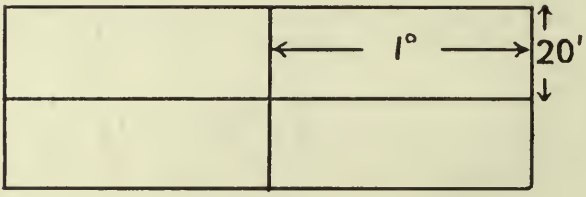


Fig. 10.

of the diagram falls on the cross-wire of the observing telescope; the telescope is then traversed till it is about central with the second barrel; if the cross-wires appear to fall within the diagram the binocular may be passed as satisfactory, but should they fall outside the diagram then the want of binocularity is too great, and the binocular needs adjustment.

There are other methods of testing the parallelism of the axes of binoculars, but the variation of Dr. Drysdale's arrangement described in the concluding paragraphs above is simple, suitable for modern prismatic binoculars of large inter-objective width as well as for ordinary binoculars, and above all is accurate and expeditious for test-room use.

Mr. C. F. Lan-Davis inquired whether this final method would enable them to adjust the prisms, which had barely been possible in the original method.

Mr. Chalmers pointed out that the binocular had been used by Drysdale in the magnifying and not in the reducing position for testing, but both positions were used in the adjustment.

Mr. T. Smith mentioned that they had tried a similar instrument at Kew, but they had not found it convenient for testing some instruments with widely separated eye-pieces.

Mr. Chalmers replied that the arrangement to which Mr. Smith referred (after Dr. Drysdale) was also described in Mr. Shackleton's paper; the apparatus had been presented to the National Physical Laboratory in consequence of the publication of Drysdale's papers.

NOTE ON THE CALCULATION OF THE SPHERICAL ERRORS OF AN OPTICAL SYSTEM OF CENTRED LENSES, BY MEANS OF THE SEIDEL FORMULÆ

By James P. C. Southall

(*Tuesday, June 25th*, 1912.)

I.—Explanation of Terms and Symbols

The five spherical aberrations of the 3rd order as originally distinguished by Seidel may be briefly enumerated as follows—

1. *The Spherical Aberration* proper, or the aberration of the image of an axial object-point; this error will be denoted by S_1.

2. The defect called *Coma*, in consequence of which the definition of parts of the image not on the axis is impaired; which will be denoted by S_2. The equation $S_2 = 0$ is equivalent also to the *Sine-Condition* which is so important in the optical theories of v. Helmholtz and Abbe. The so-called Fraunhofer-Condition discovered by Seidel is included also in the Sine-Condition.

3 and 4. *The Curvatures* of the primary and secondary image-surfaces in consequence of *Astigmatism*. This pair of spherical errors will be denoted by S_3 and S_4. If the two faults S_3, S_4 are both abolished, we obtain a flat, stigmatic image. It may be mentioned, however, that the curvatures of the image-surfaces depend essentially on the refractivities of the lenses, so that with unsuitable kinds of glass it is impossible by any choice of the geometrical dimensions of the lens-system (radii, thicknesses, distances) to obtain a plane, stigmatic image, as was first recognized by Petzval and more clearly still by Seidel.

In order to remedy as far as possible this defect of curvature, designers of optical instruments sometimes compromise on a kind of artificial flattening of the image-field; which consists in contriving so that the so-called " circles of least confusion " of the astigmatic bundles of image-rays shall all be made to lie in a definite transversal image-plane. The curvature of the surface which is the locus of the circles of least confusion is approximately equal to the arithmetical mean of the curvatures of the primary and secondary image-surfaces; accordingly if the curvature of this surface is denoted by S_3', we may write—

$$S_3' = \frac{S_3 + S_4}{2}.$$

If $S_3' = 0$, the image will be more or less plane, but in general not stigmatic.

On the other hand, again supposing that we cannot obtain an image which is both perfectly stigmatic and perfectly plane ($S_3 = S_4 = 0$), we may be content to disregard the curvature-error and to effect a compromise as to the *Astigmatism*; in which case we can endeavour to make $S_4' = 0$, this latter symbol being defined as follows—

$$S_4' = \frac{S_3 - S_4}{2}.$$

5. *The Distortion* of the marginal parts of the image, which will be denoted by S_5.*

Let M_1, $\mathbf{M}_1$ designate the positions of two points on the optical axis of a centered system of m spherical refracting surfaces, and let $A_1M_1 = u_1$, $A_1\mathbf{M}_1 = \mathbf{u}_1$, where A_1 designates the vertex of the first surface. Consider two paraxial rays which, before refraction at the first surface, cross the optical axis at M_1 and $\mathbf{M}_1$, and which, before refraction at the k^{th} surface, cross the optical axis at points designated by M'_{k-1} and $\mathbf{M}_{k-1}'$, and which are incident on the k^{th} surface at points designated by B_K and $\mathbf{B}_K$, respectively. Moreover, let the centre and vertex of the k^{th} surface be designated by C_K and A_k, respectively. In agreement with the system of notation employed in the author's book, entitled *The Principles and Methods of Geometrical Optics* (Macmillan, 1910), we shall write—

$$\mathrm{A_K M_k'_{-1}} = u_k,\ \mathrm{A_K M_K'} = u'_k,\ \mathrm{A_k B_k} = h_k,\ \mathrm{A_K C_K} = r_K,$$
$$\mathrm{A_K \mathbf{M}_k'_{-1}} = \mathbf{u}_k,\ \mathrm{A_K \mathbf{M}_k'} = \mathbf{u}'_k,\ \mathrm{A_k \mathbf{B}_k} = \mathbf{h}_k.$$

Then, if n_k' denotes the index of refraction of the medium comprised between

* The symbols S_1, S_2, etc., which are here employed are slightly different from the symbols S^{I}, S^{II}, etc., which are used in the author's book, *The Principles and Methods of Geometrical Optics* (Macmillan, 1910), page 466. The connections between the two sets of symbols are as follows—

$$S_1 = S^{\mathrm{I}};\ S_2 = S^{\mathrm{II}};\ S_3 = 3S^{\mathrm{III}} - S^{\mathrm{IV}};$$
$$S_3' = 2S^{\mathrm{III}} - S^{\mathrm{IV}};\ S_4 = S^{\mathrm{III}} - S^{\mathrm{IV}};$$
$$S_4' = S^{\mathrm{III}};\ S_5 = -S^{\mathrm{V}}.$$

It may be remarked here that the system of notation adopted in this paper is in general agreement with that employed in the volume above mentioned.

the k^{th} and the $(k+1)^{\text{th}}$ refracting surfaces, according to the well-known formula for the refraction of a paraxial ray at a spherical surface, we may write—

$$n'_K\left(\frac{1}{r_K}-\frac{1}{u'_k}\right)=n'_{K-1}\left(\frac{1}{r_K}-\frac{1}{u_k}\right)=J_k,$$
$$n'_K\left(\frac{1}{r_K}-\frac{1}{\mathfrak{u}'_k}\right)=n'_{k-1}\left(\frac{1}{r_K}-\frac{1}{\mathfrak{u}_k}\right)=\mathfrak{J}_k.$$

II.—The General Formulæ for the Spherical Errors

In order to write the Seidel-formulæ in a compact form, it is necessary to introduce several abbreviation-symbols. Thus, employing Abbe's operator Δ to indicate the difference between the value of an expression before and after refraction at a spherical surface, and putting also—

$$\mathfrak{J}_1-J_1=a,$$

We shall employ the symbols X, Y, Z as defined by the following expressions—

$$1+X_k=\frac{h_1}{\mathfrak{h}_1}\left(\frac{ah_1\mathfrak{h}_1}{h_k^2J_k}+\frac{\mathfrak{h}_k}{h_k}\right);$$
$$Y_k=h_k^4J_k^2\Delta\left(\frac{1}{nu}\right)_k;\ Z_k=\frac{1}{r_K}\Delta\left(\frac{1}{n}\right)_k;$$

and, moreover, let the symbols N, P, Q, and R be used to denote the following sums—

$$N=-\frac{1}{ah_1^4}\sum_{k=1}^{k=m}X_kY_k;$$
$$P=\sum_{k=1}^{k=m}Z_k-\frac{1}{a^2h_1^4}\sum_{k=1}^{k=m}X_k^2Y_k;$$
$$Q=\sum_{k=1}^{k=m}Z_k-\frac{3}{a^2h_1^4}\sum_{k=1}^{k=m}X_k^2Y_k;$$
$$R=\frac{1}{a}\sum_{k=1}^{k=m}X_kZ_k-\frac{1}{a^3h_1^4}\sum_{k=1}^{k=m}X_k^3Y_k.$$

The expressions of the Seidel spherical errors in terms of these magnitudes may be written as follows—*

* These results are obtained at once from the first of formulæ (356) and from formulæ (361), (363), (364) and (365) in Chapter XII of *The Principles and Methods of Geometrical Optics.*

$$S_1 = \frac{1}{h_1^4}\sum_{k=1}^{k=m} Y_k;$$

$$S_2 = S_1 - aN;$$

$$S_3 = 3S_1 - 6aN - a^2Q;$$

$$S_4 = S_1 - 2aN - a^2P;$$

$$S_3' = 2S_1 - 4aN - a^2\frac{P + Q}{2};$$

$$S_4' = S_1 - 2aN + a^2\frac{P - Q}{2};$$

$$S_5 = S_1 - 3aN - a^2Q - a^3R.$$

The practical problem consists in determining the radii, thicknesses, distances, etc., and the kinds of glass in such ways that one or more of SEIDEL's five expressions for the spherical errors will be made to vanish. If we could calculate an optical system of centred spherical refracting surfaces for which—

$$S_1 = N = P = Q = R = 0,$$

then the monochromatic image of a plane object at right angles to the axis will be at once sharply defined, flat and true throughout, so that it will indeed coincide completely with the so-called GAUSSian or theoretical (collinear) image. Although it is impossible to attain this degree of perfection, we may at least endeavour to design the optical system so as to abolish those faults, which, for the particular type of instrument in view (telescope, microscope, photographic objective, etc.), are the most detrimental to the image, and perhaps also to minimize the residual errors. Expressions for the magnitudes of these residual errors can be derived from the above equations.

When the so-called PETZVAL Condition (see *The Prin. and Meth. of Geom. Optics*, § 303, viz.—

$$\sum_{k=1}^{k=m} Z_k = 0,$$

is satisfied, we have $Q = 3P$. If in addition one of the sums denoted by S_3, S_4, S_3', S_4' vanishes also, all of them will vanish.

On account of the algebraic difficulties involved in the solution of these equations, the method is applicable only to the design of optical systems of comparatively simple structure. The analytical hindrances are very greatly reduced, provided we can neglect the thicknesses of the lenses, and a still further simplification will be introduced if we may also disregard the intervals between each pair of successive lenses.

III.—Formulæ for a System of Infinitely Thin Lenses

The expressions for the spherical errors of a centered system of infinitely thin lenses, separated by finite intervals, may be easily derived from the formulæ (356) given on page 466 of *The Prin. and Meth. of Geom. Optics*. Let us suppose that

the i^{th} lens is comprised between the k^{th} and the $(k+1)^{th}$ spherical surfaces, and let us use the letters A, B, C, D, and E as abbreviations for the following expressions—

$$\sum_{k=k}^{k=k+1} J^2{}_k \Delta\left(\frac{1}{nu}\right)_k = A_i$$

$$\sum_{k=k}^{k=k+1} J_k J_k \Delta\left(\frac{1}{nu}\right)_k = B_i;$$

$$\sum_{k=k}^{k=k+1} \left\{3J^2{}_k \Delta\left(\frac{1}{nu}\right)_k - (J_k - J_k)^2 \frac{1}{r_K} \Delta\left(\frac{1}{n}\right)_k\right\} = C_i;$$

$$\sum_{k=k}^{k=k+1} \left\{J^2{}_k \Delta\left(\frac{1}{nu}\right)_k - (J_k - J_K)\frac{1}{r_K} \Delta\left(\frac{1}{n}\right)_k\right\} = D_i;$$

$$\sum_{k=k}^{k=k+1} \frac{1}{J_K}\left\{J^3{}_k \Delta\left(\frac{1}{nu}\right)_k - (J_k - J_K)^2 \frac{1}{r_K} \Delta\left(\frac{1}{n}\right)_k\right\} = E_i.$$

Considering each lens as surrounded by air, so that—

$$n'_{k-1} = n'_{k+1} = 1, \; n'_k = n_i,$$

let us also introduce the symbols c, x and $\mathbf{x}$ to denote the reciprocals of the magnitudes r, u and u, according to the following scheme—

$$\frac{1}{r_K} = c_i, \; \frac{1}{r_{K+1}} = c'_i;$$

$$\frac{1}{u_k} = x_i; \; \frac{1}{u'_k} = \frac{1}{u_{k+1}} = \frac{x_i + (n_i - 1)c_i}{n_i}; \; \frac{1}{u'_{k+1}} = x'_i;$$

$$\frac{1}{\mathrm{u}_k} = \mathbf{x}_i; \; \frac{1}{\mathrm{u}'_k} = \frac{1}{\mathrm{u}_{k+1}} = \frac{\mathbf{x}_i + (n_i - 1)c_i}{n_i}; \; \frac{1}{\mathrm{u}'_{k+1}} = \mathbf{x}'_i.$$

If ϕ_i denotes the strength of the i^{th} lens, then, according to the well-known formulæ for infinitely thin lenses, we have—

$$x'_i - x_i = \mathbf{x}'_i - \mathbf{x}_i = (n_i - 1)(c_i - c'_i) = \phi_i;$$

and hence—

$$J_k = c_i - x_i; \; J_{k+1} = c'_i - x'_i = c_i - x_i - \frac{n_i}{n_i - 1}\phi_i;$$

$$\mathbf{J}_k = c_i - \mathbf{x}_i; \; \mathbf{J}_{k+1} = c'_i - \mathbf{x}'_i = c_i - \mathbf{x}_i - \frac{n_i}{n_i - 1}\phi_i.$$

Employing these relations, we can find the following expressions for A_i, B_i, etc., in terms of ϕ_i, c_i, x_i, $\mathbf{x}_i$ and n_i—

$$A_i = \frac{n_i + 2}{n_i}\phi_i c_i^2 - \left\{\frac{4(n_i + 1)}{n_i} x_i + \frac{2n_i + 1}{n_i - 1}\phi_i\right\}\phi_i c_i + \frac{3n_i + 2}{n_i}\phi_i x_i^2 + \frac{3n_i + 1}{n_i - 1}\phi_i x_i + \left(\frac{n_i}{n_i - 1}\right)^2 \phi_i^3;$$

$$B_i = \frac{n_i+2}{n_i}\phi_i c_i^2 - \left\{\frac{n_i+1}{n_i}(3x_i+\mathbf{x}_i) + \frac{2n_i+1}{n_i-1}\phi_i\right\}\phi_i c_i + \frac{n_i+1}{n_i}\phi_i x_i^2 + \frac{2n_i+1}{n_i}\phi_i x_i \mathbf{x}_i + \frac{2n_i+1}{n_i-1}\phi_i^2 x_i + \frac{n_i}{n_i-1}\phi_i^2 \mathbf{x}_i + \left(\frac{n_i}{n_i-1}\right)^2 \phi_i^3;$$

$$C_i = \frac{3(n_i+2)}{n_i}\phi_i c_i^2 - \left\{\frac{6(n_i+1)}{n_i}(x_i+\mathbf{x}_i) + \frac{3(2n_i+1)}{n_i-1}\phi_i\right\}\phi_i c_i + \frac{1}{n_i}\phi_i x_i^2 + \frac{2(3n_i+2)}{n_i}\phi_i x_i \mathbf{x}_i + \frac{3n_i+1}{n_i}\phi_i \mathbf{x}_i^2 + \frac{3(n_i+1)}{n_i-1}\phi_i^2 x_i + \frac{6n_i}{n_i-1}\phi_i^2 \mathbf{x}_i + 3\left(\frac{n_i}{n_i-1}\right)^2 \phi_i^3;$$

$$D_i = \frac{n_i+2}{n_i}\phi_i c_i^2 - \left\{\frac{2(n_i+1)}{n_i}(x_i+\mathbf{x}_i) + \frac{2n_i+1}{n_i-1}\right\}\phi_i c_i + \frac{1}{n_i}\phi_i x_i^2 + 2\phi_i x_i \mathbf{x}_i + \frac{n_i+1}{n_i}\phi_i \mathbf{x}^2 + \frac{n_i+1}{n_i-1}\phi_i^2 x_i + \frac{2n_i}{n_i-1}\phi_i^2 \mathbf{x}_i + \left(\frac{n_i}{n_i-1}\right)^2 \phi_i^3;$$

$$E_i = \frac{n_i+2}{n_i}\phi_i c_i^2 - \left\{\frac{n_i+1}{n_i}(x_i+3\mathbf{x}_i) + \frac{2n_i+1}{n_i-1}\phi_i\right\}\phi_i c_i + \frac{3n_i+1}{n_i}\phi_i \mathbf{x}_i^2 + \frac{1}{n_i}\phi_i x_i \mathbf{x}_i + \frac{1}{n_i-1}\phi_i^2 x_i + \frac{3n_i}{n_i-1}\phi_i^2 \mathbf{x}_i + \left(\frac{n_i}{n_i-1}\right)^2 \phi_i^3.$$

Thus, for a system of m infinitely thin lenses, we have the following expressions for the spherical errors—

$$S_1 = \sum_{i=1}^{i=m} \frac{h_i^4}{h_1^4} A_i; \qquad S_2 = \sum_{i=1}^{i=m} \frac{h_i^3}{h_1^3}\frac{\mathbf{h}_i}{\mathbf{h}_1} B_i;$$

$$S_3 = \sum_{i=1}^{i=m} \frac{h_i^2}{h_1^2}\frac{\mathbf{h}_i^2}{\mathbf{h}_1^2} C_i; \qquad S_3' = \sum_{i=1}^{i=m} \frac{h_i^2}{h_1^2}\frac{\mathbf{h}_i^2}{\mathbf{h}_1^2}\frac{C_i+D_i}{2};$$

$$S_4 = \sum_{i=1}^{i=m} \frac{h_i^2}{h_1^2}\frac{\mathbf{h}_i^2}{\mathbf{h}_1^2} D_i; \qquad S_4' = \sum_{i=1}^{i=m} \frac{h_i^2}{h_1^2}\frac{\mathbf{h}_i^2}{\mathbf{h}_1^2}\frac{C_i-D_i}{2};$$

$$S_5 = \sum_{i=1}^{i=m} \frac{h_i}{h_1}\frac{\mathbf{h}_i^3}{\mathbf{h}_1^3} E_i.$$

IV.—Abolition of Two of the Spherical Errors of a Thin-Lens System *

In the special case when the lenses are all in contact, and the entire thickness of the system may be regarded as negligible, that is, in the case of a *thin-lens system*, we have—

$$h_k = h_1, \quad \mathbf{h}_k = \mathbf{h}_1$$

* See paper by the author in *The Astrophysical Journal*, xxxiii (1911), 330–337.

and the expressions denoted by N, P, Q and R above will be found to have the following forms—

$$N = \sum_{i=1}^{i=m} \left\{ \frac{n_i}{n_i = 1} \phi_i^2 + \frac{2n_i + 1}{n} \phi_i x_i - \frac{n_i + 1}{n_i} \phi_i c_i \right\};$$

$$P = - \sum_{i=1}^{i=m} \frac{n_i + 1}{n_i} \phi_i; \qquad Q = - \sum_{i=1}^{i=m} \frac{3n_i + 1}{n_i} \phi_i; \quad R = 0.$$

Moreover, for a system of infinitely thin lenses in contact, we have—

$$x_i - x_1 = \sum_{r=1}^{r=i-1} \phi_r - \mathbf{x}_i = \mathbf{x}_1.$$

and, hence—

$$x_i - \mathbf{x}_i = x_1 - \mathbf{x}_1 = a.$$

The general expression of a spherical error of a thin-lens system may now be written in the following form—

$$S = p S_1 - a \left\{ qN - a \sum_{i=1}^{i=m} L_i \phi_i \right\},$$

where p, q have certain integral positive values (between 0 and 6), and Li denotes a function of ni. The values of p, q and L_i corresponding to each of the spherical errors S_1, S_2, etc., are exhibited in the subjoined table.

S	S_1	S_2	S_3	S_4	S_3'	S_4'	S_5
p	1	1	3	1	2	1	1
q	0	1	6	2	4	2	3
L_i	0	0	$\frac{3n_i + 1}{n_i}$	$\frac{n_i + 1}{n_i}$	$\frac{2n_i + 1}{n_i}$	1	$\frac{3n_i + 1}{n_i}$

Let S, S' and S'' denote three of the spherical errors of a thin-lens system, and according to the above formula let us write—

$$S = p S_1 - a \left\{ qN - a \sum_{i=1}^{i=m} L_i \phi_i \right\};$$

$$S' = p' S_1 - a \left\{ q'N - a \sum_{i=1}^{i=m} L'_i \phi_i \right\};$$

$$S'' = q'' S_1 - a \left\{ q''N - a \sum_{i=1}^{i=m} L''_i \phi_i \right\}.$$

If the first error (S) has been corrected ($S = 0$), the condition of the simultaneous abolition of the second error (S'), that is, the condition of the fulfilment of the additional requirement $S' = 0$, will be given by the following convenient equation—

$$\sum_{i=1}^{i=m} \left\{ l \frac{n_i}{n_i - 1} \phi_i^2 + \left(l \frac{2n_i + 1}{n_i} - M_i \right) \phi_i x_i + M_i \phi_i x_i - l \frac{n_i + 1}{n_i} \phi_i c_i \right\} = 0,$$

where

$$l = p'q - pq',$$
$$Mi = pL'_i - p'L_i.$$

If the two spherical errors have been abolished ($S = S' = 0$), the magnitude of a third residual error (S'') will depend only on the strengths of the lenses and their refractivities, and will, therefore, not be affected by any further " bending " of the lenses or by any alteration in the order of sequence of the lenses. Under the circumstances we find in fact—

$$S'' = a^2 \sum_{i=1}^{i=m} K_i \phi_i, \quad (S = S' = 0),$$

where—

$$K_i = \frac{(p'q'' - p''q')L_i + (p''q - pq'')L'_i + (pq' - p'q)L''_i}{pq' - p'q}$$

The subjoined table exhibits the values of l and M_i for the simultaneous abolition of any pair of the spherical errors denoted by S, S' and also the expression for K_i for each one of the spherical errors denoted by S''. For convenience of printing, the subscript i has been omitted from the letter n in this table. The numerals 1, 2, 3, 4, 3', 4', 5, at the head of the columns under K_i refer to the spherical errors denoted by S_1', S_2, S_3, S_4, S_3', S_4', S_5, respectively. It will be observed that in each horizontal row of the table there are always two valves $K_i = 0$; the first or left-hand one of these corresponds to $S = 0$, and the right-hand one to $S' = 0$.

As an illustration of the method of procedure in a comparatively simple case, suppose that we wish to design an ordinary telescope-objective composed of two thin lenses cemented together so that $c_1' = c_2$. The kinds of glass (n_1, n_2) will be selected with a view to the correction of the colour-faults or chromatic aberrations, but we need not concern ourselves with this phase of the problem at present. In this connection, however, it may be remarked in passing that, with regard to the correction of the colour-faults, the modern optician possesses an immense advantage over his predecessors, on account of the variety of the new kinds of optical glass that are now at his disposal. The favourable circumstance that varieties of glass of equal refractivity but of different dispersion are now available, enables the designer of a system of lenses to postpone the colour-correction until after he has effected the correction of the spherical errors.

l	M_i	K_i						
		1	2	3	4	3′	4′	5
-1	0	0	0	$\frac{3n+1}{n}$	$\frac{n+1}{n}$	$\frac{2n+1}{n}$	$+1$	$\frac{3n+1}{n}$
-6	$\frac{3n+1}{n}$	0	$-\frac{3n+1}{6n}$	0	$\frac{2}{3n}$	$\frac{1}{3n}$	$-\frac{1}{3n}$	$\frac{3n+1}{2n}$
-2	$\frac{n+1}{n}$	0	$-\frac{n+1}{2n}$	$-\frac{2}{n}$	0	$-\frac{n+1}{n}$	$-\frac{1}{n}$	$\frac{3n-1}{n}$
-4	$\frac{2n+1}{n}$	0	$-\frac{2n+1}{4n}$	$-\frac{1}{2n}$	$\frac{1}{2n}$	0	$-\frac{1}{2n}$	$\frac{6n+1}{4n}$
-2	$+1$	0	$-\frac{1}{2}$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	0	$\frac{3n+2}{2n}$
-3	$\frac{3n+1}{n}$	0	$-\frac{3n+1}{3n}$	$-\frac{3n+1}{n}$	$-\frac{3n-1}{3n}$	$-\frac{6n+1}{3n}$	$-\frac{3n+2}{2n}$	0
-3	$\frac{3n+1}{n}$	$\frac{3n+1}{3n}$	0	0	$\frac{2}{3n}$	$\frac{1}{3n}$	$-\frac{1}{3n}$	$\frac{3n+1}{3n}$
-1	$\frac{n+1}{n}$	$\frac{n+1}{n}$	0	$-\frac{2}{n}$	0	$-\frac{1}{n}$	$-\frac{1}{n}$	$\frac{n-1}{n}$
-2	$\frac{2n+1}{n}$	$\frac{2n+1}{n}$	0	$-\frac{1}{2n}$	$\frac{1}{2n}$	0	$-\frac{1}{2n}$	$+1$
-1	$+1$	$+1$	0	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	0	$\frac{n+1}{n}$
-2	$\frac{3n+1}{n}$	$\frac{3n+1}{2n}$	0	$-\frac{3n+1}{3n}$	$-\frac{n-1}{2n}$	-1	$-\frac{n+1}{2n}$	0
-3	$\frac{6n+2}{n}$	$\frac{3n+1}{n}$	$\frac{3n+1}{3n}$	0	$\frac{2}{3n}$	$\frac{1}{3n}$	$-\frac{1}{3n}$	0
-1	$+2$	$\frac{3n-1}{n}$	$\frac{n-1}{n}$	$-\frac{2}{n}$	0	$-\frac{1}{n}$	$-\frac{1}{n}$	0
-2	$\frac{4n+1}{n}$	$\frac{6n+1}{2n}$	$+1$	$-\frac{1}{2n}$	$\frac{1}{2n}$	0	$-\frac{1}{2n}$	0
-1	$\frac{2n+1}{n}$	$\frac{3n+2}{n}$	$\frac{n+1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	0	0

Since in the optical system above proposed there are three radii to be determined, we can impose three other conditions, which would probably be—

1. A prescribed focal length; so that in the equation—

$$\phi_1 + \phi_2 = \phi$$

the strength of the combination, denoted by $\phi(= 1/f)$, has an assigned value, which

is usually put equal to unity; whereby all linear magnitudes are thus expressed in terms of the focal length of the system as unity.

2. Abolition of the spherical aberration in the centre of the field of view ($S_1 = 0$): which will be expressed by the equation—

$$\sum_{i=1}^{i=2}\left\{\frac{n_i+2}{n_i}\phi_i c_i^2-\left(\frac{4(n_i+1)}{n_i}x_i+\frac{2n_i+1}{n_i}\phi_i\right)\phi_i c_i + \frac{3n_i+2}{n_i}\phi_i x_i^2+\frac{3n+1}{n_i-1}\phi_i x_i+\left(\frac{n_i}{n_i-1}\right)^2\phi_i^3\right\}=0.$$

3. Abolition of Coma ($S_2 = 0$), whereby not only the centre but the adjacent surrounding parts of the field of view will be portrayed distinctly. This condition in conjunction with the preceding involves the additional equation—

$$\sum_{i=1}^{i=2}\left\{\frac{n_i}{n_i-1}\phi_i^2+\frac{2n_i+1}{n_i}\phi_i x_i-\frac{n_i+1}{n_i}\phi_i c_i\right\}=0.$$

According to the very practical method of calculation given by E. v. HOEGH,* it is best to regard ϕ_1 as the unknown to be first determined; whence the required curvatures of the lens-surfaces can be easily computed. The first step is to eliminate x_2 and c_2 by means of the relations—

$$x_2 = x_1 + \phi_1,\ c_2 = c_1 - \frac{\phi_1}{n_1 - 1};$$

so that finally, after eliminating ϕ_2 by means of the first of the equations above, we shall obtain an equation of the fifth degree in ϕ_1 of the following form—

$$A\phi_1^5 + B\phi\phi_1^4 + C\phi^2\phi_1^3 + D\phi^3\phi_1^2 + E\phi^4\phi_1 + F\phi^5 + (G\phi_1^4 + H\phi\phi_1^3 + I\phi^2\phi_1^2 + J\phi^3\phi_1 - \phi^4 - \phi^3 x_1)x_1 = 0;$$

where the co-efficients denoted by A, B, etc., are functions of n_1 and n_2. If the object is infinitely distant, we must put $x_1 = 0$, and then (after putting also $\phi = 1$, as above stated) the equation will be simplified as follows—

$$A\phi_1^5 + B\phi_1^4 + C\phi_1^3 + D\phi_1^2 + E\phi_1 + F = 0.$$

If, for example, adopting the illustration given by GLEICHEN, we choose for the front (crown-glass) lens the value $n_1 = 1{\cdot}58$, and for the second or flint-glass lens the value $n_2 = 1{\cdot}62$, we obtain the following equation—

$$\phi_1^5 + 228{\cdot}0248\,\phi_1^4 + 6058{\cdot}5514\,\phi_1^3 - 8977{\cdot}2146\,\phi_1^2 - 3696{\cdot}4855\,\phi_1 - 55787{\cdot}2820 = 0.$$

This equation will be found to have only one real root, viz.—

$$\phi_1 = 2{\cdot}696954;$$

and hence also— $\phi_2 = -1{\cdot}696954.$

* E. v. HOEGH: "Zur Theorie der zweitheiligen verkitteten Fernrohrobjective": *Zft. f. Instr.*, xix (1899), 37–39. See also A. GLEICHEN: *Lehrbuch der geometrischen Optik* (Leipzig und Berlin, 1902), 331–334.

Thus, for the curvatures of the lens-surfaces we find—

$$c_1 = +\,1{\cdot}56599\,;\quad c'_1 = c_2 = -\,3{\cdot}08393\,;\quad c'_2 = -\,0{\cdot}34691.$$

An incidental advantage connected with v. HOEGH'S method of calculation deserves to be pointed out. Having found the equation of the 5th degree in ϕ_1 for given values of n_1, n_2 and x_1, we can derive from it a second equation similar to the first which will enable us to calculate a second set of values of ϕ_1 for a combination of the same two kinds of glass arranged in the reverse order, merely by putting $-(x_1 + \phi)$ in place of x_1 in the first equation. The reason of this is because $x_2' = x_1 + \phi$. For example, suppose that we are calculating a telescope-objective for which $x_1 = 0$, and that we have obtained v. HOEGH'S formula for a given combination of crown glass for the first lens and flint glass for the second lens; we can obtain thence a second equation by substituting $-\phi$ in place of x_1 in the first equation; whereby we can determine the dimensions of a second optical system composed of the same two kinds of glass, but with the flint glass for the first lens and crown glass for the second lens, which will satisfy the given conditions for the new object-distance.

In the case of the optical system here under consideration, we have abolished the spherical aberration and the coma-error ($S_1 = S_2 = 0$); the magnitudes of the residual aberrations can be found from the following expressions:

$$S_3 = S^5 = a^2 \sum_{i=1}^{i=2} \frac{3n_i + 1}{n_i} \phi_i\,; \qquad S^4 = a^2 \sum_{i=1}^{i=2} \frac{n_i + 1}{n_i} \phi_i\,;$$

$$S_3' = a^2 \sum_{i=1}^{i=2} \frac{2n_i + 1}{n_i} \phi_i\,; \qquad S_4' = a^2 \sum_{i=1}^{i=2} \phi_1\,; \quad S_1 = S_2 = 0.$$

The above illustration will suffice to give some idea of the methods of optical calculation on the basis of the SEIDEL formulæ for the spherical aberrations of the 3rd order. Lack of space prevents us from pursuing the subject farther in this place; but for a complete and admirable discussion of many special problems of this kind the reader is referred to A. KÖNIG'S article entitled "Die Berechnung optischer Systeme auf Grunde der Theorie der Abberationen," Chapter VII of v. ROHR'S *Die Theorie der optischen Instrumente*, Bd. I (Berlin, 1904).

A NEW METHOD OF TESTING OBJECTIVES

By S. D. Chalmers, M.A. (Northampton Institute)

(*Tuesday, June 25th*, 1912.)

The outstanding problem in connexion with the testing of objectives is the obtaining of a numerical expression for the goodness of the definition, particularly in cases where the aberration disc is not sufficiently large to allow its size to be used as a measure of the definition. It is desirable that the measurements should give either the lateral aberration for selected portions of the objective, or, better still, the phase difference for the light proceeding from the various portions of the objective to the point at which the definition is required. The measurements should be simple, but give definite numbers, and all the reductions should be easy. At the same time the method should permit of observations for all intermediate portions of the lens surface without elaborate measurements. Errors in centring and in the choice of focus should be revealed by the measurements themselves.

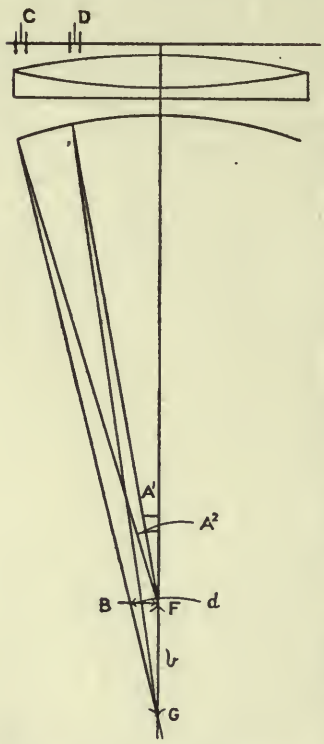


Fig. 1.—C D are the apertures, F is the focus, B the displaced position of the centre of the pattern, B F = *d*, G a point on the axis near the focus.

It has been claimed that Hartman's * method of testing does, to a considerable extent, meet these requirements, but in order to secure the necessary accuracy the measurements must be made with great care, while the demands on the stability of the apparatus are very great, and Hartmann has found it desirable to supplement the result of his tests by using the knife-edge test to reveal local peculiarities.

With a view to the simplification of the measurements and an increase in the accuracy obtainable the present author has devised a method of testing objectives, of which the following is a preliminary account. The preliminary measurements show that the necessary accuracy is obtainable, but further measurements on large objectives are necessary.

A slit or other bright line source is used as object, and a diaphragm with two narrow apertures parallel to the slit is placed in front of and close to the objective to produce an interference pattern as in Lord Rayleigh's Interferometer. When the optical paths of the two beams are equal the central white band of the diffraction pattern will appear in the centre of the image of the slit. When the paths are not

* *Zeitschrift für Instrumentenkunde*, 1904, p. 9 and 1909, p. 217.

equal the image will be displaced a distance which we denote by d; if the two rays make angles A_1 and A_2 with the axis the path difference is—

$$d\ (\sin A_1 - \sin A_2).$$

It is, however, generally simpler to express the displacement in terms of the distance between successive bands, and thus obtain the path difference directly in wave lengths.

Choice of Focus.—If the bands be focused by a high power eye-piece this will give very approximately the focus for the objective, but by observing the path difference between the central ray and any other chosen ray we can focus the eye-piece so that this path difference is zero. The effect of a change of focus b is to introduce a path difference $b\ (\cos A_1 - \cos A_2)$ where A_1 and A_2 are again the angles with the axis.

Centring.—If the difference in path between the beams through the central aperture and each of two apertures symmetrically placed with regard to it be observed we can estimate the error of centring in the direction at right angles to the slit.

Qualitative Tests for Chromatic and Spherical Aberration.—The presence of spherical aberration of considerable amount is easily discovered by using two slits at a fixed distance apart, and traversing them in front of the objective. When long slits are used the definition of the bands will deteriorate markedly as the edge is approached. By using shorter slits the nature of the aberration can be determined from the movement of the bands and also its approximate amount. Care must, of course, be taken to eliminate the effects of errors in focus and centring.

The presence of chromatic aberration will be revealed by the differences of displacement for the different colours. This will produce a change in the character of the bands and an apparent displacement of the central black band. From the apparent displacement, compared with the real displacement, the chromatic aberration is obtained.

Thus a displacement of the darkest bands three bands further on, on the interference pattern, will mean a chromatic aberration, corresponding to a path difference of three times the difference of wave length for the colours considered. In a chromatically corrected o. g. the residual secondary spectrum will also produce changes in the character of the bands, particularly in those not at the centre of the pattern.

Measurement of Spherical Aberration.—When it is desired to measure the effects of the spherical aberration and centring error it is preferable to use a comparatively short slit and to measure the displacement of the bands. The following details of a measurement indicate the procedure adopted and the accuracy obtained. The objective (12·4 cms. free aperture and 170 cms. focal length) was used with a comparatively near object (5·6 m. away) to introduce some spherical aberration. The lens was approximately, but not absolutely, squared on; the apertures were 25 mm. long by 5 mm. wide and their separation 40 mm. The bands were viewed by a microscope with a 16 mm. o. g. magnification 13·3 and a Ramsden eye-piece of 20 mm. focal length, having a scale divided to ·1 mm. at its focus. The separation of the bands at this scale was ·45 mm. The bands were focused and after a preliminary

observation to improve the focus the following observations were taken, the zero being taken from the position of the bands when the two apertures were symmetrical to the axis.

TABLE I

Position of Focus.	98	99	100	100	101
Position of Apertures in mm.	Observed Position of Central Band.				
60 (central)	0	0	0	0	0
70 ,,	0	− ·05	− ·10	− ·10	− ·18
80 ,,	0	− ·10	− ·27	− ·25	− ·38
90 ,,	0	− ·18	− ·37	− ·35	− ·55
95 ,,	+ ·10	− ·08	− ·32	− ·30	− ·55
60 ,,	0	0	0	0	0
50 ,,	0	+ ·05	+ ·10	+ ·10	+ ·18
40 ,,	− ·10	− ·05	+ ·10	− ·10	+ ·22
30 ,,	− ·40	− ·25	− ·10	− ·10	+ ·17
25 ,,	− ·80	− ·55	− ·35	·30	− ·05

The position of the focus was recorded in mm. on a scale on the observing microscope, and the zero is arbitrary.

The first column denotes the position of the carrier of the diaphragm in mm., 60 being the central reading, and the remaining columns give the displacement of the central band measured in mm. by the scale at the focus of the microscope. These figures are readily convertible into wave-length differences, since ·45 mm. corresponds to one wave length. The centring error is found by taking the mean of the values on either side of the centre position, and the spherical aberration by taking half the difference. The results are shown in—

TABLE II

Centring.

	98	99	100	100	101	Mean.
60	0	0	0	0	0	0
70 and 50	0	0	0	0	0	0
80 and 40	− ·05	− ·07	− ·06	− ·07	− ·08	− ·07
90 and 30	− ·20	− ·21	− ·23	− ·22	− ·19	− ·21
95 and 25	− ·35	− ·31	− ·33	− ·30	− ·30	− ·32

Spherical Aberration.

	98	99	100	100	101	Mean Difference
60	0	0	0	0	0	0
70	0	− ·05	− ·10	− ·10	− ·18	− ·06
80	·05	− ·025	− ·185	− ·175	− ·30	− ·12
90	·20	+ ·035	− ·135	− ·125	− ·36	− ·19
95	·45	+ ·235	+ ·015	0	− ·25	− ·22

The very good agreement between the five resulting values for the centring error shows the accuracy of the measurements (the centring error should be practically independent of the focus). The spherical aberration figures show a continuous difference due to the change of focus of ·06, ·12, ·19 and ·22 mm. for each mm. change of focus, and these may be compared with the values ·055, ·13, ·165 and ·19 mm., deduced from the expression given above for the effects of change of focus. In making the observations it was noted that there was no abrupt change of path difference; hence it is fair to assume that the lateral spherical aberration may be represented by—

$$b.\frac{x}{f}+y\left(\frac{x}{f}\right)^3+z\left(\frac{x}{f}\right)^5+\text{additional terms if necessary,}$$

where x is the distance of the portion of the aperture considered from the centre, and f is the focusing distance. The path difference between this point and the centre is—

$$b\frac{x^2}{2f}+y\frac{x}{4}\left(\frac{x}{f}\right)^3+z\frac{x}{6}\left(\frac{x}{f}\right)^5.$$

The difference between two apertures 40 mm. apart is—

$$\frac{ba40}{f}+y\frac{a}{f}40\left(\frac{a^2+20^2}{f^2}\right)+z\frac{a}{f}40\left(\frac{a^4+\frac{10}{3}a^220^2+20^4}{f^3}\right)$$

where a is the displacement of the apertures from their central position.

The observations are sufficiently accurately represented by $b = 2{\cdot}26$ and $y = 80{\cdot}00$, taking $z = 0$.

TABLE III

x	Movement of bands in mm. ·45 mm. = 1 wave length. Calculated.	Observed.	
10	−·117	−·10	−·10
20	−·174	−·175	−·185
30	−·110	−·125	−·135
35	+·015	+0	+·015

The reductions would be much facilitated by the proper choice of the separation of the apertures. It should be possible to observe the difference between the central and half aperture and also the difference between full and half aperture.

The method described above seems to be the most convenient to use, though there are some advantages in the alternative procedure in which one aperture is fixed and the other is moved right across the lens. This enables the reductions to be made very quickly, but has the drawback that the separation of the bands will vary. The expression of the results as a phase difference rather than as a lateral

aberration corresponds to practical requirements; it indicates the amount of material that must be rubbed off from each part of the objective in order to secure a perfect figure.

The method used above for making the measurements is probably sufficiently accurate for testing purposes, provided it be possible to traverse the aperture across the lens in various positions. But when the spherical aberration is small, increased accuracy would probably be obtained by using the method of compensating the displacement which is used in the Zeiss Interferometer. In this case rather longer slits could be employed, and Rayleigh's device of a cylindrical eye-piece and a fixed reference set of bands would eliminate some of the difficulties due to small movements of the apparatus.

The author has not yet had an opportunity of using these refinements of measurement, but he has found a cylindrical lens to be of considerable advantage in observing the bands.

Application to the Testing of Microscope O.G.'s.—The fact that the method gives numerical results enables it to be used in the testing of microscope objectives. The method of auto-collimation enables a slit of ordinary construction to be used as a source of light, and its image is focused by the objective on a silvered cover glass, such as the Abbe Test Plate. The reflected light passes through the objective and the two apertures are placed behind the objective, so that they are clear of the incident beam. The fact that the measurements are measurements of position and not estimates of definition makes it possible to use the method of auto-collimation in spite of the reflections from the various surfaces of the objective. The author has found it possible to observe the movement of the bands with a $\frac{1}{6}$" objective, but has had no opportunities of carrying out detailed tests. From the appearance of the bands and their movements he concludes that the method can be applied with suitably constructed apparatus, to the highest powers.

Mr. Dyson considered the method extremely interesting. Would the method make it possible to discriminate between the errors of centring due to displacement or tilting of one of the lenses?

Mr. Chalmers thought so, but had not especially studied this part of the problem.

Mr. T. Smith inquired whether the author had been able to make observations with good and with quite bad objectives, and whether he had compared his displacements with the statements made by instrument-makers. He would ask whether the observations would enable the author to separate out the various higher aberrations of a system.

Mr. E. H. Rayner asked whether it would be practicable to use an autocollimation method satisfactorily with fairly large telescopes.

There might be difficulties in the way of using a very distant source of light such as would be required if it were desired to have the source practically at infinity.

Atmospheric refraction and unsteadiness might cause trouble. An artificia distant source in the form of a collimating telescope was inadmissible, and a mercury surface would be required. Some further explanation of the meaning of the figures in Tables I and II might be given; It would appear to be better to change the first column in I and II to read "o" at centre, instead of the arbitrary 60. Perhaps the fraction of the radius would give a better idea than using a scale of millimetres.

Mr. S. D. Chalmers in replying said that he had made some measurements on a very good objective. The one table indicated the actual measurements for spherical aberration just inside and outside the focus. Differences in the displacements of different bands were to be expected. Under favourable circumstances he could measure within $\frac{1}{20}$ of a wave length. He had also tried to correct lenses of considerable aberration. An auto-collimating method could be used as he had stated in the paper, but he thought the difficulties introduced by the plane surface would generally be more serious than those with a distant source. As the aberrations were measured it would in practice be possible to make a correction for the aberration introduced by not having the object at infinity provided the construction of the telescope were known. A further advantage was that each lens could be tested separately.

DEMONSTRATION OF LABORATORY APPARATUS AND EXPERIMENTS

By G. F. C. Searle, Sc.D., F.R.S., University Lecturer in Experimental Physics, Cambridge

[Dr. Searle explained a number of methods of determining optical constants and exhibited the apparatus used by the students in his Practical Class at the Cavendish Laboratory, Cambridge, in applying those methods to optical systems.]

I.—Revolving Table Method of determining the Curvature of Spherical Surfaces *

(*Friday, June 21st,* 1912.)

For many optical purposes it is necessary to determine the radius of curvature of a convex or a concave surface with considerable accuracy. The revolving table method, designed by the author in conjunction with Mr. A. C. W. Aldis and Mr. G. M. B. Dobson, has the advantage that the radius is found directly from two readings on a straight uniformly divided scale, without corrections or calculations of any sort.

The principle of the method may be described as follows:—A table turning truly and without shake about a vertical axis is required. For the most accurate work the fit of the vertical spindle in its bearings must be as perfect as in a good lathe-head. The plane of the top of the table is normal to the axis of revolution,

* For fuller details, see *Philosophical Magazine*, Feb. 1911, pp. 218–224.

and the top carries a straight scale against which slides a carriage bearing the spherical surface (see Figs. 1 and 3). We shall assume that the scale is so adjusted on the table that the straight line described by the centre of curvature of the spherical surface when the carriage slides along the scale *intersects* the axis of revolution of the table. The position of the carriage relative to the table-top when the centre lies on the axis of revolution will be called the *first position*. If the table be turned through any angle, about the vertical axis, when the carriage is in the first position, the only effect, of the angular motion, is to substitute one part of the spherical surface for another. Hence, if rays from an object fall upon the surface, the reflected rays will be unaffected by the motion.

The carriage is now moved along the scale into a *second position* in which the axis of the table is a tangent line to the spherical surface. If, now, the table be turned about its axis, a grain of lycopodium placed at the point of contact of the vertical tangent line and the surface will remain stationary. By using a microscope for observing the image of the object in the first case and the lycopodium in the second, the settings can be made with great accuracy.

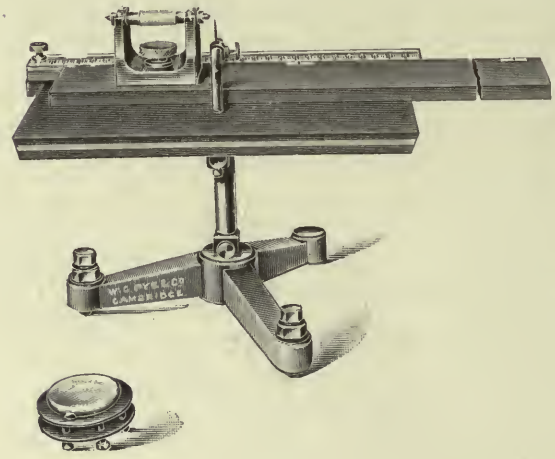


Fig. 1.

The radius of curvature of the surface is now given at once by the difference of the two scale-readings of the carriage in the first and second positions.

The adjustments of the surface to be tested are greatly facilitated by the use of a small lathe-head to form the "carriage" mentioned above, and we shall describe the method of making the measurements when the lathe-head is used. The apparatus is shown in Figs. 1 and 2.

The revolving table rests upon a tripod stand, and the table-top turns about a vertical rod carried by the tripod. The top of the table carries a scale graduated in millimetres, which can be clamped in any position on the table by the screw seen in Fig. 1.

The lathe-head is attached by a screw to a straight-edged board which rests upon the top of the table with its straight edge in contact with the scale. The spindle of the lathe-head is screwed at one end to fit a brass face-plate furnished with three screws which serve to adjust a second brass plate to which the lens or mirror to be tested is attached by wax. Each end of the spindle is turned to a conical point, and the vertex of each cone lies on the axis of revolution of the spindle.

One arm of a steel rod bent at right angles can be secured by a set-screw in a socket in the casting supporting the table-top, and a small clip carrying a pin can be fixed in any position on the other arm of the rod by a set-screw. The bent rod also serves as a means of clamping the board to the table in the manner shown in Fig. 2.

The first step is to make the axis of the spindle of the lathe-head parallel to the edge of the board. The face-plate is removed from the spindle, and the tip of the pin carried by the bent rod is brought into contact with the vertex of one of the conical ends of the spindle, the straight edge of the board being in contact with the scale. The board is then removed from the table and is replaced so that the other conical end of the spindle is near the pin. If, by sliding the board along the scale, this vertex can be made to *touch* the tip of the pin, the axis of the spindle is parallel to the edge of the board. The clamping screw enables the lathe-head to be properly adjusted on the board.

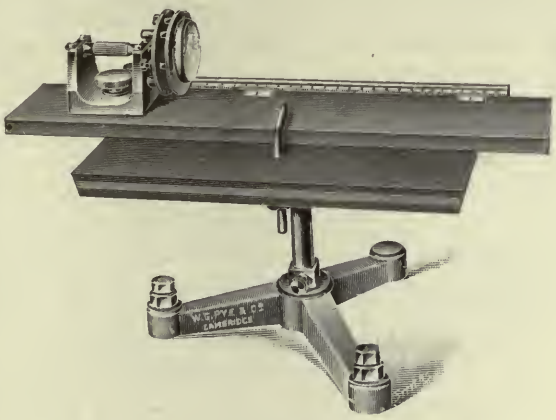


Fig. 2.

After the adjustment of the lathe-head on the board is complete, the scale must be adjusted on the table-top so that the axis of the lathe spindle *intersects* the axis of the table. The scale and the board are first adjusted roughly so that one of the conical ends of the spindle is not far from the axis of the table. A fixed microscope, with a vertical cross-wire, is then focused on the vertex of the cone when the axis of the spindle is perpendicular to that of the microscope, the image of the vertex lying on the cross-wire. The table-top is then turned through 180°; if the image of the vertex does not again lie on the cross-wire, the difference is *halved* by moving the board along the scale, and the microscope is then moved so as to bring the image again to the cross-wire. The table-top is now turned so that the axis of the spindle is parallel to that of the microscope. If the image of the vertex does not lie on the cross-wire, the scale is moved at right angles to itself until the coincidence is obtained. The vertex of the spindle then lies on the axis of the table, and the axis of the spindle will intersect that of the table for all positions of the board along the scale.

The face-plate is now attached to the spindle and the lens or mirror to be tested is fixed to the adjustable plate by wax. In the case of a lens, the back surface should be smeared with a mixture of lamp-black and vaseline to stop reflexion at that surface. The spindle is then rotated and some object is observed by reflexion

at the spherical surface. If the rotation cause a motion of the image, the three adjusting screws are manipulated until the image remains stationary when the spindle is revolved. The centre of curvature then lies on the axis of the spindle.

A piece of ground glass with a fine vertical line drawn on it forms a convenient object; the glass should be well illuminated. A fixed microscope with cross wires or a micrometer scale may be used to facilitate the adjustment. The spindle is turned into the position in which the image of the vertical line is as far to the right as possible. The plate carrying the surface is then made to move as nearly as possible about a *vertical* axis by turning the appropriate screw or screws, so that the image moves to the left through the proper distance. The success of the adjustment is then tested by rotating the spindle.

The preliminary adjustments are now complete. If they are only nearly but not quite perfect, the errors which they cause in the radius of curvature, as found by this method, will be very small, since these errors of radius depend on the *second* powers of the errors in the preliminary adjustments.

The board carrying the lathe-head is now set into the " first position " in which the centre of curvature of the surface lies on the axis of revolution of the table. A piece of ground glass with a vertical line ruled on it is set up in such a position that the image of the vertical line which is observed in the microscope is formed by rays which fall *nearly normally* upon the surface. If the image move when the table-top is turned backwards and forwards about its axis, the board carrying the lathe-head is moved along the scale until the image remains at rest. The centre of curvature then lies on the axis of the table. The scale-reading of an index mark on the board is then taken and recorded.

The board is next adjusted to the " second position " in which the axis of the table is a tangent line to the reflecting surface. A patch of lycopodium is placed on the surface and then a pointed piece of wood is held against the surface while the lathe-spindle is rotated, so as to remove all the lycopodium but a circular patch about a millimetre in diameter. If the axis of the spindle be perpendicular to the axis of the table, the tangent plane to the surface at the centre of this patch will be parallel to the axis of the table. The board is then adjusted along the scale so that the central grains of the patch remain stationary when the table-top is turned backwards and forwards about a mean position in which the axis of the lathe-spindle is *parallel* to that of the observing microscope. When the adjustment is complete, the scale-reading of the index mark is recorded.

The difference between the two scale-readings in the first and second positions gives the radius of curvature of the surface. Each of the two settings is easily made to $\frac{1}{10}$ mm. with the apparatus described.

The following experiment serves as a test of the method. Two lenses, one convex and the other concave, are selected so that wide Newton's rings are seen by sodium light when the marked face of the convex lens is placed in contact with the marked face of the concave lens. The radius of the convex surface is determined

by the revolving table and the radius of the concave surface is deduced from measurements of the rings. The radius of the concave surface is also determined by the revolving table and the two results for this surface are compared.

II.—Goniometer Method of measuring Focal Lengths of Optical Systems

The simple goniometer illustrated in Fig. 3 was designed by the author, in conjunction with Messrs. W. G. Pye and Co., of Cambridge, England, to provide a means of measuring angles up to about $\frac{1}{4}$ radian (about 14°) with an accuracy of $\frac{1}{4000}$ radian. The base is formed of a strip of wood furnished at one end with a spherical pivot and at the other with a cross-bar carrying a scale. Angles are measured by means of a movable arm which turns at one end about the pivot, while the other end moves over the scale on the cross-bar. The optical system consists of a lens fixed to the arm near the pivot and of a fine vertical wire attached to the other end of the arm and adjusted to be in the focal plane of the lens.

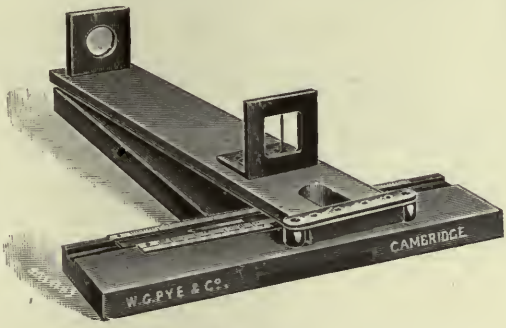


Fig. 3.*

The spherical pivot is a phosphor-bronze ball attached to the base by a fitting which allows the distance between the ball and the scale to be adjusted. The ball enters a conical hole turned out of a block of brass attached to the arm. This arrangement destroys three out of the six degrees of freedom of the arm relative to the base. The other end of the arm carries two brass feet which rest upon the cross-bar and thus destroy two degrees of freedom. The remaining degree of freedom allows the arm to turn about an axis through the centre of the ball and perpendicular to the plane of the surface of the cross-bar.

The scale on the cross-bar is divided into millimeters, and the ball is adjusted so that its centre is 40 cms. from the edge of the scale. The readings are taken by means of a fine wire passing across an opening in the arm and stretched by a spring. The scale is engine-divided on white metal and is provided with an anti-parallax mirror. For small angles, one centimetre along the scale corresponds to $\frac{1}{40}$ radian; as the scale can be read to $\frac{1}{100}$ cm., the angle can be read to $\frac{1}{4000}$ radian, or to about $\frac{1}{70}$ degree.

The lens attached to the movable arm is achromatic and has a focal length of about 35 cms. The vertical wire is held in an adjustable frame attached to the arm and is kept tight by a spring, and this frame is adjusted so that the wire is in

* Registered Design, No. 526858.

the focal plane of the lens. The image of a distant point will then fall upon the wire, if the arm be properly directed. If a plane mirror be placed so that the lens lies between it and the wire, the image of the wire formed by two refractions through the lens and one reflection at the mirror may be made to coincide with the wire itself.

In some optical experiments it is convenient to be able to turn a plane mirror through a small measured angle. For this purpose it is only necessary to attach the mirror to the arm of the goniometer near the pivot.

The instrument may be used to measure the angle between two beams of parallel light. The arm is moved so as to bring each beam in turn to a focus on the wire; the angle turned through by the arm is then equal to that between the two beams.

One of the uses of the goniometer is its application to measure the focal length of any coaxal system of lenses by a method depending upon the properties of the nodal points.

In the diagram (Fig. 4) let F_1P and F_2Q be the two focal planes of the system,

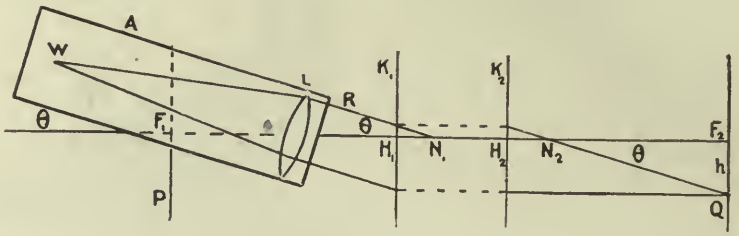


Fig. 4.

H_1K_1 and H_2K_2 the two principal or unit planes, and N_1 and N_2 the two nodal points. The lenses forming the optical system under test are *not* shown in the diagram. Let $F_1H_1 = f_1$, and $F_2H_2 = f_2$; then f_1 and f_2 are the two focal lengths of the system. The nodal points are defined by the property that any incident ray QN_2, whose direction passes through N_2, gives rise to an emergent ray N_1R whose direction passes through N_1 and is parallel to the ray QN_2. It can be shown that $N_2F_2 = f_1$ and that $N_1F_1 = f_2$. When the medium is the same at both ends of the system, the nodal points N_1, N_2 coincide with the principal points H_1, H_2, and then $f_1 = f_2 = f$. The goniometer can be applied without difficulty to determine the *two* focal lengths in the general case in which the two focal lengths are *not* equal.

The point Q in the focal plane through F_2 will emit a pencil of rays which will give rise to an emergent beam of parallel rays, as indicated in the diagram. If the goniometer arm A be properly adjusted, this parallel beam is brought to a focus on the wire W by the lens L. Similarly, a point at F_2 gives rise to a parallel beam which may be brought to a focus at W by turning the arm through θ radians, the angle between the two beams. But, by the property of the nodal points, the angle QN_2F_2 is also equal to θ. But, if $F_2Q = h$, we have, for small angles, $\theta = h/N_2F_2 = h/f_1$, and thus $f_1 = h/\theta$. Since we know θ, f_1 can be calculated as soon as h is known.

If a plane mirror be mounted on the goniometer arm, the image of Q may be made to coincide with Q itself by properly adjusting the arm.

The distance h or F_2Q is measured by a glass scale placed with its divided face in the focal plane; it is in adjustment when one of the dividing lines is sharply focused (without parallax) upon the wire W. The adjustment is easily made if the scale be mounted on a sliding carriage similar to that shown in Fig. 5, and described below. A scale divided to millimetres may be used for optical systems with focal lengths exceeding 5 cms. Satisfactory results may be obtained for telephoto systems of any focal length.

The goniometer method may be used for finding the focal length of a microscope objective if a micrometer scale, divided to $\frac{1}{10}$ mm. and provided with an eye-piece, be substituted for the glass scale. When the micrometer is in adjustment, the image of the goniometer wire will be sharply focused (without parallax) on the micrometer scale. Two readings are taken of the position of the image of the goniometer wire on the micrometer scale for two positions of the goniometer arm, and the focal length is calculated as before.

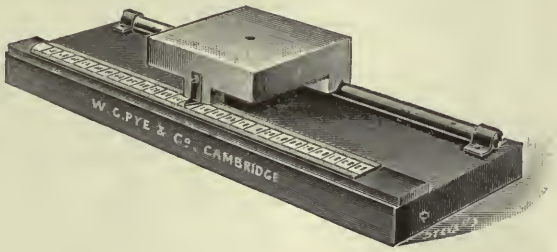


Fig. 5.*

The simple sliding carriage shown in Fig. 5 was designed to satisfy the need felt at the Cavendish Laboratory for a simple contrivance for producing a straight-line motion of a small carriage; it has been found to be a thoroughly useful addition to the equipment of the Laboratory.

The base-board carries a steel rod and a steel scale 30 cm. in length divided to millimetres, the rod and the scale being parallel to each other. The carriage is of cast-iron and has three projections on its lower side. In two of these a V groove is cut and these two V's rest upon the steel rod, the remaining projection resting upon the scale. There are thus five points of contact; the one remaining degree of freedom allows the carriage to slide along its track. If the bearing surfaces are slightly oiled, the carriage moves so freely that it is easily adjusted by hand to $\frac{1}{10}$ mm. A pointer attached to the carriage enables readings to be taken to $\frac{1}{10}$ mm. by estimation.

A vertical hole is bored through the carriage and a suitable bolt with a convenient nut is provided. By means of this bolt any object can be securely attached to the carriage just as a tool is secured to the slide-rest of a lathe. For many purposes it is convenient to fit a vertical steel rod to the carriage, as in Figs. 6 and 7, and to attach objects to the rod by the aid of suitable clamps.

* Registered Design, No. 526859.

III.—Revolving Table Method of Measuring Focal Lengths of Optical Systems

Among the methods available for the measurement of the focal length of a thick lens or of a system of lenses, those depending upon the property of the nodal points rank high for elegance and for accuracy. Of nodal point methods, the one in which a revolving table is used to locate the positions of the nodal points is the most direct, and involves the least arithmetic; it also illustrates the principle of the panoram camera.

The simple revolving table shown in Figs. 6 and 7 has been designed by the author in conjunction with Messrs. W. G. Pye and Co. for use in this method. The table rests on a tripod stand, which can be levelled by two screws fitted with lock-nuts to avoid shake. The table-top turns about a vertical steel rod carried by the tripod. The design secures both freedom of revolution and absence of side-shake. The latter condition is essential for accurate work.

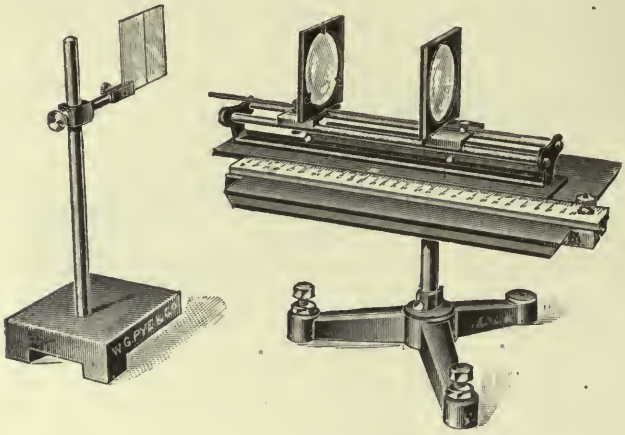


Fig. 6.

The table carries a bar graduated in millimetres, which can be clamped in any position on the table by the screw seen in Figs. 6 and 7. A cranked steel rod can be secured by a set screw in a socket in the casting which supports the table-top. This rod, as shown in Fig. 7, is fitted with an adjustable clip, in which a scale can be held.

With some exceptions, *any* system of lenses may be arranged for use with the revolving table by attaching the system to a small base-board in such a way that the optical axis is parallel to one edge of the board; this edge, which must be straight, is kept in contact with the graduated bar and must be provided with an index mark by which its position relative to the bar may be observed. To avoid mistakes, the opposite edge of the base-board should have no index mark.

The focal lengths of telephoto systems or of systems of *very* short focal length cannot be conveniently found by the revolving table method. The focal lengths of such systems are best found by the goniometer method described above.

In the process of finding the focal length of a lens system, the first step is to make the axis of the system intersect the axis of revolution of the table. The lens system is placed on the table, its base being in contact with the graduated bar. The

axis of the system is then directed towards a *distant* object, whose image is received on a ground-glass screen, placed as in Fig. 6. If the table-top be now turned backwards and forwards through a small angle, the image will, in general, move. If the image move in the same direction as the end of the table nearer to the screen, the system is moved on the table-top further from the screen, and the screen is moved so as to re-focus the image; the tripod itself must *not* be moved during the experiment. After a few trials, a position will be found such that the image of the distant object does not move from a vertical line ruled on the ground glass, when the table is turned through a small angle. The nodal point now lies in a plane containing the axis of revolution and perpendicular to the direction of the distant object, but does

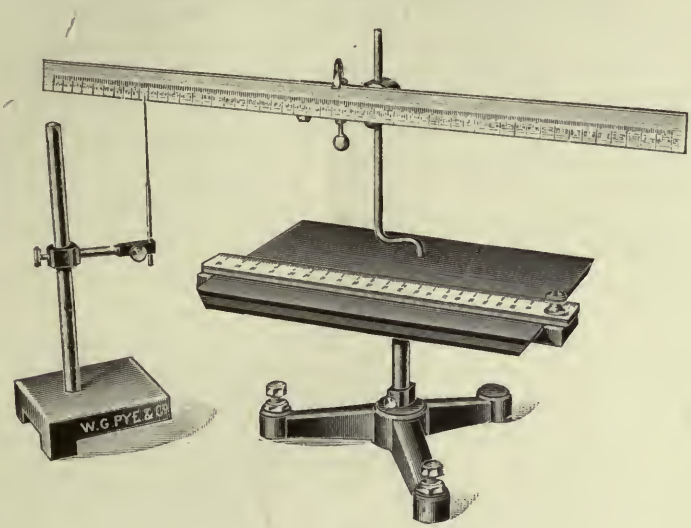


Fig. 7.

not necessarily lie *on* the axis of revolution.* The table-top is turned end for end, and the lens system is moved on the table so as to re-focus the image on the screen, which has not been moved. It will probably be found that, although the image does not move when the table-top is slightly turned, the image does not fall again upon the vertical line on the screen. The distance between the image and the vertical line is measured, and the graduated bar is moved at right angles to itself through half this distance, and is then clamped. The axis of the lens system will then intersect the axis of the table when the base of the system is in contact with the graduated bar.

The lens system is re-adjusted so that the image of the distant point does not move when the table is turned through a small angle, and the reading of the lens

* The image of the distant object point lies on the straight line which passes through the nodal point of emergence and is parallel to the straight line joining the distant point to the nodal point of incidence. The image is, therefore, undisturbed only when the nodal point of emergence does not move at right angles to the two parallel lines.

index on the graduated bar is recorded. The nodal point of emergence now lies on the axis of the table. The table-top is then turned end for end, a similar adjustment is made, and the new index reading is recorded. The difference between the readings gives the distance between the nodal points.

The focal length is now easily measured. The system is adjusted, as above, so that a nodal point lies on the axis of revolution, and a pin is placed so that its tip coincides with the image of the distant object. The lens system is removed from the table, and the cranked rod carrying the scale is secured to the table-top as in Fig. 7, the rod and scale being so adjusted that the divided face of the scale intersects the axis of revolution; this adjustment is easily made by the help of a set-square as a straight line intersecting the axis is ruled on the table-top. The table-top is then turned so as to bring first one and then the other end of the scale into contact with the pin, and the two readings of the pin are taken. Half the difference of these readings gives the focal length. No corrections of any kind are required.

When the distances of the principal foci from the end surfaces of the lens system are known, as well as the distance between those surfaces, a diagram can be made showing the positions of the nodal points and foci relative to the system.

After a little practice, the student will prefer to discard the ground-glass screen and to use the pin throughout the experiments.

When a *distant* object is not available, a plane mirror may be used to form an image of the pin coincident with the pin itself.

In the ordinary case, the media at the two ends of the system are the same, viz., air, and then the principal points coincide with the nodal points, and the two focal lengths are equal. The revolving table method can, however, be applied to find the two focal lengths, when the two media are different.

The lens system shown in Fig. 6 has been designed to meet the convenience of teachers. Two plano-convex lenses are held in metal frames, the convex surfaces facing each other. The frames can be clamped together at any chosen distance, and the pair, so clamped, can be moved with a simple straight-line motion along two metal rods attached to a small base-board.

This optical system is convenient in practical work. The focal length, F, of the system is given accurately by the equation

$$F = \frac{f_1 f_2}{f_1 + f_2 - a},$$

where f_1 and f_2 are the focal lengths of the first and second lenses, and a is the distance between the convex surfaces. The focal length f_1 is accurately equal to the distance of the image of a *distant* object from the convex surface of the first lens, when the light falls first on the plane surface of that lens, and similarly for the second lens.

The two lenses are sufficiently large to allow their radii of curvature, r_1 and r_2, to be found by a spherometer. From r_1 and f_1 and from r_2 and f_2 the refractive indices of the first and second lenses can be found. When the thicknesses t_1 and t_2 are known, the distance between the principal points of each lens can be calculated;

these distances are required in calculating the distance between the principal planes of the complete system. The calculated values of F, and of the distance between the principal planes may be compared with the values found by means of the revolving table.

IV.—Determination of the Astigmatism and Curvature of Field of a Lens System

The apparatus is arranged for the investigation of the " defects " of astigmatism and curvature of field when, as in landscape photography, the objects of which the lens is to form images are at infinite or very great distances from it. The arrangement is represented diagrammatically in Fig. 8.

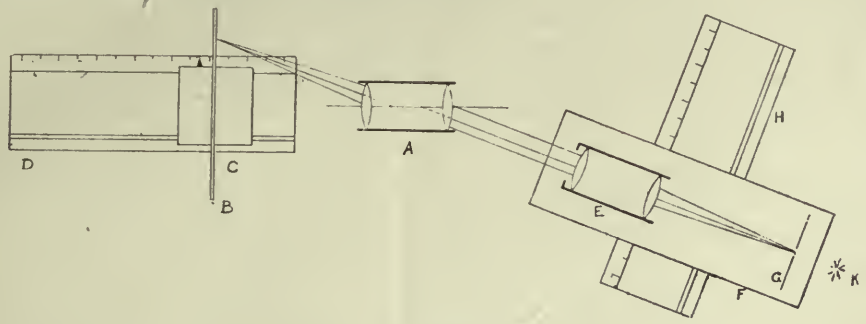


Fig. 8.

The lens A, which is to be tested, is firmly supported, with its axis horizontal, upon a suitable stand. A ground-glass screen B is mounted on a sliding carriage C which moves on the track D, as in the arrangement shown in Fig. 5. A horizontal line divided to half centimetres is ruled on the ground-glass. For convenience, the frame supporting the ground-glass is clamped to a vertical steel rod rising from the carriage (see Figs. 6 and 7). To obtain a beam of parallel light a collimator is used. A projection lens E, with its aperture diminished by a stop, is mounted with its axis horizontal on a board F. A pair of horizontal and vertical wires stretched across an opening in a metal plate intersect at G, the focus of the lens E. The board is clamped to a vertical rod fixed to a sliding carriage which moves on the track H, and the axis of E is adjusted to the same height as that of A. A flame K is placed behind the cross-wires to give the necessary illumination.

The two tracks and the stand supporting A rest on a table. Care must be taken that the axis of A is parallel to the direction of the track D, and that the plane of the ground-glass is perpendicular to that direction.

The collimator is first placed so that its axis approximately coincides with that of the lens system A. The carriage C is then adjusted so that the cross-wires are sharply focused on the ground-glass screen B, and the index reading of the carriage on the scale of the track is taken and also the reading of the image of the vertical wire upon the scale on the ground-glass screen. The collimator is then moved into

an oblique position, as in Fig. 8, and the optical effect of the cross-wires on the screen is examined. Unless the lens be free from astigmatism and curvature of field, there will be no clear image of both wires. It will, however, be possible to adjust the screen so as to obtain (1) a sharp image of the vertical wire, formed by primary focal lines, and (2) a sharp image of a part of the horizontal line, formed by secondary focal lines. The two readings of the carriage C will probably differ from each other and from the reading found when the collimator was in the symmetrical position. The observations are repeated for a number of positions of the collimator on either side of the symmetrical position, the obliquity being increased as far as the mount of the lens A allows. The difference between the two readings for (1) and (2) for a given obliquity of the collimator is the astigmatic difference for that obliquity of the incident parallel beam.

The track H is not *necessary* for the experiment, though it is convenient, since it allows the collimator to be readily adjusted so as to fill the lens A with light.

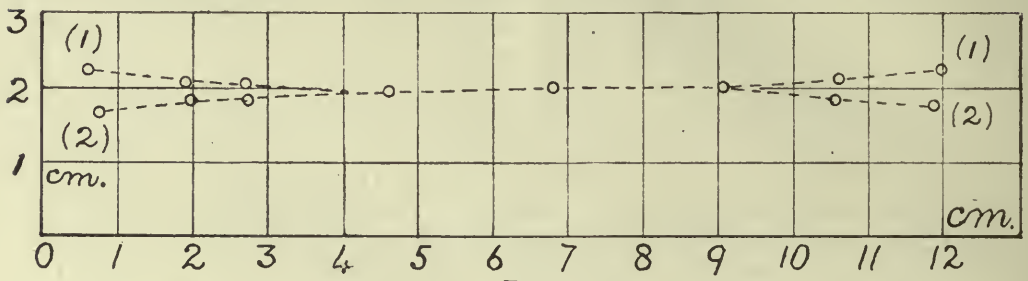


Fig. 9.

Unless the lens A is well-corrected, it will be necessary to stop it down considerably in order to obtain a sharp image of the vertical wire, when the obliquity of the incident beam is considerable. It will not always be possible to obtain a sharp image of the *whole* of the horizontal wire, though it will be possible to get a sharp image of any particular point on the wire by suitable adjustment of the screen.

Two curves (1) and (2) are then plotted. In (1) the abscissa is the reading of the image of the vertical wire upon the scale on the ground-glass screen and the ordinate is the reading of the carriage C. In (2) the abscissa is the reading of the *sharply focused point* of the image of the horizontal line on the scale on the ground-glass screen and the ordinate is the reading of the carriage C.

The diagram obtained in a test of a projection lens of 21·7 cm. focal length is shown full size in Fig. 9. The two image surfaces are slightly curved but in opposite directions. The primary image surface, on which the image of the vertical wire comes to a focus, is concave towards the lens. With a single thin lens at A, the stop being in contact with the lens, both image surfaces would be concave towards the lens, the primary surface being the more curved.

Prof. S. P. Thompson said that they would be glad indeed to have things so well done in their Laboratories.

DIFFRACTION IMAGES

By J. W. Gordon

(*Friday, June 21st,* 1912.)

In this paper an attempt is made to exhibit the theory of diffraction patterns in a comprehensive form and by means of an elementary mode of treatment. The object aimed at is to put the theory of diffraction upon the same textbook level as the theories of refraction and reflection.

The mode by which this simplification in the treatment of the subject is effected consists essentially in substituting as the surface of resolution the envelope which forms the boundary of an aplanatic pencil in place of wave front which occupies its aperture. By this alteration in the mode of analysis a great simplification in the processes of the analysis is effected, the analogies between different forms of aperture which have little resemblance to one another become apparent, and the elementary form of exposition is rendered possible.

By reason of the limited space within which this paper is restricted the subject is treated only illustratively and in outline, the mathematics involved being for the most part reserved for discussion in another paper.

The general doctrine is developed in seventeen propositions, as follows—

(i) The diffraction pattern produced by an aplanatic pencil of polygonal cross section when that pattern is formed and observed in an apertural plane remote from the focus is a series of Fresnel bands bordering the edge of the aperture of the pencil in that plane.

(ii) The diffraction pattern appearing in the focal plane is essentially a radiate pattern, and when the aplanatic pencil is of polygonal cross section the pattern takes the form of a star, in which every side of the polygon is represented by two opposed rays in the star.

(iii) The rays of the radiate pattern are broken lines of light showing maxima and minima of brightness in regular succession and having a definite centre-to-centre distance for the segments of the line. This centre-to-centre distance is proportional to the reciprocal $\frac{1}{\sin u_1}$: u_1 being the angle subtended at the focal point by the edge of one face of the polygon.

(iv) When, by the multiplication of sides, an aplanatic pencil of polygonal cross section becomes indistinguishable from one of which the cross section is a curve, the radiate pattern passes into an annular form, and from that cause a quasi-annular, or oval (circular) diffraction pattern may arise in the

focal plane as, for example, when the image of a star is observed in the focal plane of a telescope.

(v) Whatever the shape which the diffraction pattern in a focal plane may assume that shape is always bilaterally symmetrical.

(vi) The diffraction pattern in the focal plane formed by a concave radiant surface is identical with that so formed by the convex surface which fits into the concave.

(vii) A pencil of aplanatic light is perfectly transparent to an impulse of diffracted light in tune with it; and, therefore, is transparent to the diffracted light from any other segment of the same homocentric pencil.

(viii) Successive segments of a diffraction pattern, taken radially, are in opposite phase.

(ix) In the case of an oval (circular) pattern in the focal plane the maximum amplitude in various rings varies approximately in inverse proportion to the square of the radius of the ring.

(x) In the radiate pattern the maximum amplitude varies approximately in inverse proportion to the radial distance of the segment.

(xi) The ray velocity of diffracted light is equal to the ray velocity of aplanatic light in the region beyond the boundary of the aplanatic pencil.

(xii) Within the boundary of a homocentric pencil the ray velocity of diffracted light varies with the angle of the ray to the wave front through which it passes and is equal to $\frac{1}{\sin \theta} V$ if $\frac{\pi}{2} - \theta$ be that angle and V equal to the velocity of aplanatic light.

(xiii) The diffraction pattern due to the periodic structure of a regular compound aperture tends to settle down to the pattern of a double aperture, although the pattern of a compound aperture, comprising only a small number of components, may show marked deviations from the double aperture type of pattern.

(xiv) In the diffraction patterns of compound apertures of regular structure the diffraction pattern of the simple component aperture is found as a bright field upon which the secondary diffraction pattern, due to the periodic structure of the compound aperture, is displayed.

(xv) Every compound aperture exhibits three diffraction patterns, that is to say—

1. A primary pattern, which results from the form and dimensions of the simple component aperture.
2. A secondary pattern, which results from mutual interference of a number of primary patterns, and
3. A tertiary pattern, which is determined by the form and dimensions of the entire compound aperture considered as an undivided whole.

(xvi) The diffraction pattern visible within the aperture of an aplanatic pencil near the focal point is produced by visual projection upon the plane examined

of the diffraction pattern formed by optical projection in the focal plane.

(xvii) The diffraction pattern external to the aperture of an aplanatic pencil which has its ρ interval proportional to $\frac{m}{\sin u}$ is formed upon a sheet disposed at right angles to the surface of the aplanatic pencil.

The bearings of the theory thus outlined are illustrated by reference to the optical problems connected with the formation of a spectrum by a diffraction grating: also by reference to the effect of stopping out the centre of an object glass and again by the problem presented by the ultimate limit of resolving power in optical instruments.

The theory of diffraction, as developed in works on optics at the present time is in a strangely incondite condition, so much so, indeed, that the imperfections of the accepted textbooks in this respect afford ground for a serious reproach to our twentieth-century Science. Two classes of diffraction phenomena are described, and are called by the names of Fresnel and Fraunhofer respectively; the latter exhibited in focal planes, the former in what Sir Almroth Wright has aptly called the apertural planes of optical instruments—of the telescope, for example. But these two classes of phenomena are usually treated apart and investigated on different lines, so that the connection between them is obscure. The only successful attempt to bring them into clear relationship with one another known to the present writer is that made by Professor von Helmholtz in his paper in *Poggendorf's Annalen* of 1874 on the theoretical limits of the resolving power of the microscope. In that paper the principle which systematises the whole body of phenomena is rather implied than explained and, perhaps for that reason, it has not obtained such currency in scientific literature as its appearance under such conditions would seem calculated to secure. In fact, Fresnel's method of investigation is practically the only method in use for explaining diffraction phenomena, and Fresnel's method, for a reason to be presently mentioned, is wholly unsuited to the investigation of Fraunhofer phenomena. As a consequence of this inadequate discussion of the subject the phenomena discussed are very imperfectly dealt with.

But the faults of omission in the existing textbooks are even more serious, for one-half of the visible phenomena are passed without so much as mention: those, namely, which are seen by visual, as distinct from optical, projection. The most familiar example of what I am here calling visual projection is the image seen in a plane mirror. Such an image is commonly called a virtual image because it is not real, that is to say, it would not appear delineated on a screen if a reflecting screen were placed in the position occupied by that image. Its formation is due to the presence between that place and the observer's eye of an optical instrument; to wit, the mirror; from which a beam of light is received into the eye. The image

itself is not to be found in the surface of the mirror but a certain texture of light impulses—an undulation front; as I may call it by analogy to the familiar term wave front. The virtual image is what would be the optical projection of that undulation front if it were travelling away from the eye and focused by a suitable lens. But since the travel of the undulation front is towards the eye the real image is formed in the observer's retina and is by visual projection referred to its apparent position in space.

In like manner virtual images are formed by negative lenses and under certain well-known conditions by positive lenses. In analogous cases virtual images are formed by convex and concave mirrors, and all these phenomena of virtual image formation are duly and fully treated in works upon the doctrines of the reflexion and refraction of light.

Similarly, virtual diffraction images are formed, observed and call for explanation, being, indeed, among the most noteworthy of diffraction phenomena. But curiously enough they are entirely ignored—so far as the present writer's information goes—by writers on the theory of diffraction. They are consequently a stumbling-block and a rock of offence to the naturalist who is constantly mistaking for objects the créatures of visual projection, and more often than enough suspicious of appearances which are perfectly trustworthy and correspond to the most solid realities, simply because the physicists cannot teach him how to distinguish diffraction patterns from material objects in the field of the microscope.

It is proposed in the present paper to attempt a systematic outline of the whole subject of diffraction. More than an outline would be impossible within the limits of space which a paper imposes, and this limitation will, it is hoped, be accepted as a sufficient excuse for a cursory treatment of the subject as a whole. It is of more importance in the present state of the Science to indicate clearly what may be accomplished by the methodical treatment of the subject than to deal exhaustively with any part of it. I shall make no scruple, therefore, of selecting for discussion those parts which appear to me best adapted to serve as illustrations of this main proposition. Full proof and detailed investigation I reserve for another occasion.

A few words by way of preface may be addressed to the subject of Fresnel's method of investigating diffraction phenomena. Allusion has been already made to its limitation to certain types of diffraction phenomena and it will be convenient at starting to trace the limits of its applicability.

It is well known that Fresnel's method of investigation consists in tracing down the optical projection of a light impulse from its source—its primary source in the original focus, or its secondary source in some selected wave front or image of the focus, as the case may be—to an object point in the diffraction pattern which is under investigation. If this deduction of the light impulse through its successive stages has been successfully carried out it yields in the result a numerical value which expresses the light intensity at that object point.

Now it is obvious without discussion that this method of inquiry can yield no information directly concerning virtual diffraction images. It is concerned with

real images only, and this is probably why the virtual images, which appear as diffraction patterns in the telescope, although they have attracted much notice and have been largely utilised by telescope-makers in the testing of their lenses, have hitherto remained without scientific explanation. They cannot be directly investigated by Fresnel's method, and physicists are so dependent upon Fresnel that they fail to explain even the most obvious diffraction phenomena when these result from visual projection.

There is, however, another limitation which restricts the scope of Fresnel's method of investigation, and this limitation applies within the sphere of optically projected patterns.

Fig. 1 will serve to illustrate the point. Here, a pencil of light is shown diagrammatically by a series of plane wave fronts delineated by full lines perpendicular to the x–x axis. The light transmitted along the ray x–x, and along any ray parallel to it, is aplanatic light. Another system of undulation fronts is shown in broken lines. These are not wave fronts, for they are obviously polyphasal, but they are, or may be, capable of transmitting an impulse along the y–y axis. The impulse so transmitted will be diffracted light, for the rays parallel to y–y cross the *wave fronts* at an angle (say $\pi/2-\theta$) other than a right angle. Such a polyphasal surface I propose to call an undulation front. If, then, the angle between the wave fronts and the undulation fronts be called θ we shall have an impulse of diffracted light transmitted at an angle θ to the axis of aplanatic light.

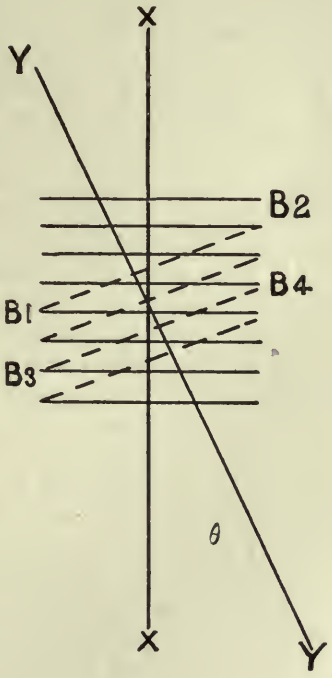


Fig. 1.

Fresnel's method of investigating the magnitude of this diffracted light impulse consists in dividing one of these polyphasal undulation fronts into a number of zones, so small and so shaped that every one of them may be treated as a monophasal surface, the phase of which is at any moment identical with and, therefore, determined by the phase of the wave front of which it forms a part. These separate impulses may be called elements of the impulse of diffracted light and expressed by the symbol $d\mathsf{L}$. Then every such element being treated as a component force, the resultant impulse is found by combining all these forces according to the mechanical law for the composition of coplanar forces. This resultant is then substituted for its components, and treated as if it were a wave front, having amplitude L and occupying the position of the undulation front which it replaces. It is, therefore, taken to be propagated as a wave front would be propagated along the axis y–y, and if, by the laws of optical projection, a wave front so propagated would condense upon a given

point, then the resultant amplitude L is taken to be the amplitude of the diffracted light at that point.

In all cases in which we can show that all optical paths between a given undulation front and object point are equal, Fresnel's method applies, and, subject to difficulties of computation—which are sometimes very formidable—it will solve the problem. Thus, if the object point be supposed to be situated in the axis y–y at a distance from B_1, B_2 equal to infinity, then, since all optical paths from plane surfaces perpendicular to that axis to such a point are equal, we may conclude that the impulse of diffracted light propagated to that point will have the amplitude L given by Fresnel's formula. So, again, if at any finite distance along the y–y axis we have a lens placed so as to reproduce in its principal focal plane the image at infinity we shall, at its principal focal point, have an impulse of that same magnitude. So much results from treating the undulation front B_1, B_2 as if it were a wave front with the resolved amplitude L. And this result is justified by observation.

It will be convenient here to digress for the purpose of explaining the Helmholtz theory of the diffraction pattern, which branches off from Fresnel's theory at this point.

The optical system defined by introducing a lens, as just described, upon the y–y axis is an uncentred system; that is to say, it has no principal optical axis. As in a panorama, or in a panoramic picture, the optical axis is everywhere, and runs in a different direction through every different point in the picture. Such a scheme of construction does not afford us much information concerning actual images, for most actual images are formed by centred optical systems in which the centres of all the refracting surfaces lie in one line. Helmholtz pointed out, accordingly, that for the determination of the diffraction patterns which actually occur in focal planes it is necessary to treat the y–y axis, not as being the principal optical axis of a system having its central points along the line y–y, but as an oblique axis belonging to an optical system centred along the line x–x. Then, by applying the known laws of optical projection for oblique pencils, he found to what point in the focal plane the resultant impulse, determined according to Fresnel's law, should be referred. That, shortly stated, is the Helmholtz theory of Fraunhofer diffraction patterns, and it may be observed in passing that it establishes a clear and definite relation between the Fresnel and the Fraunhofer phenomena.

Coming back now to the question of the limits imposed upon the application of this method of investigating diffraction phenomena, let it be observed that we are compelled for the purpose of Fresnel's theory to assume a higher rate of transmission for the light impulse along the diffracted ray than along the aplanatic ray. It is essential to the validity of Fresnel's reasoning that nothing should depend upon the selection which he makes of a particular undulation front to serve as his surface of resolution, that is to say, as the surface in which he finds the data for his calculation of the resultant impulse. For example, if in passing from B_1–B_2 to B_3–B_4 the component impulses change in magnitude or phase distribution then it will be necessary to perform a second computation in order to derive the resultant in the second plane

from the resultant in the first. It is because in all the elements of the calculation every one of these undulation fronts is exactly like every other, and because this applies throughout the whole series, that Fresnel was able to determine the value of his resultant impulse in his selected surface of resolution and to transfer the result so computed to the object point in the diffraction pattern. Now, in order that this may be so, the light impulse must travel along the y–y axis faster than it travels along the x–x axis and faster in the ratio of $1/\cos\theta : 1$. Stated in other words, the ray velocity of diffracted light must be greater than the velocity of aplanatic light in the ratio just named; or, putting the same thing again in other terms, the wave length measured along the diffracted ray exceeds in this proportion the wave length when measured along the aplanatic ray.

So long as we have only to deal with plane wave fronts this point is of no consequence. The element of ray velocity does not enter into the calculation for the simple reason that, whatever that velocity is, it is exactly the same along all the optical paths in question; and therefore its magnitude, great or small, does not affect the resultant impulse at the object point.

But now let it be supposed that, instead of plane undulation fronts of diffracted light, as in Fig. 1, we have to deal with undulation fronts of concentric curvature as in Fig. 2. It is at once evident that very different considerations arise. We can no longer afford to disregard the ray velocity of diffracted light because it affects the phase distribution in successive fronts. Thus, if we perform Fresnel's operation upon the undulation front B_3–B_4 in Fig. 2, we shall clearly obtain a different resultant impulse from that which would be obtained by basing the computation on the undulation front B_1–B_2. In this case, therefore, the result secured by Fresnel's method is purely arbitrary. Its magnitude depends entirely upon the particular surface selected as the surface of resolution. It is, therefore, quite inapplicable except in those cases in which the curvature of the undulation front in question is so small that the undulation front may, without sensible error, be identified with a plane surface. Stated in other words, the Fresnel method applies only when variations in the ray velocity of diffracted light are so small as to be negligible. This circumstance limits the application of Fresnel's method to focused pencils of light in which the divergence angle is so small that writing u for that angle, we may without sensible error, write $\sin u = u$. It is only to such cases that Fresnel's method has been successfully applied; the reason is now evident.

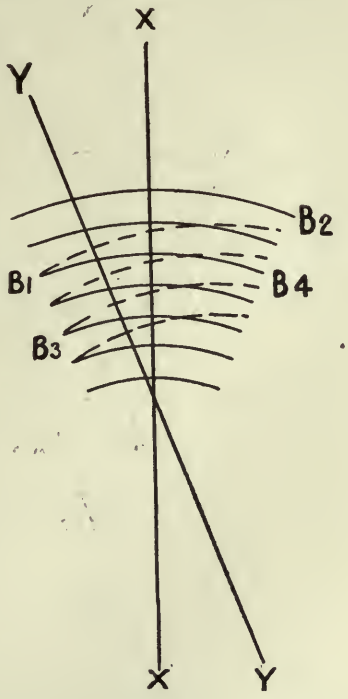


FIG. 2.

A similar difficulty has been encountered in applying the undulatory theory of light to explain the propagation of light by plane wave fronts. If we suppose, for instance, that any point in one of the aplanatic wave fronts shown in Fig. 1 receives its light by radiation from the whole area of any one of the surfaces shown above it, we have again the difficulty that the optical paths involved are unequal; and if, therefore, we are to account by calculation for the propagation of this impulse from an extended area to a selected point with a velocity equal to the velocity of light, we must assume unequal ray velocities from parts of the radiant surface which are unequally distant from the object point. This difficulty has been in practice met by an ingenious contrivance which is known as Archibald Smith's Law of the Secondary Wave. It would involve too much digression to enter upon the exposition of that law here, but it is useful to refer to it in order to point out that the same difficulty which limits the application of the Fresnel method to the theory of diffraction has been encountered and successfully surmounted in applying the undulatory theory to the more familiar phenomena of aplanatic light propagation. Suffice it only to add here that Smith's Law of the Secondary Wave is of very special character and limited application. It cannot be applied in the case we are now discussing to get over the difficulty of the varying ray velocity of diffracted light. That must be surmounted by another expedient.

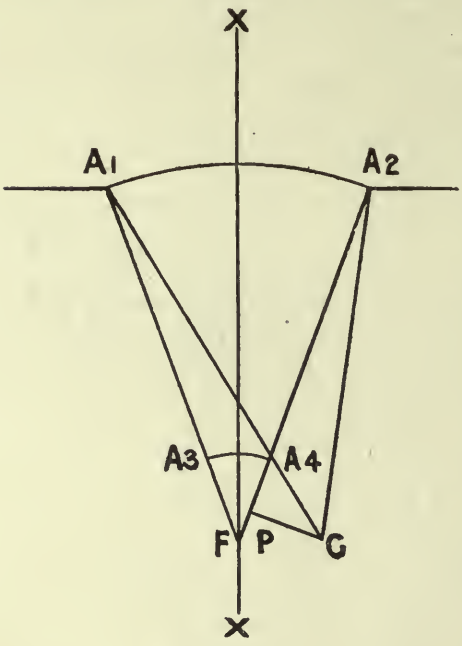


Fig. 3.

The expedient which has been adopted as the basis of the system unfolded in this paper, is the substitution of the enveloping surface of the aplanatic pencil for the wave front as the surface of resolution. Fig. 3 is a diagram which serves to illustrate the change so made. In this diagram A_1–A_2 is the aperture through which the focused pencil of aplanatic light is received. G is the object point at which the amplitude of the diffracted light is to be computed. By the Fresnel method this is done by taking a wave front in the aperture A_1–A_2 as the surface of resolution, and assuming that the resultant impulse is transmitted to the point G by rays included in the angle A_1GA_2. It is obvious, in view of what has been pointed out above, that the result so arrived at must be subject to very considerable correction. For instance, it is quite certain that the wave front marked A_3–A_4 in the diagram would send diffracted light to the point G. But for the purpose of this calculation its contribution to the whole result is ignored. Again, the ray

velocity of the diffracted light must obviously vary, not only from one ray to another but at different points along the same ray, for the angle θ, as above defined, which determines the ray velocity, is constantly varying along the length of any given ray. In fact, as has been above stated, this method of calculation is never applied to such a case as is illustrated by Fig. 3. Only when the points F and G lie so close together that these sources of error can be neglected, is the Fresnel method applicable.

The alternative method here adopted is to substitute, as above stated, for the wave front in A_1–A_2, the enveloping surface of the aplanatic cone between the points A_2 and P. The line GP meets the edge ray A–F, at right angles, and, therefore, none of the wave fronts between P and F can send any diffracted light to G, since that would involve their radiating a light impulse backwards—a manifest impossibility. Upon this method, however, all that part of the aplanatic cone which can radiate any light upon G, is accounted for, and, furthermore, a proper allowance is made for varying ray velocity within the aplanatic pencil, because the data from which we calculate, being derived from the state of phase distribution known to subsist in the envelope, are themselves duly modified by the action of the varying ray velocity. An incidental advantage arises from the use of this method in the circumstance that the required integration over the conical surface is, in fact, a very much simpler operation than the integration over the circular surface which constitutes the base of the cone, and, therefore, the calculations involved in what is the most ordinary case—the case, that is to say, of a light pencil of circular aperture—are greatly simplified, while at the same time the result is corrected.

It may be supposed at first sight that the difficulty arising out of varying ray velocity would still present itself even when the enveloping surface is chosen as the surface of resolution. It is not proposed here to go into the purely mathematical problem involved, but the answer to this difficulty may be briefly indicated. What we know about the enveloping surface of a focused beam of light amounts to this: within that surface the light impulse can be divided up into certain component parts which are called secondary waves, and the propagation of the aplanatic light impulse can be fully explained by treating these secondary waves as so many component forces, and working out their resultant upon that hypothesis. These secondary waves, however, or component forces, are all arrested by the envelope and cannot travel beyond it into outer space. The envelope, however, does not arrest the resultant impulse. The resultant impulse passes freely through the envelope while the component impulses are confined within it. This is all that we know about the envelope, but it is sufficient for the purposes of computation. It follows from this that whereas within the envelope we must never lose sight of the fact that the system of component impulses exists and determines the conditions of light propagation within the enclosed region; beyond that envelope the components have ceased to exist, and we have only to deal with the resultant impulses which we are now at liberty to treat as so many separate pencils of light. Of the correctness of this view very striking experimental proof will presently be adduced.

Postulating, then, the radiant surface which envelops an aplanatic pencil of

light as the source of diffraction, we may proceed to deduce certain general properties of the resulting diffraction patterns. It will be assumed, in the first place, that a bright star is the object of which the diffraction fringe is the optical image, and that the image-forming pencil is transmitted by a simple—that is to say, an undivided —aperture. Fig. 4 will serve the purposes of discussion.

Let N_1 be the nodal plane of an objective system; N_2 the nodal plane of the eye-lens, F the focal point and F–G the aplanatic focal plane. $A_1A_2A_3$ is the aperture, having one straight side A_1–A_2, and bounded otherwise by the circular segment $A_1A_3A_2$.

It is obvious, upon inspection of the diagram, that the envelope $A_1A_2A_3F$

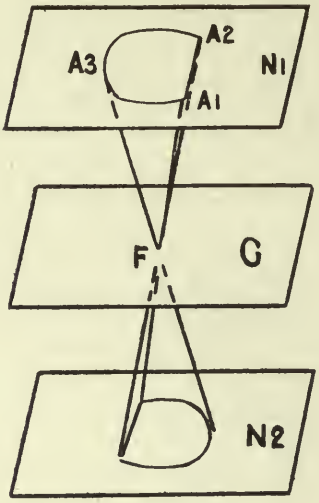


Fig. 4.

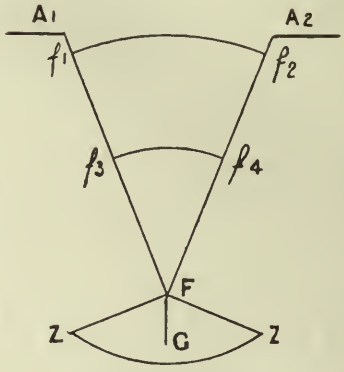


Fig. 5.

must necessarily be of the nature (1) of a cone or (2) of a pyramid; or (3), as shown in the figure, it must be a compound of elements derived from both. That is to say, it must be either a faceted or a convex surface, if we disregard the alternative that the surface might be concave. This last is a case which it is not necessary to distinguish, for it will presently appear that in the focal plane the pattern thrown by a concave surface is the same as that thrown by a convex. In Fig. 4, then, we have all the possible varieties of radiant surfaces illustrated, since every aplanatic ray must pass through the focal point F. The problem, therefore, is to determine how many varieties of diffraction patterns can arise from a triangular flat surface and a conical segment respectively.

To deal first with the triangular flat radiant surface: Let Fig. 5 represent such a surface—a facet of a pyramid enveloping the aplanatic pencil transmitted by a polygonal aperture, upon which certain isophasal lines *f*, *f*

DESCRIPTION OF PLATE

A. Diffraction pattern formed within the aperture of the aplanatic pencil and remote from the focal plane. This is a Fresnel pattern (p. 183).

B. The like pattern formed in a pencil of rectangular section and exhibiting four symmetrically disposed Fresnel patterns (p. 183).

C. Diffraction pattern formed within the aperture of the aplanatic pencil and near to the focal plane. This exhibits the projection upon the apertural plane of the Fraunhofer pattern of the focal plane (p. 187).

D. Focal plane pattern of six rays produced by the three-sided prismatic pencil of aplanatic light transmitted by a triangular aperture (p. 186).

E. The like pattern of four rays yielded by a pencil of square section (p. 186).

F. The like pattern of ten rays yielded by a pencil having the form of a regular five-sided prism (p. 186).

G. Focal plane pattern formed by the conical surface of the pencil of aplanatic light transmitted by a circular aperture (p. 187).

H. The like pattern formed by the pencil transmitted by a semicircular aperture exhibiting the radiate pattern—two-rayed star—of the flat face in combination with the ring system of the hemicone (p. 188).

I. The like pattern formed by the two hemicones resulting from the division of a circular aperture by a cross wire (p. 189).

J. Focal patterns formed by a compound aperture comprising two parallel slots. The upper figure is the image of a star, the lower the image of a bright object line (p. 197).

K. The like patterns formed by a compound aperture comprising three parallel slots (p. 197).

L. Apertural pattern seen near the focal point in the prismatic pencil transmitted by a triangular aperture. The photograph is defective. If perfect it would show strongly curved segments of those rays which originate in the internal faces of the prism, thus illustrating the increased ray velocity of diffracted light within the aperture of the aplanatic pencil (p. 190).

A

B

C

D

E

F

G

H

I

J

K

L

[*To face page* 183.

have been traced. Let it be assumed, in the first place, that the spectator's eye is focused on a surface perpendicular to the facet where it is traversed by the isophasal $f_1 f_2$, and that his axis of vision passes through the line F–G, traced upon the complementary pyramid ZFZ, and disposed symmetrically with reference to the radiant surface A_1FA_2. The pyramid ZFZ is a pyramid placed, as shown, so as to be coaxial and covertical with the pyramid of which A_1FA_2 is one face. The angles A_1FZ and A_2FZ are right angles.

It is obvious upon inspection that, under these conditions, the radiant surface which produces a diffraction pattern in the isophasal surface through f_1–f_2 is identical with the surface producing diffraction in the well-known case of a pencil of unfocused light passing a straight edge, for the rays are in all parts of the region in question perpendicular to the isophasal f_1–f_2, and are disposed within the boundary of a straight edge, extending from A_1 to A_2. We ought, therefore, under these conditions, to see the well-known Fresnel diffraction pattern produced by a straight edge. In fact, we do see such a pattern. Figure A in the photographic plate is a photograph obtained under the conditions specified. Next, let the focus of the eye be drawn back towards the focal plane, and suppose that a halt is made at the isophasal f_3–f_4. The conditions of image formation are obviously unchanged, save in one respect, namely this, that the light amplitude is increased in the proportion of $\frac{\text{radius of } (f_1\text{–}f_2)}{\text{radius of } (f_3\text{–}f_4)}$. We find, accordingly, that the breadth of the image produced by diffraction in this place is less, and its brightness greater, but its characteristic features are unaltered.

If we suppose the aperture of the aplanatic pencil to be square in section, we shall obviously obtain four such Fresnel patterns arranged as sides of the square (see photographic plate B); and since the characteristic feature of such a pattern is a group of parallel bands arranged about a centre of figure, I propose to speak of this as a pattern of the annular type. If the polygon be one having very numerous sides, then the pattern becomes indistinguishable from a system of oval bands, if the word " oval " is used in a sense large enough to include the circle. As the pencil of circular section is the most important which we have to consider, the term annular is appropriate enough to be used with the extensive meaning which will make it apt when the perimeter of a polygon is meant.

The first result, then, of this discussion of the subject is that—

I. The diffraction pattern produced by an aplanatic pencil of polygonal cross section, when that pattern is formed within the aperture of the pencil in an apertural plane remote from the focal point, is a series of Fresnel bands bordering the edge of the aperture of the pencil in that plane.

This proposition is stated in terms that cover the case of the diffraction pattern in the plane N_2 as well as in the plane N_1, and in the two apertural planes actually discussed in connection with Fig. 5. And it is quite rightly stated in this comprehensive form, for it is evident that the same geometrical considerations apply in all cases. The only difference that requires to be noted is, that beyond the focal

plane F–G the sides of the aperture are reversed, so that the figure seen beyond the focal point is the inverted replica of that seen within the focal point.

But this rule is obviously inapplicable to a focal plane, for in the focal plane the aperture of the aplanatic pencil contracts to a point, and there is no edge about which the Fresnel bands can be formed. A point, however, although it cannot have sides or an edge, can have different aspects. Thus we may define the line F–G of Fig. 5 as being a line of symmetry with reference to the surface A_1FA_2, and with reference to the isophasals drawn upon that surface. We have seen that the Fresnel bands contract gradually as the isophasals contract towards the point F, and therefore if we suppose these bands to continue to contract until the line F–G is reached, we shall obviously have a broken line of diffracted light formed here. What is so seen is the condensed light of the Fresnel bands. In fact, a phenomenon very much like this does occur, but not accurately as described, for the pattern seen in the focal plane is the optical projection, not of those bands, internal to the aperture, which are shown in the photographs of Figs. A and B, but of the external diffraction bands —too feebly luminous to appear in the photographs—which edge the aperture outside and lie in the external shadow.

We are to think, then, of the radiant surface A_1FA_2 as being the source in which a series of diffraction bands take their rise almost invisible in their extended form, but rising into visibility when condensed in the focal plane. From this plane the internal fringes disappear, for the undulation phase is opposite in direction in adjacent rings, and consequently as they approach the apex of the envelope they blend and cancel one another. Only the outermost of these internal bands survives to reach the aplanatic focal point, and by its condensation it supplies the light which is transmitted to the expanding pencil which emerges from the focal point. The external bands, condensing simultaneously upon the line of symmetry F–G, give rise there to a line of light radiating from the focal point. Thus, whereas Fresnel bands, parallel to the edge of the aperture, are the characteristic feature of the diffraction pattern in an apertural plane, the characteristic feature of the diffraction pattern in the focal plane is a broken bright line radiating from the focal point. I shall speak of the pattern constructed on this basis as the radiate type of diffraction pattern.

There is another distinction between the patterns of the apertural and focal planes—equally characteristic but not equally pronounced,—namely this, that the focal plane pattern is always and essentially bilaterally symmetrical. In a sense the same thing may be said of the diffraction fringe of the apertural plane, for it has been shown that every feature which is found on one side of the optical axis in front of the focal plane can be observed on the other side of the axis beyond the focal plane. But these two halves of the annular pattern are developed in separate apertural planes—the two halves of the radiate pattern must, on the contrary, be observed in the same focal plane. The explanation of this is very simple. The radiating line on one side is formed by optical projection, as has been just described, taking its rise in the radiations given off from the face of the envelope of the convergent pencil

on the objective side of the focal point. A radiating line, which is the image and continuation of that line, is seen by visual projection when the diffracted light from the diverging pencil on the ocular side of the focal point is received into the eye and projected by a focusing effort upon the focal plane. The geometry of the optical projection which produces the real radiate pattern is identical with the geometry of the visual projection which produces the virtual radiate pattern. The two elements, therefore, are inseparable, and the radiate pattern is always bilaterally symmetrical.

The radiating bright line now under consideration differs from Fresnel bands not only by reason of its direction in relation to the focal point, but also by the circumstance, already referred to, that it is a broken line—whereas they are continuous. The law according to which this breaking up of the radiate pattern into radiating segments takes place must now be considered. It will be easily understood that this formation results of necessity from the origin of the line in question if it be considered to be the condensation upon a line of symmetry in the focal plane of a system of bands in the apertural plane. But a more precise statement is desirable.

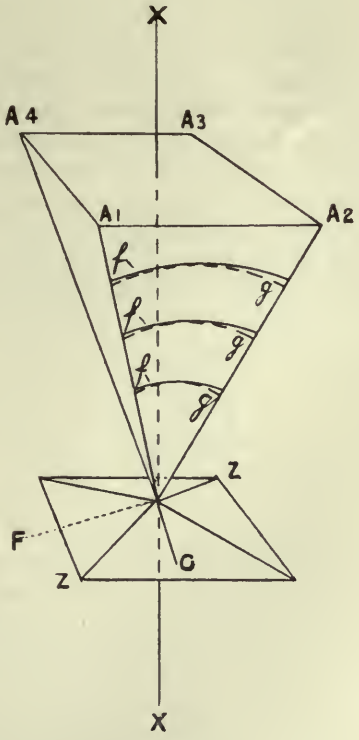


Fig. 6.

Let the centre-to-centre distance of these segments of the radiate line be equal to ρ. The question is, what influence determines the magnitude of ρ? To solve this problem let Fig. 6 be considered. This diagram shows two faces, A_1FA_2 and A_1FA_4, in perspective and the line of symmetry F–G, with reference to the first-mentioned face. On A_1FA_2 the isophasals *f*, *f* are traced and curves *g*, *g*, which are clearly not isophasal lines. But these would be isophasals if the point G were a radiant point and the surface A_1FA_2 were illuminated by the radiation from that point. I shall speak of these as anisophasal lines traced in the A_1FA_2 surface. It is obvious that the distance intercepted between *f* and *g* on any radius through F, if expressed in terms of $\frac{\lambda}{2\pi}$, is a measure of the retardation of phase with which light from a given point upon the isophasal reaches the point G. Let that intercept be equal to $d\phi$. Then ϕ obtained by integration over the whole surface A_1FA_2 will be an angle expressing the range in phase of the impulses of diffracted light which at any instant of time reach the point G, and it can be shown that $\frac{\sin \phi}{\phi}$ expresses the amplitude of the light undulation so set up in G compared with

the amplitude of the undulation which would be there set up if the anisophasals g, g were isophasal lines. It is not proposed to investigate in this paper the magnitude of ϕ, but one property of the integral may be noted which is perfectly obvious on the face of the diagram. It is clear that the value of ϕ must be greater for any given position of the point G when the angle A_1FA_2 is large than when that angle is small, for the largest intercepts are those which are cut off in the radii of widest angle. Now, by the definition of the value ρ, we learn that ρ equals the distance in which ϕ passes from the value $\phi = 2n\pi$ to the value $\phi = 2(n + 1)\pi$, n being a positive integer. Therefore $\frac{F-G}{\rho} = \phi$, and the periodic interval ρ must be inversely proportional to the integral ϕ. We thus arrive at a rule that increase of angular aperture of the radiant surface is accompanied by reduction in the periodic distance ρ between the centres of the segments into which the radiate pattern is cut up. Observation shows that this distance varies inversely as the breadth of the surface which produces the pattern, and that for all ordinary focal lengths—that is to say, when the focal length is equal to a large number of wave lengths—increase of focal length makes no difference to the pattern. We may, therefore, assume an arbitrary focal length, and express the breadth of the surface in angular measure. If, then, we write l for this focal length, and take the angle $A_1FA_2 = 2u_1$, we shall have for the breadth of the radiant surface $2 \sin u_1\ l$, and therefore the magnitude of ρ, being inversely proportional to this breadth, will vary as $\frac{1}{\sin u_1}$.

The form of the diffraction pattern has, so far, been determined only with reference to the line of symmetry F–G, and it is obvious that any polygonal aperture must yield a radiate pattern in which the various radiant faces give origin to as many optically projected radiate lines in the pattern. As every such optically projected line is balanced by a visually projected line, it follows that the numbers of radii in the pattern is potentially double the number of sides in the polygon. Thus a single straight edge produces two radii, which lie in the same straight line and meet in the focal point. A three-sided pyramid yields a six-rayed star, and so on. But in the case of a square pyramid only four rays appear in the radiate pattern because the four visually projected rays coincide with the four optically projected, the opposite sides of a square being parallel to one another. For the same reason a regular six-sided pyramid yields a six-rayed star, precisely like that yielded by the pencil which passes an aperture having the cross section of an equilateral triangle. Moreover, it is quite immaterial that the aperture is eccentric in the apertural plane, however great its eccentricity may be. Thus all the apertures shown in Fig. 7, A, yield the same diffraction pattern in the focal plane; that, namely, which is illustrated in Fig. 7, B. The effects produced by the varying eccentricities of the component apertures are so very subordinate and inconspicuous as to be hardly noticeable. The typical pattern remains quite unchanged.

When, however, the polygon passes into a curve we find a line of symmetry in every azimuth. Thus the rays of the radiate pattern become so numerous that they

constitute a new type of annular pattern; and it, therefore, becomes necessary to distinguish this pattern from the annular pattern of the apertural plane.

The distinction is easily drawn. The diffraction pattern in the apertural plane lies upon the bright field of the aperture; and there is here, therefore, no gradual falling off of light except, as will be presently explained, in the outer half of the outermost ring. When the system of bands occupies the whole aperture, full light and full shadow are to be found in all the bands respectively, and there is no distinction between them in this respect. When the banded fringe occupies part only of the aperture, and is disposed as an edging just within the boundary, it is in the dark bands, not in the bright, that a gradual loss of intensity is noticeable.

In the quasi-annular system of the focal plane these conditions are reversed. Here the pattern consists of bright bands on a dark field, and the brightness of the bands falls off rapidly as we pass from the centre outwards. Highly characteristic is the appearance of the outermost ring, which in the focal plane is a narrow band of extremely low illumination. In the case of the truly annular pattern of the apertural plane, on the contrary, this outermost band is of greater breadth than any other, and shows the full local brightness of the aplanatic light in the region adjacent to its inner edge. By these characteristic features the annular pattern of the apertural plane can be easily distinguished from the quasi-annular pattern of the focal plane. The photographs C and G of Plate will serve to illustrate these points of difference.

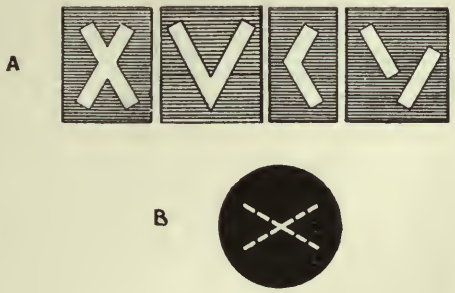


Fig. 7.

Summarizing the facts so far ascertained concerning diffraction patterns in the focal plane we have the following result:

II. The diffraction pattern appearing in the focal plane is essentially a radiate pattern, and when the aplanatic pencil is of polygonal cross section, the pattern takes the form of a star, in which every side of the polygon is represented by two opposed rays in the star.

III. The rays of the radiate pattern are broken lines of light, showing maxima and minima of brightness in regular succession, and having a definite centre-to-centre distance for the segments of the line. This centre-to-centre distance is proportional to the reciprocal $\frac{1}{\sin u_1}$.

IV. When, by the multiplication of sides, an aplanatic pencil of polygonal cross section becomes indistinguishable from one of which the cross section is a curve, the radiate pattern passes into an annular form, and from that cause a quasi-annular or oval (circular) diffraction pattern may arise in the focal plane, as, for example, when the image of a star is observed in the focal plane of a telescope.

V. Whatever shape the diffraction pattern in a focal plane may assume, that shape is always bilaterally symmetrical.

It will be convenient now to take up again the consideration of the forms of diffraction patterns in apertural planes; but, as preliminary to the further discussion of that topic, it may be pointed out that an important general rule, which applies equally to both classes of patterns, may be deduced from the rule now established—that the conditions of visual projection are similar to those of optical projection, with the result that the images visually projected and the images optically projected from the same light pencil are similar. From this it follows that the diffraction pattern thrown off from a concave surface is identical with the pattern thrown off by the convex surface which fits into the concavity of the surface first mentioned. This will be obvious if it be considered that any pencil of aplanatic light which presents a convex surface towards the observer, as it approaches the focal point, must present a complementary concave surface towards him as it emerges from the focal point and vice versâ. If then we find, as we do, that the visually projected half of the diffraction pattern in the focal plane is always similar to the optically projected half, we may infer that the concave surface produces the same pattern as the convex.

It may be objected to this proof, thus stated, that the two things compared are not equivalent because, although the form of the surface changes, the position of the aplanatic light with reference to its envelope does not change. The pencil, solid before focusing, emerges as a solid pencil from the focal point, and a pencil which is hollow to start with finishes as a hollow pencil. It may be added that the ray velocity is one thing on the darkened side of the boundary, another thing on the bright side of the boundary; and that, therefore, the argument is inconclusive, inasmuch as the light impulses are propagated on the two sides of the surface at different rates of speed and in conformity with different rules.

So much is perfectly true, but the point of the criticism is met by a further law of the propagation of diffracted light—namely this, that an aplanatic pencil is perfectly transparent to impulses of diffracted light which are "in tune" with it. This, which may be described as a particular example of the great law of resonance, gives rise to some very striking phenomena.

Among the most striking—and, at the same time, most easily intelligible—is the following. The diffraction pattern exhibited in the focal plane by a pencil of light transmitted by a semicircular aperture, is of the form shown by Fig. H of the photographic plate. It comprises, in accordance with the laws above explained, the quasi-annular pattern due to the curved surface and the radiate pattern due to the flat surface of the aplanatic pencil. If we put two hemicones together, so as to form one homogeneous cone, the flat face, of course, disappears and with it disappears the radiate pattern. We have only the circular pattern left of Fig. G. But now let the cone be divided into two hemicones by a fine wire drawn across the aperture. The shadow of this wire will, of course, give rise to two flat surfaces facing one another and very nearly adjacent to one another, but still separated by the short space cor-

responding to the thickness of the wire. It is found that under these conditions the radiate pattern reappears as shown in Fig. I. Save in this respect, that its false disc is of circular not of oval form, the diffraction pattern so produced by two hemiconical pencils complementary to one another is indistinguishable from the pattern due to one of them alone.

It obviously follows from this observation that the one hemicone is perfectly transparent to the diffracted light given off by the other hemicone. And this is by no means hard to be understood. The ray velocity of the diffracted light is, as we have seen, different from that of aplanatic light so long as the impulse of diffracted light is transmitted within the aplanatic system. But when it escapes from that system the diffracted light is transmitted in all respects—velocity included—like an impulse of aplanatic light. It follows that in crossing the shadow of the wire the diffracted light from the one hemicone must get out of phase with the aplanatic light of the other hemicone. It, therefore, produces interference, and since both hemicones are parts of one homocentric pencil the phase difference between the two halves sets up the phenomena of regular interference. Thus, the recipient half becomes endowed with the power of radiating from its curved surface the diffracted impulses proper to a flat surface; in other words it is transparent—and, observation shows, very perfectly transparent—to the diffracted light of the active half. The pencil being symmetrical, each half is, of course, active and receptive at the same time.

The transmission of this impulse of diffracted light without modification through the curved external surface of the hemicone shows that nothing in the nature of refraction occurs at the enveloping surface of an aplanatic pencil. The component impulses do not pass at all, but the resultant impulses pass without change. The influences, therefore, which determine the disposition of the diffracted light are the shape and disposition of the surface; and inasmuch as shape and disposition are the same, whether the surface be convex or concave, it follows that the pattern in the focal plane must be the same on both sides of the surface.

This proposition is completely true for patterns formed in the focal plane, but it needs to be guarded when applied to patterns formed in an apertural plane. The periodic spacing of the concentric rings of an annular pattern is the same on both sides of the envelope, but the law of brightness is different, as we have seen. Outside the envelope the darkness is uniform; the bright rings fall off, and rapidly, in brightness. Within the envelope the bright rings are of uniform brightness and the dark rings fall off in intensity. Moreover, the external bright rings do not follow the outline of the aperture with such fidelity as do the internal dark rings. This last distinction brings us back to the discussion of the diffraction pattern in the apertural plane. But it will be convenient first to summarize in two rules the result of the last half-dozen paragraphs.

VI. The diffraction pattern in the focal plane formed by a concave radiant surface is identical with that so formed by the convex surface which fits into the concave.

VII. A pencil of aplanatic light is perfectly transparent to an impulse of diffracted light in tune with it; and, therefore, is transparent to the diffracted light from any other segment of the same homocentric pencil.

Coming back now to the case of the internal pattern in an apertural plane it may be observed that this rule, as to the transparency of the aplanatic pencil to diffracted light, produces a very curious effect upon this internal pattern in the case of a triangular aperture. Such a pattern is photographed in Fig. L of the photographic plate,[1] and it will be observed that the bands are strongly curved and about different centres of curvature. The result is that the ρ interval varies in different parts of the pattern thrown off by any one of the three faces.

It will be found on examination that this phenomenon of the greatly increased ρ interval is confined to three of the rays; those, namely, which appear to come off from the points of the triangle. Now the points of the triangle are not, of course, radiant surfaces, and these rays are, in fact, due to diffracted light from the inner faces of the sides which subtend the angles at the points; which light has been transmitted through the aplanatic pencil itself. It has been already pointed out, in connection with Figs. 1 and 2, that diffracted light is transmitted with accelerated velocity within the limits of the aplanatic pencil, and the conclusive proof of this proposition will be given at a later point in this paper. Assuming that proposition, for the purpose of the present explanation, it is easy to see that those rays will have undergone most acceleration which have travelled farthest through the aplanatic pencil; and bearing this in mind we can see, upon reference to Fig. 5, that such an acceleration would tend to bring the anisophasal lines into the condition of isophasals. The consequence is, that under these conditions, we may observe a lengthening—even an extreme lengthening—of the ρ interval, which gives to the rays affected by it the character of unbroken lines for a long distance from the focal point. This peculiarity in the diffraction pattern from a pencil of triangular aperture attracted the notice of Schwerd, and was the subject of remark by him in his paper on "Diffraction Phenomena." But he overlooked the fact that this property was confined to those rays above mentioned, as, indeed, he overlooked the increased ray velocity of diffracted light within the aplanatic pencil; and in the diagrams which illustrate his paper he shows the diffraction patterns of a triangular pencil with six unbroken rays, instead of three unbroken and three opposite to these, showing the segmented structure with the regular ρ interval.

The dependence of the periodic interval between the segments of a radiate pattern, or the rings of an annular pattern, upon the breadth of the radiating surface, is a fundamental law of diffraction pattern design, and may be usefully considered a little more at large. It has been shown that the length of this interval varies inversely as the breadth of the flat face when a flat face constitutes the radiant surface. Taking for investigation a pencil of unit focal length we have been able to express the breadth of this face in angular measure as equal to 2 sin $u_1 l$ where $2u_1$

[1] My photograph of this object has come out so badly that the reader must be asked kindly to examine the phenomenon in a telescope.—J. W. G.

is the angle subtended at the focal point by the face in question. We thus obtained the result $\rho = \frac{m}{\sin u_1}$, m being a constant. Let it be supposed that this flat face is one side of a pyramid, and that the pyramid is a regular polyhedron. If the aperture be square, and if $2u$ be the angle between two opposite faces of the pyramid, then $u = u_1$, and we may write $\rho = \frac{m}{\sin u}$.

It is well known, and has been pointed out in the earlier part of this paper, that the varying light-intensity in a diffraction pattern, which results in this periodic design, is due to the varying phase range of the radiation given off by the radiant surface along different axes. The periodic distance, ρ, may, therefore, be connected

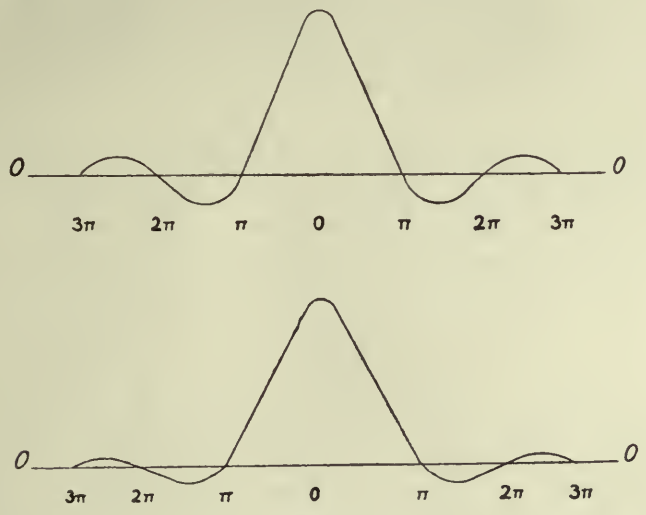


Fig. 8.

with this change of phase range and, taking the known function $\frac{m \sin \phi}{\phi}$ to represent the light amplitude at various points in the diffraction pattern, we may say that the zero value corresponding to the division between bright segments of the pattern must represent those values of the phase range angle ϕ for which $\sin \phi$ equals zero; that is to say, that in these cases ϕ must equal π or some integral multiple of π. If ϕ equals zero we have $\frac{\sin \phi}{\phi} = 1$, the commonly employed expression for the amplitude of focal light.

The expression $\frac{m \sin \phi}{\phi}$ serves to express the light amplitude in the diffraction fringe produced by an aplanatic pencil of square section. In that case the diffracted

light is condensed, as has above been pointed out, into four lines of symmetry, whereas in the case of the circular fringe the diffracted light is distributed in all azimuths. It is manifest to the eye that a pencil of square section yields a pattern in which the amplitude falls off much less rapidly than in the case of a pattern produced by a circular aperture.

It will be convenient now to give a graphic representation of the light intensity calculated by means of this expression for a pencil of square, and pencil of circular, section having the same vertical angle. These are given in Fig. 8.

The curves of the last diagram may for practical purposes be simplified. It will be found that in the following discussion nothing turns upon the precise form of the sine curve, and as it is now proposed to indicate a practically serviceable

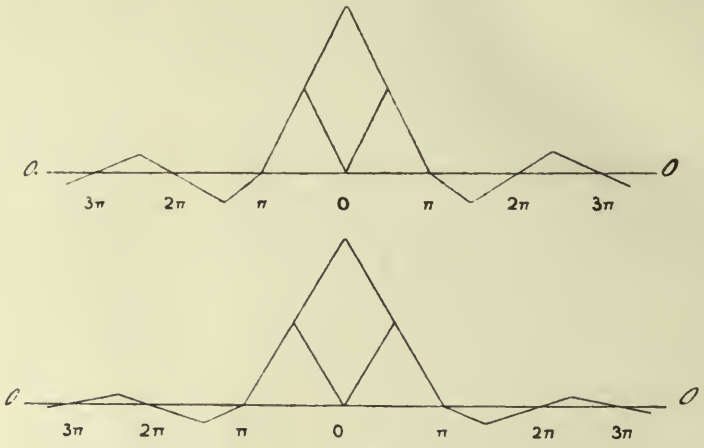


Fig. 9.

method of graphically compounding diffraction patterns of compound apertures a convenient diagram for use for this purpose may here be introduced.

Fig. 9 is a simplified form of Fig. 8, in which the curves are represented by angular forms, the angular forms being near enough for our present purpose to the more accurate curved forms. The curves in Fig. 8 are drawn to the formula $\gamma = \frac{\sin \phi}{\phi}$ where $\phi = \frac{2\pi x}{\lambda}$. In Fig. 9 alternative forms of the curves are shown, one in which the aplanatic light is, as in Fig. 8, equal to 1 and the other form in which it is equal to 0. The latter form represents, as nearly as it can be represented in this way, the diffracted light pure and simple; apart, that is to say, from the aplanatic light. It represents, therefore, the whole of the light from a non-luminiferous segment of a homocentric pencil, and the whole also of that portion of the light from a luminiferous pencil which can be affected by interference. It is in this latter

form, therefore, that the curve will enter into the following computations. The first-mentioned curve, that into which the aplanatic light enters, is drawn approximately to the formula $y = \frac{\sin \phi}{\phi} = \frac{2 \sin \phi}{2\phi}$; the latter to the formula $y = \frac{2 \sin \left\{(n+m)\frac{\pi}{2}\right\}}{(n+1)\pi}$.

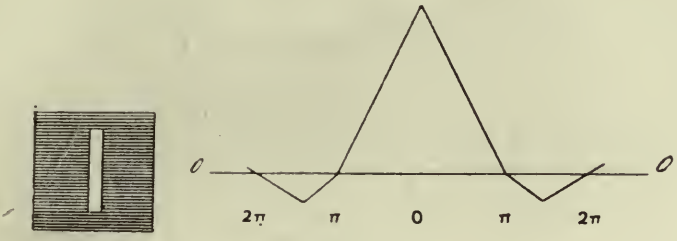


Fig. 10.

Here $(n+m)\frac{\pi}{2} = \phi$, and n is the integral, m the fractional, part of the coefficient $(n+m)$. It will be observed that the essential features of these curves are these—

VIII. Successive segments of the pattern taken radially are in opposite phase.

IX. In the case of the circular pattern the maximum amplitude in various rings varies approximately in inverse proportion to the square of the radius of the ring.

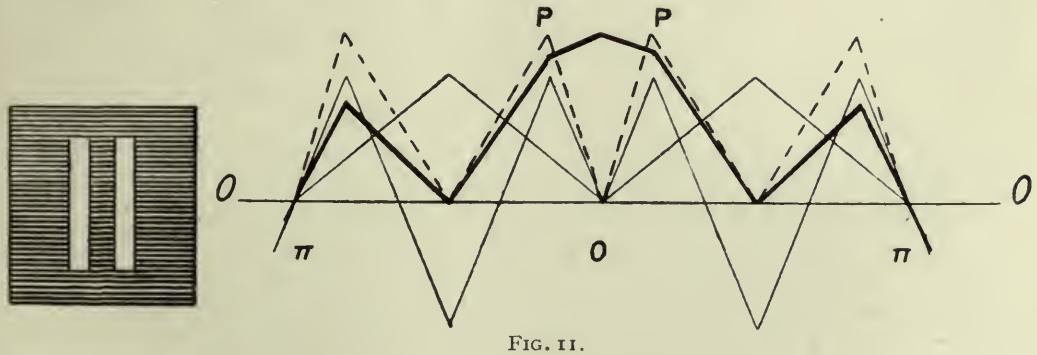


Fig. 11.

X. In the radiate pattern the maximum amplitude varies approximately in inverse proportion to the radial distance of the segment.

We come now to the case of an aperture comprising several divisons, and will first consider the case of a compound aperture in which there are several parallel rectangular openings. It is required to determine the form of the diffraction pattern due to a focused pencil of light transmitted by such an aperture.

The principle by which it is proposed to solve this problem is the principle already established: that every segment of the homocentric pencil will be transparent to the diffracted light from the surface of every other segment (above, VII).

To illustrate the application of this principle let Figs. 10, 11 and 12 be considered.

Fig. 10 represents a simple aperture and, side by side with it, the amplitude curve of its diffraction pattern. Here there is no question of a compound image, and, therefore, the form of curve is shown into which the aplanatic light enters as a component.

Fig. 11 shows a double aperture, comprising two equal and similar simple apertures separated by a non-luminiferous segment of equal angular magnitude. Thus, the whole pencil is bounded by four surfaces, two inner, which form the boun-

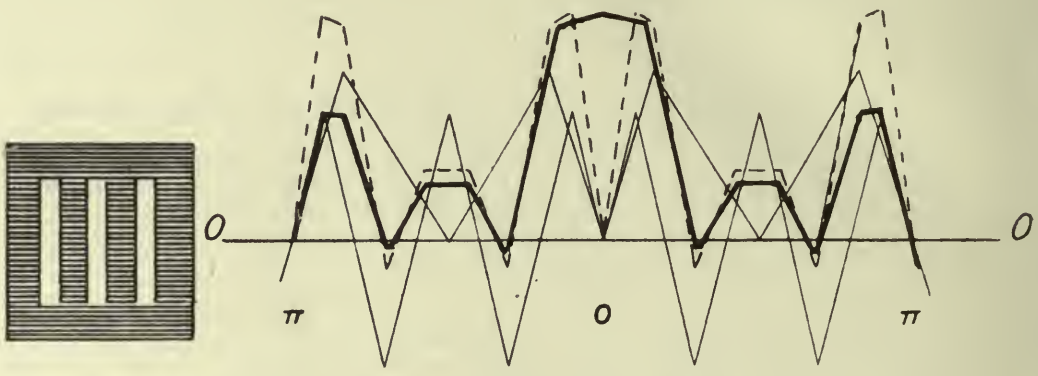


Fig. 12.

daries of the interior non-luminiferous segment, and two outer, which form part of the envelope of the whole pencil. The end walls, which complete the envelope, will be passed over without discussion in the present connection.

Here we have two diffraction patterns to be combined; namely, that yielded by the interior segment and that yielded by the whole pencil. The angles being small we may take the ratio between the sines to be the same as a ratio between the angles; that is to say, 1 : 3. We have, therefore, to combine two patterns similar in type, but having, the one a ρ interval three times as great as that of the other. It is to be noted in connection with the composition of these two patterns that it takes place, not in the focal plane itself but before the focal plane is reached; that is to say, the component impulses are combined where first they meet in the aplanatic pencil itself. This circumstance has an important bearing on the form of the component curves, which must be employed for finding the form of the resultant curve. For it is to be noted that $\frac{\sin \phi}{\phi}$ is the expression for a resultant amplitude; that is to say, it is a

mean value. The component amplitudes of which it is the mean are individually equal to $\cos\theta \mathsf{L}$, if L equals the amplitude of the aplanatic light. If then we wish to give a graphic representation of these component impulses which, under the conditions specified, enter into combination with one another we must modify the curve of Fig. 11 by equating it to $2\cos\theta \sin\left\{(n+m)\frac{\pi}{2}\right\}$ simply, that is to say, by giving to all the components substantially the same amplitude (since $\cos\theta$ may be taken to be equal to 1), so that the only difference between them is in the ρ interval. The two components are drawn accordingly in Fig. 11.

It may be remarked that, in order to prevent the diagram from assuming unmanageable dimensions, a reduced scale has been adopted for the ordinates, and for greater clearness an increased scale for abscissæ in the construction of this curve.

By adding together the ordinates of these component curves we obtain the resultant curve shown by a broken line in Fig. 11.

This resultant curve shows the resultant impulse, not as it should be plotted in the focal plane, but as it may be thought of as potentially existing within the envelope of the aplanatic pencil itself. In order to pass from this form to the form which it assumes when in the focal plane, we must perform upon this curve the same operation which under similar conditions we perform upon the simple sine curve of the single aperture, that is to say, we must divide it by ϕ, or what is approximately equal to ϕ, *i.e.* $(n+1)\frac{\pi}{2}$. Carrying out this operation we obtain, as a final resultant, that part of the curve shown by the heavy line in Fig. 11 which lies outside the two points marked P and P.

There remains to be added the central segment which represents the aplanatic light, for if we put $n=0$ and $m=0$ we shall have $\frac{2\sin\left\{(n+m)\frac{\pi}{2}\right\}}{(n+1)\pi}=0$; that is to say, the ordinate representing aplanatic light has disappeared. Its measure is, however, easily found. For it is to be noted that compared with the amplitude of the components, the amplitude of the unreduced resultant is in the proportion 4 : 3. Now, the components have an amplitude proportional to the sine of the whole divergence angle, equal, that is to say, to three times that of the radiation from the surface of the simple aperture. Thus, the unreduced resultant has a maximum amplitude of four times that of the simple aperture amplitude; equal, that is to say, to twice the amplitude of the aplanatic light transmitted by the simple aperture. But twice the amplitude of the aplanatic light transmitted by the simple aperture is the amplitude of the aplanatic light transmitted by the double aperture. It follows that the aplanatic light in the new diagram must be represented by an ordinate having the same height as that of the ordinate representing the maximum of diffracted light in the unreduced resultant curve.

Passing now to Fig. 12 we have a compound aperture containing three simple

apertures, and, accordingly, we have three radiant surfaces to be considered. The first and second apertures are bounded by the same enveloping surfaces as those already discussed in connection with Fig. 11; and, therefore, the diffraction pattern, so far as it is due to them, may be taken to be represented by the unreduced resultant —the broken line curve—of Fig. 11. We must in this connection make use of the unreduced resultant for two reasons. First of all, because the quantity of aplanatic light will obviously be different in this case from what it is in Fig. 11, since we have now to deal with three simple apertures instead of with two. In the next place, we must recur to the unreduced form, and not to the reduced form of resultant curve, because it is required now to combine with the two patterns already considered a third pattern; that, namely, which is due to the outermost envelope; having a divergence angle five times as great as the divergence angle of the single aperture, and, therefore a ρ interval equal to one-fifth only of the original ρ interval. The components, consisting of these two curves are shown by thin lines in Fig. 12, the unreduced resultant by a broken line, the reduced resultant by a heavy line, in which last-named has been incorporated an ordinate representing the aplanatic light; the magnitude of which has been determined by the application of the rule already explained.

It will now be evident that a general type of diffraction pattern can be identified as belonging to the diffraction grating type of aperture. Thus, let it be supposed that a number of similar apertures entering into the compound aperture is very large. It will then be evident that the first maximum of diffracted light tends to become equal to the aplanatic light and that the second maximum tends to a magnitude equal to half of the foregoing. Between these two maximum points there will be a region of depressed illumination approximating to zero. Beyond the second maximum there will be a minimum point at which the illumination reaches zero, and this point will correspond to the first minimum point in the pattern of the simple aperture. Beyond this point we get a repetition of the pattern on reduced amplitude and of opposite phase. In this description the reader will recognize the well-known form of diffraction pattern thrown by a diffraction grating.

It is not possible, within the limits of this paper, to investigate with any completeness the application of this method of plotting down diffraction patterns. Enough has been said to illustrate the matter, and we may now formulate the following results—

XI. The ray velocity of diffracted light is equal to the ray velocity of aplanatic light in the region beyond the boundary of the aplanatic pencil.

XII. Within the boundary of a homocentric pencil the ray velocity of diffracted light varies with the angle of the ray to the wave front through which it passes, and is equal to $\frac{1}{\sin\theta}\,V$, if $\left(\frac{\pi}{2}-\theta\right)$ be that angle, and V = the ray velocity of aplanatic light.

XIII. The diffraction pattern due to the periodic structure of a regular compound aperture tends to settle down to the pattern of a double aperture, although

the pattern of a compound aperture comprising only a small number of components may show marked deviations from the double aperture type of pattern.

This is an important principle, for it enables us to understand that a well-marked secondary pattern must necessarily be conformable to the ρ interval of the primary pattern. Thus, if in the instance just discussed, there were no common measure of all the ρ intervals, there could be no diffraction pattern at all. The salient features of one would obscure the salient features of another, and so soon as any considerable number of patterns were exhibited in one view the whole effect would merely be an appearance of diffused illumination. But since one measure is common to the whole series of ρ intervals that measure is of necessity the largest—that is to say, the ρ interval of the simple aperture, and we thus arrive at an important rule controlling the design of the diffraction patterns of compound apertures, namely this—

XIV. In the diffraction patterns of compound apertures of regular structure the diffraction pattern of the simple component aperture is found as a bright field upon which the secondary diffraction pattern, due to the periodic structure of the compound aperture, is displayed.

The ideas of primary and secondary diffraction patterns being thus introduced it may be well to interpose the observation that—

XV. Every compound aperture exhibits three diffraction patterns; that is to say: (1) the primary pattern, which results from the form and dimensions of the simple component aperture; (2) a secondary pattern, which results from the mutual interference of a number of primary patterns; and (3) a tertiary pattern, which is determined by the form and dimensions of the entire compound aperture, considered as an undivided whole.

This analysis of the diffraction pattern of a compound aperture may be illustrated by the case of the diffraction image, yielded by any diffraction grating. Figs. J and K, Plate I will serve the purpose of illustration. Here the object is a line and the aperture, being rectangular, produces a radiate diffraction pattern. Each segment of that radiate pattern is itself an image of the light source, and the breadth of that segment is determined by the size of the grating as a whole. The broader the grating—that is to say, the larger the number of openings of a given size that it contains—the more sharply defined will be the image of the bright line which forms the source of light. But, however sharply defined, that image of the line is edged off by a diffraction fringe, the breadth of which is inversely proportional to the total angular breadth of the diffraction grating. Hence the wider the grating the sharper the definition which it produces.

It is a common thing to use, for purposes of precise work, diffraction gratings so wide that the image of the illuminated slit which forms the source of light is measureably a mathematical line. In other words, the microscopical diffraction fringe due to the aperture of the grating, considered as a whole, is for practical purposes invisibly small. This is the fringe above described as the tertiary diffraction pattern.

Passing now to the secondary pattern we come to what is ordinarily known as the diffraction pattern of a grating—that is to say, the repeated images of the light source,

slit or star as the case may be, which in a spectroscope constitute the spectrum. It is well known that if we use monochromatic light these images are placed at a certain interval, the magnitude of which depends upon the spacing of lines in the grating, and is inversely proportional to that periodic distance. It is found, however, that these repeated images are not of uniform brightness nor do they fade off uniformly.

If the diffraction pattern of any compound aperture be examined, it will be observed that the secondary pattern is visible only upon the illuminated field of the primary diffraction pattern.

It thus appears that these three elements in the diffraction pattern of a grating determine the three principal properties of the resulting spectrum; that is to say—

1. The diffraction pattern of the aperture occupied by the whole grating determines the definition;
2. The smallness of the periodic distance of the ruling determines the dispersive power; and—
3. The breadth of the opening, in comparison with the periodic distance, determines the relative brilliancy of different parts of the spectrum.

We may now turn to another matter involving the theory of diffraction patterns, and consider a question which, though long since settled by the practical method of trial, can hardly yet be said to have passed out of the region of discussion.

It has from time to time been proposed to attempt improvement in the resolving power of optical instruments by introducing a stop into the middle of an object glass; the notion being that, inasmuch as the diffraction pattern of an annular opening has a smaller ρ interval than that of the corresponding unstopped aperture, it should be possible by means of a stop to obtain a diminished antipoint, and, therefore, better definition. Many attempts have been made from time to time to put this idea into practice, but always without success. It can now be shown that the idea is radically unsound, and that it is in the nature of things impossible to improve definition in that way. It is generally said that the difficulty of realizing this programme arises from the circumstance that the outer rings produced by an annular aperture are brighter than those produced by an unstopped aperture, and that the brightness of these remoter rings is the cause of failure.

That explanation, however, leaves out of account the main source of the difficulty, the real nature of which will at once appear if we examine the diffraction pattern of a stopped aperture. Fig. 13 will serve the purpose. In Fig. 13, A, the aperture is shown side by side with its curve, and in B the same aperture is shown almost entirely stopped down. Side by side with B is the diffraction curve produced by the interior faces, which, having an angle, differing only by a negligibly small quantity from that of the external pair of faces, yield what is substantially the same diffraction curve, subject only to this, that the aplanatic light is reduced to zero. Of course, when the aperture is thus very largely obstructed, only a small quantity of aplanatic light can get through, and hence in the curve of A the aplanatic light is represented by a very small quantity. Side by side with this new component the old component from A

is shown, and the resultant curve is shown by the heavy line. It will here be seen that the resultant pattern shows rings of approximately double brightness, because diffraction comes from four radiant surfaces instead of two, while at the focal point there is now minimum instead of maximum brightness. Compared with A the resultant curve of B does, indeed, appear to have reduced radial dimensions; but only because, by the disappearance of the aplanatic maximum, the two first maxima of diffracted light, marked P in the diagram, have become conspicuous, whereas they were unconspicuous before, being outshone by the aplanatic light. It is quite unnecessary, with these two diagrams under the eye, to enter into a discussion as to why the apparently diminished diffraction pattern of a stopped aperture, such as B, should yield an antipoint in-

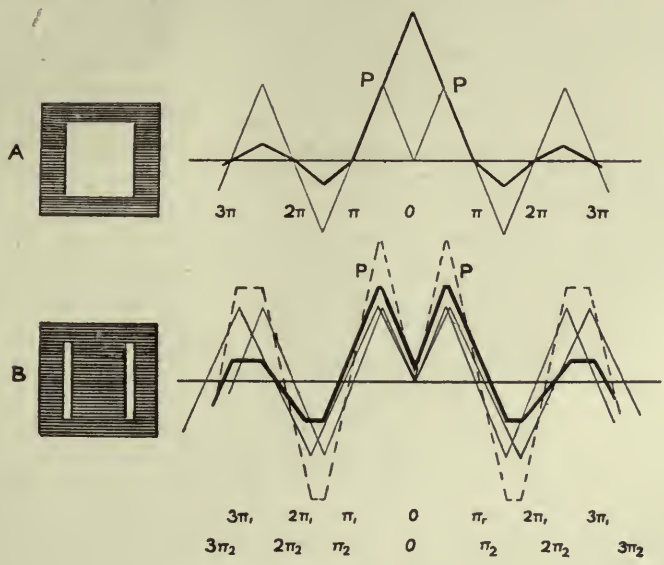


FIG. 13.

ferior for the purposes of definition to the apparently broader diffraction pattern of diagram A.

The case which has so far been discussed is that of a number of segments which have a common centre. There remains to be considered the alternative case of a number of segments which are not concentrically situated in the aplanatic pencil. We may, for example, have a case such as that of the triple aperture of Fig. 14, where we have three similar apertures, one placed upon the axis of collimation, the other two disposed eccentrically to it. These apertures are assumed to be all circular and all of the same diameter. It follows that every one of them will yield the same diffraction pattern and the whole result is that we have the ring system corresponding to any one of these simple apertures reinforced threefold. A very simple way of

obtaining a strong diffraction pattern in the focal plane of a small aperture is to multiply apertures in this way. If the small apertures are carefully arranged in azimuth, so as to avoid the formation of a radiate pattern, we may get in this way a strong image of the pattern of the small component aperture.

It is not, however, possible to arrange so small a number as the three apertures shown in Fig. 14 so as to produce this effect. Being unevenly distributed in azimuth they will necessarily produce a radiate pattern, and the question arises: What will that radiate pattern be ?

There is a very simple way of solving that problem. Radiate patterns, as we know, are yielded by flat radiant surfaces. We must, therefore, consider how the segmental pencils transmitted by these three component apertures can be arranged as component parts of flat faces. In other words we must draw as many straight lines as possible connecting the three apertures. The secondary radiate pattern then will contain a pair of rays lying at right angles across every one of those lines.

In the case shown there are three such lines; and, therefore, the secondary pattern is of the radiate type, the same that would be produced by a pencil of the three-sided prism type. It is, however, not to be supposed that the ray structure is as pronounced as it would be if the aperture were undivided. The primary diffraction pattern controls the illumination; and, consequently, the secondary pattern is cut up by it in such a way as to present to the eye not its own general contour, but that of the primary pattern. The general effect is that the secondary pattern appears to be a diapered pattern wrought over the primary.

Fig. 14.

There is one remaining class of diffraction patterns still to be considered: those, namely, which are seen when the aperture of the aplanatic pencil is examined near the focal point. This pattern is of the annular type, and when formed in a pencil of circular section it takes the form shown in the photographic plate (Fig. C). It is very familiar to the makers of astronomical objectives as it affords one the means of testing the corrections of their glasses. This pattern is the result of visual projection, and for its explanation we must bear in mind that this region can only be examined by looking at it through the diffraction pattern developed in the focal plane. Whatever is seen must, of course, be the visual projection, upon the focal plane of the eye, of that part of the diffraction pattern which constitutes the optical projection of the point on which the eye is focused.

It does not, of course, follow that what is seen by visual projection is not also actually constituted by optical projection, and in this case it has been shown that at any great distance from the focal point the internal diffraction pattern conforms to the Fresnel type, and can be explained by Fresnel's method of investigation. It is, however, more practically useful when we are discussing the patterns formed in the near neighbourhood of the focal plane to treat them as virtual than as real images, a mode of treatment which is illustrated by Fig. 15. Here the observer's eye is taken to be focused on a point p, which is seen by looking through the focal plane

F–G, occupied of course, by the diffraction pattern of the focal plane. Let F of Fig. 16 be a plan of that diffraction pattern on an enlarged scale, and let G be the projection on that pattern of the observer's eye when focused on the point *p*. It will be seen that the projection covers two complete zones, and takes in part of a third. The central

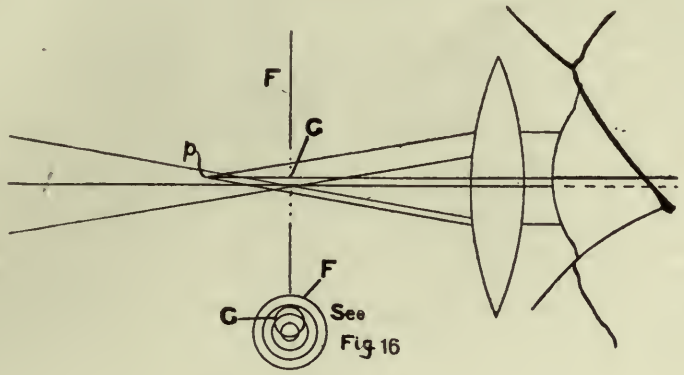


Fig. 15.

and next adjacent zones, being opposite in phase, will exactly counteract one another, and the point *p* will appear to be illuminated by the light received through the partially admitted third zone. It will, therefore, appear bright, but not of full brightness. If the observer selected a point slightly higher than *p*, so that his projection circle coincided with the circle traced in broken lines, he would cut out some of the negative second zone and admit a little more of the positive third zone. The new point would consequently appear more brightly luminous than the first. If his projection circle were made to coincide with one of the dividing rings between adjacent zones, the spot in which he then focused would appear bright with full illumination, or completely dark according as the number of included zones was odd or even. It is obvious that, as the focal point travels over the apertural plane, the projection circle will travel correspondingly over the diffraction pattern in the focal plane, and will yield focal points of varying illumination in such an arrangement that we shall have a ring system on the apertural plane, with the same ρ interval as the ring system of the focal plane.

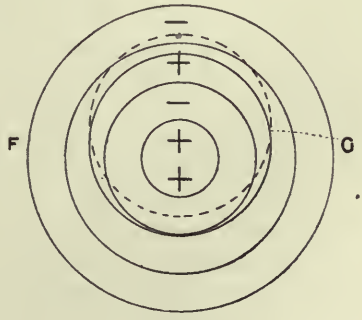


Fig. 16.

To this rule of the equivalence of the ρ interval in the two systems there must obviously be one marked exception. For it is to be observed that, as the projection circle traverses the diffraction pattern, it passes from maximum to minimum illuminating capacity and back to maximum in the space of one ρ interval measured in the

focal plane. This fact it is which secures the repetition of the ρ interval in the projected image in the apertural plane. But this proposition does not hold good when the edge of the projection circle is traversing the central zone, for in the central zone we have two adjacent ρ intervals which transmit light in the same phase. Moreover, when that central zone is passed a marked change occurs in the light-giving capacity of the projection circle. It is then intersected by zone segments, which, being alternately positive and negative, and all very limited in angular magnitude, come very near to cancelling one another, so that when the central zone is excluded from the aperture of the projection circle there can be no bright illumination, and scarcely any illumination at all, of the object point. The whole region so defined is a region of darkness.

Now it will be obvious, on reference to the diagram, that when the edge of the projection circle stands upon the centre of the diffraction pattern the object point p must stand upon the edge of the aperture of the aplanatic pencil. This must be so, because the divergence angle of the eye and the divergence angle of the aplanatic pencil are necessarily the same angle. Whatever limits the one, limits the other. Hence the diameter of the projection circle in the focal plane is the same as the diameter of the aplanatic pencil in the plane of the object point p. It follows, therefore, that in carrying the eye from a point within the aperture to a point without the aperture the observer must pass across a broad bright ring, the outermost edge of which lies outside the boundary of the aplanatic pencil itself and constitutes the innermost of the external diffraction rings. This broad bright edge is, in fact, the most striking feature of these internal diffraction patterns.

It will now be evident why the internal diffraction pattern of the aplanatic pencil near its focal point does not present the features of a Fresnel system. It stands in striking contrast with the Fresnel system of the remoter apertural planes in respect (1) of the uniformity of its ρ interval and (2) in the uniform intensity of both its bright and its dark rings. In fact, as seen, these patterns near the focal point to all appearance are not Fresnel patterns at all, but projections of the Fraunhofer pattern in the focal plane itself. We thus arrive at a further rule:

XVI. The diffraction pattern internal to the aperture of the aplanatic pencil, in the near neighbourhood of the focal plane, is the image formed by visual projection of the Fraunhofer pattern in the focal plane.

In the discussion of the visually projected diffraction patterns it has been convenient to speak of them as derived from the Fraunhofer pattern in the focal plane. But here it is necessary to take note that this is not the identical pattern which has been made the subject of discussion in the earlier part of this paper. That pattern was formed, it will be remembered, along lines of symmetry situated at right angles to the surface of the aplanatic pencil. The focal plane, on the contrary, lies at right angles to the optical axis. The one pattern may, however, be very simply derived from the other. The process of derivation is illustrated by Fig. 17. Here x–x is the optical axis; F–G the line of symmetry on which the radiate pattern is formed; F_1–G_1 and F_2–G_2 are lines of symmetry parallel to F–G; and Fg_1–g_2 is a line traced

in the focal plane. The rules above given enable us to plot down the diffraction pattern along any one of the lines F–G, F_1–G_1, and F_2–G_2. We find the diffraction pattern in the focal plane by selecting the point g_1 from the F_1–G_1, pattern; the point g_2, from the F_2–G_2 pattern, and by selecting, on the same principle, the remaining points from the remaining patterns which intersect the focal plane. In the case of a conical aplanatic pencil of circular section—the common case presented by the telescope and microscope—this mode of proceeding leads to a very simple rule, for all the patterns in question are alike, except that they expand as they travel away from the focal point in the direction of the light source. This expansion, however, does not affect the ρ internal, which is the same in all. Consequently in this case the diffraction pattern in the focal plane has the same form as that already discussed by a ρ interval greater in the proportion $\frac{1}{\cos u}$: 1.

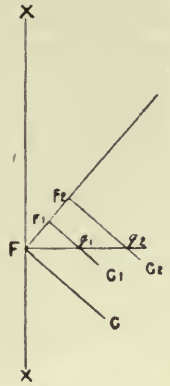


FIG. 17.

It is now evident that it is important to distinguish between the focal plane strictly so called and the focal surface for diffracted light which lies everywhere at right angles to the edge ray of the aplanatic pencil. We may therefore formulate a new rule thus—

XVII. The diffraction pattern external to the aperture of the aplanatic pencil is formed upon a sheet which stands in every azimuth perpendicular to the surface of the aplanatic pencil.

This last formulated law suggests a doubt as to whether the aplanatic focal plane is the plane in which the visible image is most sharply defined. If the diffraction fringe is more compressed in a plane lying slightly beyond that theoretical focal plane than in that focal plane itself, then there will hardly be room for doubt that this lower plane is that in which our empirical focusing is done. If this be so, then it is clear that calculations as to the ultimate limit of resolving power, based upon the assumption that the aplanatic focal plane is that in which the picture is developed, are quite untrustworthy, and this whole subject stands in need of thorough reinvestigation.

Prof. S. P. THOMPSON said that Mr. Gordon's approximate method had the advantage of extreme simplicity, though it might not contain anything very new. He had had the pleasure of seeing some of the experiments and the telescopic images of artificial stars in Mr. Gordon's house. He would have thought that it was hardly possible to throw these phenomena on the screen, considering the extremely small intensity of the light. But Mr. Gordon had had the hardihood to attempt it, with the aid of the epidiascope which Messrs. Zeiss had placed at the disposal of the Optical Convention, and they had seen the beautiful results.

Mr. C. P. BUTLER said, with reference to the diffraction patterns from small apertures, there was one practical application which deserved more notice than it

had hitherto received. It was well known that if two diffraction gratings were crossed, so that the spectra were produced along two lines perpendicular to each other, then there were also produced other series of spectra along the oblique angles. It was not so well known, however, that whereas in the ordinary spectra there was generally a considerable amount of scattered light, especially between the direct image and the violet of the first orders, in these extra orders there was little or no scattered light perceptible. This feature had been pointed out by Mr. Thomas Thorp, of Whitefield, at a meeting of the Manchester Literary and Philosophical Society (January 23, 1911). It would thus appear feasible to use this property of crossed gratings for various researches in the ultra-violet which were at present rendered imperfect by this dilution with scattered light. The arrangement was generally used with transmission gratings, but it would probably be equally suitable for reflection gratings with some slight modifications of their disposition.

PRINCIPAL SURFACES OF A LENS

By P. Everitt

(*Abstract*)

(*Thursday, June 20th*, 1912.)

The customary definition of Principal planes only applied when points quite near the axis were considered. If in a practical system a series of rays parallel to the axis were calculated and the point found where the ray, after passing through the lens, was at its original distance from the axis, these points would lie on a surface. This surface might be called the Principal Surface of the lens. In the case of a system, corrected for spherical aberration the radial distances from the focus would be given by $\frac{h}{\sin A}$ where h was the distance from the axis and A the final angle between the ray and the axis. In the special case where the coma was corrected the surface would be a sphere with its centre at the focus.

Such a Principal Surface was more convenient than the Principal plane of the ordinary definition when dealing with corrected systems of large aperture and the graphical constructions were more accurate.

Mr. T. Smith thought the author had done well to call attention to the fact that the Principal Surfaces of a Lens were often very far from plane. The principal planes of elementary geometrical optics could with advantage for some purposes be assumed to be accurately plane at large distances from the axis of the lens, as in

that way geometrical constructions could be made with some exactitude which gave results that applied accurately to paraxial rays, though not to rays such as those used for purposes of geometrical construction. Such a construction also typified the actual paths of rays in that unattainable ideal, the perfect lens. In such ways the use of principal planes was of value, particularly as the properties of the planes were independent of the positions of object and image. But when aberrations were taken into account, and the planes became surfaces of revolution, it was not evident that the principal surfaces were of much importance. In these images, as well as in others, there would exist spherical aberration, Coma, Curvature, Astigmatism, Distortion—in short, all the ills that images of any kind were subject to. It followed that the use of these surfaces when aberrations were taken into consideration for any purposes for which the planes, when aberrations were disregarded, might be used, would require justification. For example, it might be found that owing to the presence of these aberrations a different surface would be obtained for each different direction of the incident light: the difference might be slight, but without investigating the amount and influence of this difference, it would be undesirable to use the surfaces in connexion with rays at large apertures and appreciable obliquities in the way that the planes could be used for paraxial rays.

Mr. S. D. Chalmers said that the principal surface referred to by Mr. Everitt might be regarded as the wave-surface of the originally parallel beam taken at the place where the aperture of the emergent beam was equal to that of the incident beam. If we assumed there were no aberrations, this would correspond exactly to Mr. Everitt's principal surface.

Dr. R. S. Clay said that, following upon the remarks of Mr. Chalmers, he would like to point out that there was one case where the principal surfaces defined in that way would be of value. If an optical system had a pair of conjugate aplanatic points, the spherical surfaces with those points as centres and which pass through the principal points of the system, would be surfaces of unit magnification, and might be used for the geometrical construction connecting an image with its object in the same way that the principal planes were used in the ordinary construction for rays making small angles with the axis.

Mr. Chapman Jones said he would like to refer to a common misconception; they did not as a rule deal in photography with parallel rays, rather with near objects.

Mr. Everitt replying, said his object had been to call attention to the advantages of the principal sphere when the surface was a sphere.

THE MEASUREMENT OF COLOUR

By A. C. Jolley and A. J. Bull

(*Friday, June 21st*, 1912.)

The earliest experimental results on the subject of the measurement of colour were obtained by Koenig, and subsequently, Clerk Maxwell; and still later Sir W. Abney and many others have contributed valuable researches. In most cases the attempts were made on the basis of the three-sensation theory as developed by Young and Helmholtz.

According to this theory we may specify any hue in terms of the percentages of three fundamental primaries—Red, Green, and Blue; white light corresponding to 100 per cent. of each constituent.

Unfortunately nearly all the methods adopted by these early experimentalists were not generally applicable to the commercial problem. It is, for instance, very desirable, or may at least greatly facilitate the standardization of the quality of a product, to be able to express its colour metrically; this is particularly so in such industries as Dyeing and Textile work, Printing, Colour-grinding, Steel manufactory, Brewing, Sugar and Oil refinery, and Chemical manufacture. Lovibond was among the first to feel this necessity, and set himself the task of producing an instrument, now so well known under the title of the Tintometer, which could be applied to the numerical expression of colour.

Frederick Ives has also produced a commercial colorimeter based upon the three-colour theory, and except for these two instruments there is but very little which actually meets the commercial want.

The ideal instrument should be capable of giving numerical expression to any ordinary colour in such a way that a complete specification of it may be stated, and the hue reproduced at any future time without reference to any manufacturer's standards.

Now it will be seen at once that the instruments mentioned above labour under the disadvantage of depending upon arbitrarily chosen colour-screens, and therefore the uniformity of the results given by various instruments is dependent upon the care and uniformity with which these screens are manufactured, and this, together with the difficulties introduced by the laws of absorption for them which we shall deal with later, make such instruments fall far short of the ideal.

The following investigations were undertaken not so much with a view to producing a commercial instrument, as to see if better methods could be applied.

In considering all the possibilities we have recognized that only those methods

based upon the Young-Helmholtz theory are likely to be useful and simple, employing either additive primaries or subtracting them by selective absorption. At the outset we have found that there is only one set of narrow primaries in the normal spectrum which, when suitably admixed, give the complete range of colour from black to white, and although a number of modern text-books assert there are an indefinite number of such triplets, we have failed to find more than the one set after careful experiment, although it is easily possible to find triplets which will cover a fairly wide range. And, indeed, this is only reasonable if we are to regard colour-vision as being due to the stimulation of three resonating bodies each possessing suitable but not necessarily identical damping.

The adoption of these three narrow primaries would be important in specifying the composition of a hue, since it does not involve the use of any arbitrary standards of colour, were it not for one unfortunate circumstance, viz., that although the colour matches are perfect for any particular individual, there are considerable variations in the expressions given by different persons with the same apparatus, due to differences in the colour-sensitiveness of individual eyes.

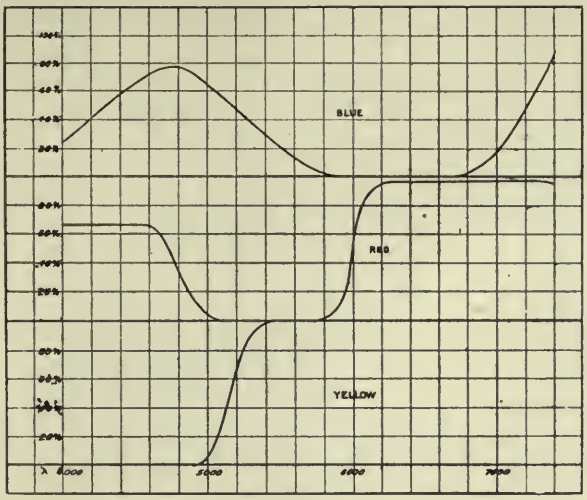


Fig. 1.

We have, however, tried to overcome this difficulty by broadening out the regions of the primaries, so that they between them comprise a large proportion of the visible light, but are yet individually pure enough to match any ordinary colour, and this process has the advantage of lessening any disturbing effects due to the Purkinje phenomenon.

Instead of superposing or adding the primaries we may progressively abstract them from white light by selective absorption, as Lovibond does in his Tintometer, but in this case the difficulty of expression becomes a serious one, since, quite apart from the fact of the absorption being geometric, it is not possible to secure dyes which will absorb one primary and transmit the remainder of the white light in equal proportions. This is well shown by the absorption curves in Fig. 1 of a set of dyes that we have carefully selected for this purpose.

Turning now to the experimental methods adopted to examine these possibilities, we have found the following arrangements convenient.

Fig. 2 shows the means adopted for testing the possibility of employing monochromatic primaries. Here the colour under test is placed over a transmission grating, or, in the case of transparent bodies, a white slip is used, and illuminated by light passing through the body under test. The slip together with the grating were surrounded by a screen of black velvet in order that reflection from surrounding surfaces shall be reduced to a minimum. The lamp, L, provides the illumination for the colour-patch whose image half fills the field of the lens, R, when the observer's eye is at the slit, S. At a considerable distance in front of the grating three straight filament lamps are placed, in such positions relative to the grating that each filament and the observing slit are conjugate for some particular wave-length.

Owing to the fact that it is not convenient to use the observing slit very narrow, a short range of wave-lengths were transmitted, usually about some 50 tenth metres. Thus these three regions, ranging for blue from ·463 to ·466 μ, for green from ·529 to ·535 μ, and red from ·634 to ·644, filled the other half of the lens, R, and were superposed at the observing slit, so that the field was of uniform tint.

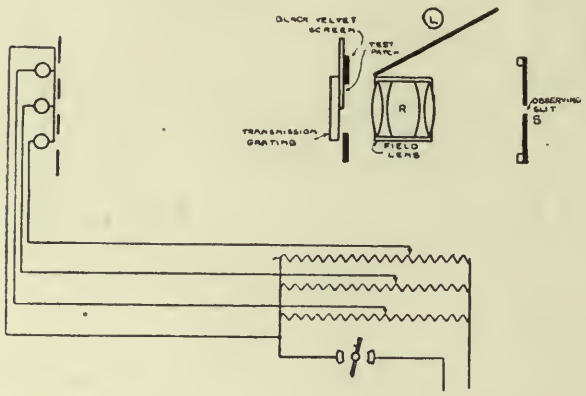


Fig. 2.

One of the greatest difficulties in all colour-measuring devices is to provide an accurate method of modifying the intensity of the component primaries. In all our work we have been careful to eliminate anything which involves intermittent illumination, as our experience shows that such devices introduce serious physiological complications and involve great loss of accuracy.

In this instance we adopted the device of reducing the voltage at the terminals of each straight filament lamp by connecting it across a suitable potentiometer resistance, which could be placed conveniently under the observer's hand and gave complete control.

The law of variation of intensity of the light from an incandescent filament can be found by simple photometric tests and is quite definite, whilst the change in the nature of the radiation between the limits of temperature over which it was necessary to work is more apparent than real, probably owing to the Purkinge effect, and does not appear to introduce serious errors, and in any case suitable corrections could be made.

An easy and convenient method of varying the intensity of the primaries is thus available, and may be made direct reading in percentages by providing each potentiometer rheostat with a suitable scale.

As indicated in Fig. 2, the distance between the grating and the lamps was made large in comparison with that between the patch and observing slit, in order that the light incident upon the grating should be sensibly parallel.

The wave-lengths of the transmitted primaries were determined by setting up a wave-length spectroscope at the observing slit and reading off the limits of the transmitted bands. Were it not for the individual variation previously mentioned we believe this to be one of the most perfect and convenient colour-matching arrangements.

Turning now to alternative methods of broadening the primary regions, two systems are available.

Firstly, we may use another reversed spectroscopic arrangement, as shown in Fig. 3. Here the screen, A, having a white diffusing surface, sends light from the lamp, L, through three broad slits in the screen, B, these slits corresponding to the Red, Green, and Blue-violet regions of the spectrum that would be formed by the grating, C, if light were sent in the reverse direction from the observing slit D. The grating, C, is half covered with the test-patch as in the previous arrangement, and is illuminated by reflecting light from the source, L, on to it by means of a surface silvered mirror.

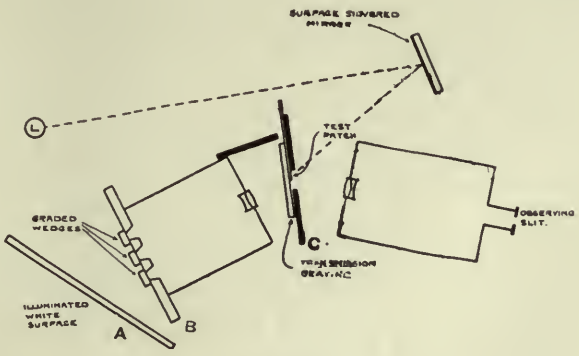


Fig. 3.

The modification of the primaries is effected by sliding across the apertures in B three long graded wedges of lamp-black in gelatine, whose absorption curves can easily be determined.

In this case the patch of comparison colour was not perfectly uniform owing to the use of the absorbing wedge, but this was not sufficient to be troublesome to the eye, and might have been rectified by using two wedges sliding over one another in reverse directions, as in the Féry absorption pyrometer. But the arrangement was good enough to determine that the matches obtained were correct for eyes differing considerably in their colour-sensitiveness, and to show, therefore, that instruments employing broad bands are more likely to yield comparative results.

In addition to using matching colours derived from the spectrum of white light, we have also tried primaries produced by selective absorption. We have already referred to the defects of such colour-filters, but the manufacture has reached such a high stage of perfection that we have no doubt that filters for this purpose could be supplied with sufficient exactness.

Of the three we have selected the red primary is the best; it can be made so as

to transmit a large proportion of its own coloured light (see Fig. 4), and the termination of the transmitted band is fairly abrupt.

In the case of the green, the filter cannot be obtained so transparent if a sufficiently narrow band is to be used, and then the absorptions, especially on the red side, are not nearly so abrupt.

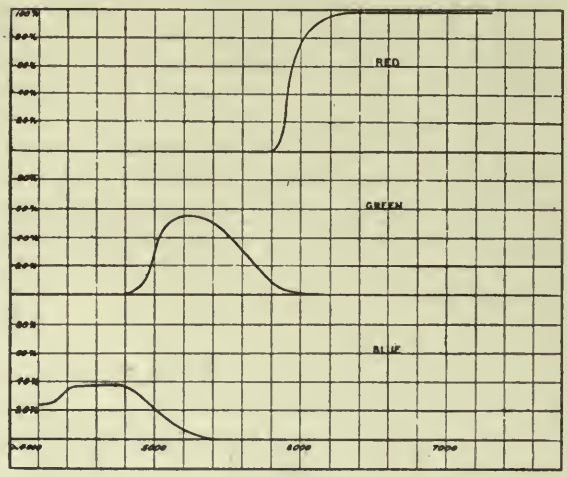


Fig. 4.

The effect is still more marked in the case of the blue primary, where the want of abruptness of absorption on the red side is liable to allow too much green light through.

Such primaries could not be usefully employed were it not for the fact that the hue changes slowly in the primary regions.

The method of superposing primaries obtained in this way that we have found most convenient is indicated in Fig. 5.

The two components of a large rectilinear lens are mounted in a framework, and the screen A, with three variable apertures covered by selected Red, Green, and Blue broad-banded filters, is placed symmetrically between them. A photometer, P, devised by Dr. Drysdale, and consisting of two right-angled prisms mounted with their edges in contact, is placed in such a position that the lens, R/R, focuses the illuminated surface, B, at the photometer; and, therefore, one half of the photometer is filled with light transmitted by the apertures in A; the other half receives light reflected from the test patch, which is illuminated from a lamp in parallel with those illuminating B.

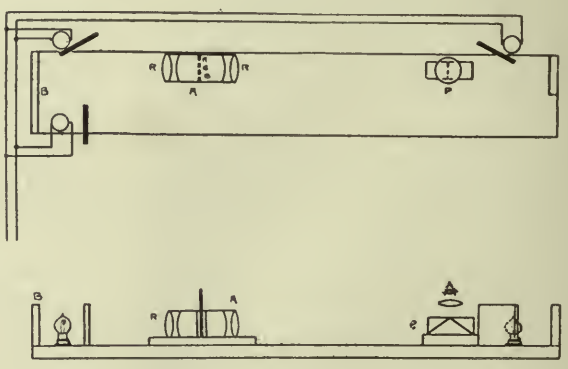


Fig. 5.

By varying the apertures in A various proportions of the Red, Green, and Blue may be mixed in one side of the photometer.

This is much the simplest method of measuring the proportions of Red, Green,

and Blue in a given hue, because the light transmitted is sensibly proportional to the areas of the openings, and as far as we have gone in the matter, this appears to be the simplest arrangement for the purpose; but, of course, it is open to the previously mentioned objection that the primaries are arbitrary filters whose absorptions are by no means abrupt.

The method of constructing a colour by abstracting the primaries is one of the most easily applied, and is merely a question of superposing suitable thicknesses of absorbing dyes. This may be accomplished in many ways, as, for instance, by varying thickness of solutions (a patented method), by glasses of increasing depth of tint, as in the Lovibond Tintometer, or by sliding wedges of gelatine containing the dye one over the other, a method we have used in our work.

If, however, the measurements are to be expressed in percentages of the primaries and not by a mere arbitrary scale, then the method presents very distinct difficulties, because dyes cannot be secured which will transmit two broad primaries perfectly and also progressively absorb the remainder of the white light. This is well shown in the set of curves in Fig. 1, which are from a set of dyes used in our best wedges, and for this reason we have found the method unsatisfactory and been obliged to abandon it.

Dr. F. W. Edridge-Green said that he quite agreed with the authors, that the use of any method in which flicker was employed was most undesirable on account of the errors it introduced. He also strongly objected to any projection method when accuracy was required. The eye must be considered as a whole : at the back of the eye was a photo-chemical liquid, in which the nerve terminals were, and what we saw was a projection outwards from the brain. As an example of the necessity of considering the eye as a whole he might instance the fact that, if a piece of ultramarine blue paper be placed on one side of a photometer and illuminated by electric light, it would exactly match a piece of orange-brown paper illuminated by daylight on the other side of the photometer. On taking the blue paper out of the photometer and examining it by exactly the same electric light by which it was previously illuminated, it would then appear a definite dark blue instead of orange-brown. Designers of colour-mixing apparatus ought to state that their work was based on the facts of colour-mixing and not on theory. The facts were correct, but there was a large number of other facts which showed that the theory was erroneous. For instance, he had conclusively shown that yellow was a simple sensation and not compounded of a mixture of red and green sensations, though red light and green light when mixed made yellow, but this was quite another matter. The perception of relative difference was the underlying principle in all colour-perception.

Mr. F. Twyman drew attention to the necessity, in obtaining good results with colour-mixing apparatus, of ensuring that the beams of light entering the eye should be accurately superposed. In this connexion he described an arrangement devised by F. Stanley and himself, whereby a continuous spectrum formed by an ordinary prism spectroscope was reflected back from a curved mirror, the said mirror being of suitable

radius and so disposed as to cause the various component rays of the spectrum to retrace precisely their original paths, recombining once more in a white light image. By interposing immediately in front of the curved mirror apertures adjustable both in width and position the various desired portions of the spectrum alone were permitted to retrace their original paths and take part in the reformation of the slit image. This apparatus seemed to the speaker to avoid several defects common to many forms of colour-mixing apparatus.

The authors had dealt only with colour-measuring apparatus in which three colours were mixed. Another method had recently been put into practical use by P. G. Nutting and had been found by the speaker to give excellent results—namely, to measure colour by matching it with white light mixed with a pure spectrum light, the colour being then defined in terms of the intensity of the white light and the intensity and wave-length of the spectrum light in the following way—

Colour under test = Ax white light + Bx spectrum colour of wave-length λ.

The speaker had not yet found any colour which could not be matched very well in this way, even with the provisional apparatus which he had set up in accordance with Mr. Nutting's instructions.

It will be noted that in the measuring of colour by this method three factors were necessary, as in the case of matching by mixing three primaries.

In the case of some colours (purples) the procedure briefly described above requires modification, the spectrum colour being added to the one to be matched in such a manner as to result in a match with white. The above expression then becomes—

Colour under test = Ax white light − Bx spectrum colour of wave-length λ.

Prof. S. P. Thompson asked whether the statements just made by Mr. Twyman were not really Abney's principle, according to which white consisted of three primaries, and any tint could be matched by diluting light of a certain definite wave-length with white light. He was himself not sure, however, that this theory expressed the whole truth.

Mr. A. P. Trotter said that he had been working on somewhat similar lines with a Maxwell colour box, and agreed that no satisfactory mode of describing any given colour had been arrived at, but that such a mode was greatly to be desired. The authors had alluded to the fact that when a colour was matched, by some such apparatus as they were using, so as to appear exact to one observer, it might not satisfy some one else. Another difficulty arose in connexion with the light used for illuminating the colour to be matched. What was white light? When a fairly white and constantly reproducible white light was provided, it would be found that the colour of daylight was continually altering. Not only on a sunny day did the light come sometimes from the sun, sometimes from the blue sky, and sometimes be reflected from a cloud, but on a grey day he had found the colour continually changing.

Mr. A. J. Bull, replying to the speakers, said that several interesting points had been raised. The matching by dilution with white light was certainly not

suitable for colour measurements. There was a great difficulty as regards crimson and purple. They had obtained some surprisingly accurate matches, however, when experimenting with narrow primary bands; but in that case the two observers had not always agreed. When using broad primary bands they had agreed better, quite well, even, when they had been utilizing seventy-five per cent. of white light. They had been able to match yellow—which Dr. Thompson, he had just stated, had been unable to do—by taking narrow bands. The difficulty was individual, the superposition of the colours in the eye. They had experienced the trouble to which Mr. Trotter had alluded, that there was no standard of white light. Intermittent illumination was unreliable; in that respect they agreed with Dr. Edridge-Green, whom they could not follow as to other points, however.

THE DESIGN AND CONSTRUCTION OF LARGE POLARISCOPES

By Professors E. G. Coker and S. P. Thompson, of the City and Guild of London Technical College, Finsbury

(*Monday, June 24th,* 1912.)

In optical investigations, for which a beam of polarized light is required, the best means yet devised for obtaining plane polarized light is the prism of Iceland spar first invented by Nicol and known by his name.

Such prisms are unsurpassed for the purity and intensity of the light transmitted through them; and they are almost always preferred for optical investigations wherever the conditions admit of their use.

Nicol's prisms have been made giving a clear view of an object as much as four inches in diameter, and this appears to be very nearly the greatest size for which spar has been found available, except in one or two isolated instances, where full advantage was not taken of the size and perfection of the crystal.

For most scientific purposes Nicol's prisms, of the size mentioned, would be more than ample if they could be readily obtained; but, at the present time, the number of large prisms in existence is very limited, and there is a complete dearth of spar of a suitable kind, owing to the stoppage of the supply from Iceland, and the lack of any other supply of sufficiently good optical quality.

Other means are, however, available for producing polarized light, and the object of the present paper is to describe certain forms of apparatus using sheets of glass and mica.

Although the instruments described are somewhat inferior in their polarizing and analysing properties to those using Nicol's prisms, yet they have many advantages of their own which make them almost indispensable for some kinds of investigation. The polarizing properties of glass plates are extremely useful in cases where a large doubly refracting object requires examination; and their behaviour has been examined

by several physicists, and especially by Stokes,* who has obtained expressions for the intensities of the reflected and refracted rays for a pile of plates made of a glass which reduces the intensity of transmitted light by absorption.

If ρ is the fraction of the incident light of unit intensity reflected at the first surface of a single plate, as determined by Fresnel's laws, the intensity r of the reflected ray may be expressed by—

$$r = \rho + \frac{\rho(1-\rho)^2 g^2}{1-\rho^2 g^2};$$

and the intensity t of the refracted ray by—

$$t = \frac{(1-\rho)^2 g}{1-\rho^2 g^2},$$

where g is the fraction to which the intensity is reduced by absorption in a single transit.

If there is no absorption $g = 1$, and we have the sum of the intensities of the reflected and refracted rays equal to that of the incident beam; but in general the absorption causes a diminution in the intensity of both beams. If a pile of m plates is used, and $\phi(m)$, $\psi(m)$ are the intensities of the reflected and refracted rays respectively, the corresponding formulæ are shown to be—

$$\frac{\phi(m)}{\sin m\beta} = \frac{\psi(m)}{\sin \alpha} = \frac{1}{\sin(\alpha + m\beta)},$$

which reduces to—

$$\frac{r}{\sin \beta} = \frac{t}{\sin \alpha} = \frac{1}{\sin(\alpha + \beta)},$$

when $m = 1$, and α and β are imaginary since in general $r + t < 1$.

The intensities, corresponding to values of $g = 1$, 0·98 and 0·90 respectively, are shown in the accompanying diagram for reflection and refraction at the polarizing angle: $\omega = \tan^{-1}\mu = \tan^{-1} 1{\cdot}52$.

The forms of the ϕ curves show that the intensity of the reflected light falls off very rapidly as the absorption increases, and that there is little advantage in using a very large number of plates; the ψ curves show similar results.

The refracted ray will, in general, consist of two components polarized in directions at right angles, and the ratio of these is a measure of the effectiveness of the polarization. These ratios are shown by the dotted curves; the one corresponding to perfect transparency coincides with the ψ curve for $g = 1$.

These ratios show the important result that although light transmitted through a pile of plates is materially reduced by absorption, the polarization effect is improved somewhat. If, therefore, the illumination is sufficiently intense it is not necessary to use glass that is of the clearest quality and therefore expensive.

* On the Intensity of the light reflected from or transmitted through a pile of plates. *Collected Papers*, vol. iv. p. 145.

The formulæ and curves form a general means of estimating the probable polarizing effect obtainable for a given design of polariscope, constructed with sheets of glass having definite values of ρ and g, ascertained by experiment.

In the construction of polariscopes we may make use of reflected or refracted rays, or both; and since plates of glass may be obtained of any convenient size at a low cost, they afford a cheap and easy means, when suitable illumination is provided, of obtaining, in polarized light, a field of view of almost any extent, thereby offering an advantage over Nicol's prisms, in which the field of view is always a very limited one.

In practice it is found that light which falls on a sheet of black glass at the polarizing angle is almost perfectly plane polarized, and although its intensity is only about one-seventh of the incident light, yet it is not as a rule worth while to modify this arrangement to produce a more intense reflected beam at the expense of the perfection of the plane polarization usually entailed, especially as the initial intensity can be made very great. It is, therefore, convenient to use such a sheet and illuminate it by a bank of lamps.

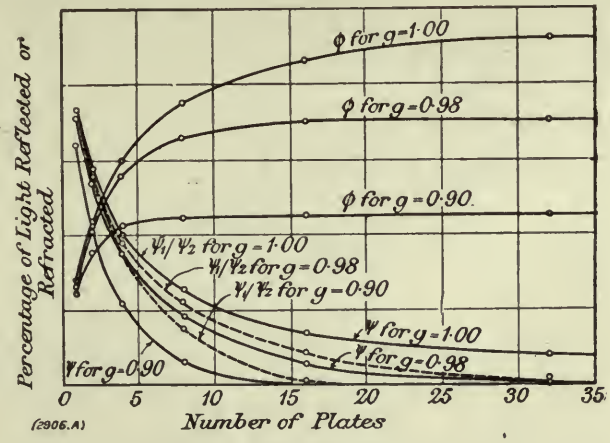


FIG. 1.—Reflection and Refraction of Light at the Polarising Angle.

To obtain equal illumination at all parts of the field the light must be of uniform intensity, and if filament lamps are used some means of diffusing the light must be employed. The interposition of a sheet of ground glass, or one or two sheets of white tissue paper, between the lamps and the reflecting surface gives an effect which is uniform enough for most purposes. The illumination may be intensified by reflectors behind the lamps, and also at the sides after the manner of a kaleidoscope.

An object, possessing doubly refracting properties, when viewed in this illuminated field by a suitable analyser, shows interference colours; and, since the analyser may be of moderate dimensions, it is possible to use a quite small Nicol's prism for this purpose. An example of a polariscope of this kind is shown by Fig. 2 in sectional elevation, in which the light from a bank of lamps, A, passes through a diffusing screen, B, of tissue paper, and is reflected at the polarizing angle by a sheet of black glass, C, to afford a horizontal beam for illuminating an object placed in the space, D, and viewed by a Nicol's prism, E.

An apparatus of this kind was used in an investigation for determining the

distribution of stress in a plate subjected to shear * where a field of view of about 120 sq. ins. was required; and it replaced an earlier arrangement of Nicol's prisms giving a field of view of barely two square inches. This latter arrangement, on account of its small size, required to be constantly shifted to new positions for making observations, and this involved a serious waste of time and liability to error, owing to the frequent adjustments required. Its replacement by a large polariscope diminished the time of labour of observation to about one-quarter of that formerly needed.

For many investigations a circularly polarized beam of light is necessary, and if only a small field of view is required, the well-known Fresnel rhomb offers the great advantage with white light that waves of varying length are all nearly accurately circularly polarized. In the present case a glass prism of sufficient size would be

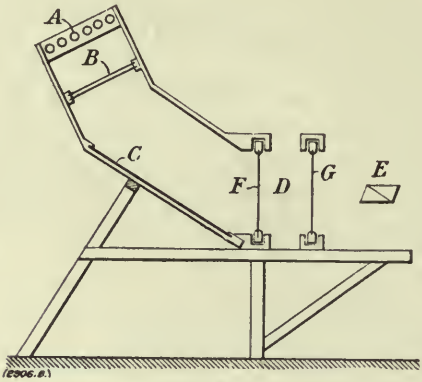


Fig. 2.

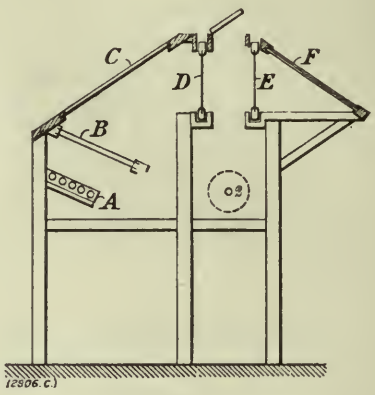


Fig. 3.

almost impossible to construct and use, but an admirable substitute is found in sheets of mica, which may, with some practice, be split into laminæ of large size to give approximately a quarter-wave phase difference to the two components into which a plane polarized beam is divided in passing through a lamina at perpendicular incidence.

Quarter-wave plates F and G are shown in this polariscope between the reflector and the analyser, and in the space D, therefore, we obtain circularly polarized light. An object set in this space will not, therefore, show a black cross effect when viewed by the analyser.

It may be interesting to state that quarter-wave plates up to 20 ins. in diameter, and also of rectangular form 40 ins. by 10 ins., have been constructed by one of us by joining two sheets of mica of the same thickness. It is believed that these dimensions have not been approached before.

* "An Optical Determination of the Variation of Shear Stress in a Thin Rectangular Plate Subjected to Shear," by E. G. Coker, *Proc. Roy. Soc.*, **A**, vol. lxxxvi. (1912.)

For some experimental purposes it is convenient to construct of transparent materials, engineering and other models, and to load them in the way the actual structures and machines are loaded in practice; the stresses may then be obtained by methods which have already been described.*

A general optical examination of a model often shows the places where maximum stress occurs, and thereby affords information of value in the design of a large structure, especially in cases where calculations cannot be made owing to the complexity of the stress system. In many cases of construction, as in roofs, plate and lattice girders, ships and the like, the over-all dimensions are very great in some one direction, and a long and narrow field of view is required. Moreover, it may be necessary to make arrangements to apply and release the load at regular intervals.

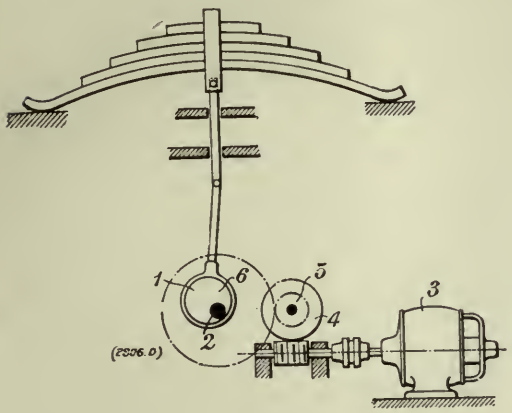


FIG. 4.

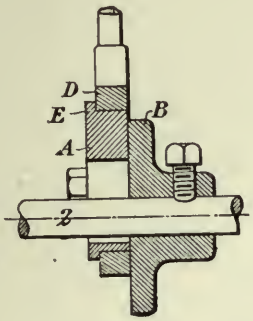


FIG. 5.

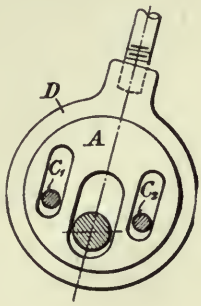


FIG. 6.

A polariscope designed for these conditions is exhibited, and a cross-sectional view of it is shown, in the accompanying Fig. 3. Light obtained from a bank of filament lamps, A, is diffused by a tissue paper screen, B, before reflection from a black glass plate, C, 40 ins. by 21 ins., inclined at the polarizing angle. The light afterwards passes through two quarter-wave plates, D and E, each affording an unobstructed view 40 ins. wide and 10 ins. high. The analyser consists of several sheets, F, of clear glass set at a suitable angle. In this way an observer in front of the polariscope can view directly the interference effects produced by the application of load to the object under examination.

Cyclical loadings of the models, like the spring shown in Fig. 4, are obtained by means of eccentrics (1) keyed to a shaft (2), running the whole length of the polariscope. This shaft is driven from a motor (3) by means of a train of gearing comprising

* "The Optical Determination of Stress," by E. G. Coker, *Phil. Mag.*, October 1910. "The Distribution of Stress in Plates of Variable Section," by E. G. Coker, *Trans. Inst. Naval Architects.* (1911.)

a worm reduction gear (4), driving a pinion (5), which latter gears with a spur wheel (6) on the main shaft; the reduction effected is from 2000 to ten revolutions per minute. A feature of interest is afforded by the eccentrics, which have a throw capable of variation between 0 and 1½ ins.

This is accomplished by the construction shown in section by Fig. 5 and in side elevation by Fig. 6. The main sheave (A) is not keyed to the shaft directly, but is bolted to a flange-plate (B) secured to the shaft. The sheave may be fixed at any definite eccentricity by means of a pair of studs (C_1, C_2) screwed into the flange-plate and clamping the sheave to it. Slotted holes are provided to accommodate the studs and shaft within the assigned limits of throw. The strap encircling the sheave is in one piece, and is prevented from moving sideways in one direction by a projecting collar (E) on the sheave, and in the opposite direction by the flange-plate. This arrangement has been found very useful for laboratory and demonstration purposes. We desire to express our warmest thanks to Messrs. Mould, Withycombe and several other members of the College staff, and to many students for the skill and care they have devoted to the construction of this apparatus.

Mr. T. Smith said he had been much interested in watching the machine placed by the authors in the Exhibition. It appeared to him that in the models exhibited the strain was much greater than would usually be permissible in actual structures, and he would be glad to hear from the authors whether the distribution of the stresses in an actual structure, where the strain was proportionately much less, could be deduced from those of the experimental models, and if so, what the limits were within which such deduction might fairly be made.

Dr. W. Rosenhain asked whether calculations could be based on these qualitatively most interesting results.

Dr. A. S. Percival inquired how it was possible to make quarter-wave plates of such enormous sizes as shown (21 in. in diameter). He remembered spending an inconceivable time in making a quarter-wave plate one inch in diameter.

Dr. S. P. Thompson, in reply, said that it would be quite impossible now to obtain material of uniform thickness, if it were not that Messrs. Wiggins & Sons had been collecting specimens of exceptional quality. They had also helped him with their experience. A little dexterity and experience were all that was required, and he had no doubt that the quarter-wave plates could be repeated over again.

Prof. Coker stated in answer to Mr. T. Smith and Dr. Rosenhain that the points raised were, of course, very important. The strains were no doubt much greater in xylonite than in iron and steel; yet if the physical constants were taken into consideration—modulus of elasticity of xylonite 300,000, of steel 30,000,000; Poisson's ratio for xylonite 2·8, for steel 9·0—there was a fairly good agreement. The strains, moreover, appeared larger than was actually the case, owing to the movement of the

colour bands which gave the eye a false impression of considerable strain, although measurement showed nothing abnormal. Quantitative results had been obtained; test cases like bending rectangular section beams, eccentrically loaded tie bars, and other specimens yielded values in transparent materials which were quantitatively correct within five per cent. or less, and agreed with similar experiments on steel. Glass was better for such experiments than xylonite. The neutral axis, generally in the middle of the beam, should go up as the stress from above increased. When shear was applied to a long plate, the quantitative results of the calculations based upon the optical examination were again fairly accurate. With regard to the production of the quarter-wave plate, Prof. Coker wished to add that his share in this had practically been nil, and that Prof. Thompson had hardly told the meeting all the skill that was required.

COLOUR-MIXING APPARATUS

By Thomas H. Blakesley

(*Monday, June 24th*, 1912.)

In a paper which I had the honour to present to the Optical Convention of 1905, upon Spectroscopes of Constant Deviation, I foreshadowed some of the uses to which these instruments readily lend themselves with advantage over the older forms of Spectroscopes. Among these was that of mixing colours selected from various parts of the spectrum, and the instrument before this meeting is an instance of this application of constant deviation prisms. It was made by Messrs. Beck for the Mercers' Company, and was given by that Company to the Finsbury College, and is now very kindly lent by Prof. Silvanus Thompson for the purposes of the Optical Convention.

The prisms are three in number placed in a pile, but each capable of being separately actuated by rods from the eye-piece end of the telescope. Each prism is derived from the triangle in which, keeping to the notation of my former paper—

$$A = 18° \qquad B = 80° \qquad C = 82°.$$

Hence the angle of constant deviation for any ray which for the moment occupies the centre of the field of view is 82°, and the telescope and collimator are fixed permanently at that angle.

The equivalent refracting angle is $\overline{B—A}$ or 62°, and the angle of refraction, constant for all rays, is half this, or 31°.

The shape is one which admits of truncation of the angle of 18°, so that the resulting form of the prism is a quadrilateral having angles equal to 80°, 82°, 59°, 139°, in order, and the sides are 5·461, 9·468, 3·736, 7·135 centimetres respectively in the same order, the first being between the first two angles mentioned.

The indices of refraction of the glass employed are—

For A′ 1·68270
,, C 1·68972
,, D 1·69607
,, F 1·71225
,, G′ 1·72606

The angle of incidence for any ray is obtained from the formula—

$$\text{Sin } I = \mu \,.\, \sin 31°.$$

Hence the incidence will be—

For A′, 60° 4′ 20″
,, G′, 62° 44′ 46″

The difference between these angles, or 2° 40′ 26″, is the angle through which the prism has to be turned to make the ray G′ replace the ray A′ at the centre of the field of view.

The width of the beam of light, in cross section, received by the prism is given by the formula—

5·461 Cos. I. Centimetres—

which for G′ will be 2·50 centimetres and for less refracted light a somewhat greater value.

The lenses of the collimator and telescope are similar in all respects.

Each lens is composed of a concave meniscus of heavy silicate flint cemented to a convex of silicate crown of high index.

The chief optical details are appended in a table in which all lengths and distances are given in terms of the central focal length for the D ray.

The calculations are for an aperture of $\frac{1}{15}$ made upon the published lists of the manufacturer. The actual aperture of the constructed instrument is $\frac{1}{12·7}$.

The changes in the results due to values in the new meltings of the glasses were calculated for the D ray, but were inconsiderable.

Synoptical Table of the Principal Aberrations of the Mercers' Company's Objective, in terms of the Central Focal Length for the Ray D, for an Aperture of $\frac{1}{18}$, calculated from the published Jena Lists.

Ray.	Central Focal Length F.	Marginal Intercept f.	Difference $F - f$.	Central Focal Distance Vc.	Marginal Focal Distance Vm.	Difference Spherical Aberration $Vc - Vm$.	Angle between Marginal Ray and Axis.
A'	1·0016422	1·0015539	·0000883	·9825474	·9824874	·0000600	1° 54′ 26·1″
C	1·0004392	1·0003877	·0000515	·9813287	·9813052	·0000235	1° 54′ 34·1″
D	1· New Meltings	1·0000033 ·9999677	− ·0000033 ·0000324	·9808689 ·9808280	·9808972 ·9808144	− ·0000283 ·0000136	1° 54′ 36·7″ 1° 54′ 37·0″
F	1·0004013	1·0005239	− ·0001226	·9812040	·9813277	− ·0001237	1° 54′ 33·2″
G'	1·0022277	1·0024463	− ·0002186	·9829597	·9831667	− ·0002070	1° 54′ 20·0″

The actual focal length of the constructed lenses is believed to be 77·696 cm., or 30·590 inches.

The telescope when used with an eye-piece which makes the magnifying power 20·8, shows the satellite Titan of Saturn quite well under ordinary circumstances.

The apportioning of the light for the various prisms is effected by a slide containing three sliding shutters, each having its opening measured by a scale and vernier. When the proportions of the lights are unimportant the shutters can be left fully open, or the whole slide may be removed. The proper place for this apportioning slide is between the collimating lens and the prisms. Another slide can be inserted between the prisms and the telescope objective. Its function is to separate and exhibit apart from one another the three spectra. It contains three horizontal strips of optical glass, of which the top and bottom ones are prisms of small angle turned in opposite directions, the centre strip being a plate. By this means the top and bottom spectra are moved up and down respectively, the central spectrum retaining its proper place.

There is also a blocking slide, which can be inserted in the place of the analysing slide. By its means any one or any two of the spectra can be suppressed temporarily. Thus each spectrum can be accurately adjusted, so that any desired light, as defined by absorption lines or otherwise, may be placed exactly upon the crossed lines of the eye-piece diaphragm.

The mixture of two colours only can also obviously be obtained by the blocking of one spectrum. This end could be obtained by the shutters of the apportioning slide, but when any given proportions are under examination it might be inconvenient to shift one of the shutters for another purpose.

The collimator slit is controlled by three movable shutters. One controls the width of the slit in the usual way, the others the extremities top and bottom; and

these latter have such an amplitude that, even when the analysing slide is in position, the top and bottom spectra may be made to overlap each other. By this means very beautiful chromatic effects may be produced.

There is a device furnished with this instrument which can be well employed to give particular spectrum positions when the source of light is not solar. A ring cap, which can be fixed in front of the slit, carries a small uniform sheet of mica set at an angle of 45° to the axis of the collimator. A small mirror parallel to the mica throws the light from the source upon the mica, which then reflecting it from both its surfaces produces lines of interference running vertically across the spectra, seen in the spectroscope. The bands retain a constant position upon each spectrum, moving with it as the prisms are rotated. Once for all these positions may be determined in solar light. For instance, counting from the position of D along the spectrum, E is 28 spaces away, F is 51, G′ 87, G 89, C is −24·5, B −34, and so on.

The position of D can be shown by any sodium flame, and the remainder of the operation will then merely consist in counting carefully the spaces which pass the centre of the field of view until the positions required are obtained, when the mica and its carrier can be removed. The diaphragms of the eye-piece, which settle the size and shape of the patch of light presented to the eye may be varied indefinitely, and the patch reduced to the size which the smallest pinhole would appear to have. The intensity of the light seen can be varied by operating upon the width of the one slit which alone is necessary in this instrument. Want of definition may be produced by putting the eye-pieces out of focus. Proportion of lights of different colours is effected by the apportioning slide, and the lights themselves are varied by turning the separate prisms. One source of light only is used, and, therefore, the difficulty of providing many sources of similar and similarly intense light for as many slits is avoided. Moreover, all these variations are made by arrangements which are independent one of the other.

The actual constitution of the light seen on any one occasion may be recorded and faithfully reproduced by noting the three scale readings attached to the three prisms, the three readings of the apportioning slide shutter, the width of the slit, the diaphragm, and the source of light. Thus I find a record made during the short time I had the instrument under observation, as follows—

Upper prism	4° 7′	a blue,
Middle ,,	3° 25′	a green,
Lower ,,	2° 57	an orange.

The shutters were fully open, and the slit long so as to make the extreme spectra overlap with the analysing slide in place. The source of light was an incandescent electric lamp with frosted globe.

The results put down are: Gamboge where green and orange overlap; Cambridge blue where blue and green overlap; and in the centre a grey which when the green

is shut off is rose. That is an observation from an experiment which can be repeated exactly with perfect ease.

An almost infinite variety of arrangements is possible, and in a genuine examination for colour-blindness it could be made impossible to predict the impression which was going to be presented. The variety of coloured wools cannot be compared with it. The safety of travelling by railway or by sea greatly depends upon a man recognizing the colour of a lamp small in extent and isolated in a dark background. In a crisis there can be no such operation possible as matching the light seen with any other colour, though that may be a useful operation in a draper's shop. A skilful observer of light signals should be able to detect a colour even when the intensity of the light is low, as must often be the case from mere distance. A colour of low intensity seen alone is not easily named. What, for instance, is the colour produced by a particle of radium acting upon platinocyanide of barium, only to be seen in quite a dark room? Many a person would say it was much the same colour as that of a common candle. But the spectroscope pronounces the light to be near the line E, and E when of sufficient intensity would be called green. Would the smaller stars of the binaries γ Andromedæ, and the head of the Swan be called blue if seen alone, and without the contrast of their companions? I think it is very doubtful. Large telescopes detect the colour of a star by bringing others into sight, and so obtaining the necessary contrast. On railways the absolutely isolated coloured light is probably rare, but at sea the case is different. What should be tested is the power of pronouncement of the colour of a weak isolated light, and I doubt if any better means of testing sight for signal purposes, and at the same time of evading deception, can be found than the spectroscope.

Many people remember the case of Captain Trattles, a skipper who passed through all the vicissitudes of fortune in connection with his certificate on account of the different views held by a variety of officials on the subject of his eyesight. He may, indeed, be congratulated that on the last occasion he was sent afloat again with his certificate at least in order, but the arbitrator, Sir Francis Mowatt, took occasion to decry the use of Spectroscopes for sight-testing, and to discourage those who have attempted to work some improvement in a technical matter which the case of Captain Trattles proves to be one of some difficulty. Unless some reason is given for this adverse opinion it must be regarded as medieval and unsatisfactory.

METHOD OF TRACING CAUSTIC CURVES.

By Archibald Stanley Percival, M.A., M.B. Cantab.

(*Monday, June 24th*, 1912.)

In all mathematical books on Optics there are some very troublesome sections, dealing with Caustic Curves and with the aberration of lenses. Many mathematicians have devoted their energies to discovering the general equation to a Caustic Curve without success. The usual method is first to find a curve that is called the secondary caustic, and then to find the evolute of this curve. It is a most laborious and tedious method, and if the incident wave-front is plane it involves elliptic functions, so that practical men have for the most part neglected these recondite mysteries, especially as no method is given of finding the equation to the Caustic Curve of a lens or a system of lenses.

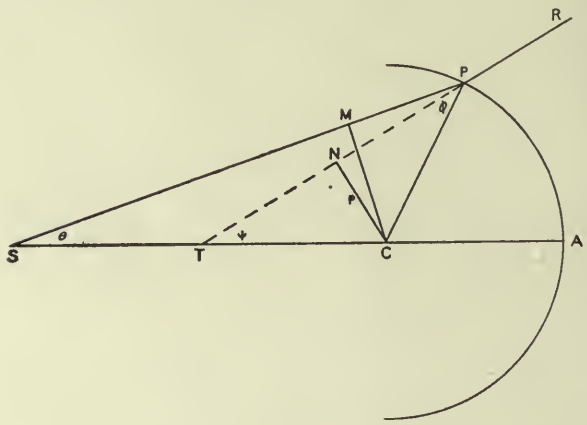


Fig. 1.

Although I cannot give a general equation to a Caustic, yet I have pleasure in bringing forward a method by which the Caustic due to either reflection or refraction at a single spherical surface can be traced with ease, expedition and accuracy, and corresponding points can be marked after refraction at another surface, so that the Caustic formed by a lens can be plotted out in a reasonable time. Now different lenses show differently shaped Caustics, and it will be found that some of the general rules dealing with the size of the least circle of aberration are not universally true.

A Caustic Curve is the locus of all the primary focal lines formed by the intersection of two contiguous reflected or refracted rays.

We will take the case of refraction at a single spherical surface of which the centre is at C. Let S denote the source of light and consider the ray SP, making an angle ASP or θ with the axis, incident at P and refracted as PR, so that CPS is the angle of incidence, or ϕ, and CPT is the angle of refraction, or ϕ^1. From C drop the perpendiculars CM and CN on SP and TP. Let us denote the angle ATP by ψ, CP by r, CS by a, NC by p, and therefore MC by μp.

Now
$$\theta = \text{CSP} = \sin^{-1}\frac{\text{MC}}{\text{CS}} = \sin^{-1}\frac{\mu p}{a}$$
$$\phi = \text{CPS} = \sin^{-1}\frac{\text{MC}}{\text{CP}} = \sin^{-1}\frac{\mu p}{r}$$
$$\phi' = \text{CPT} = \sin^{-1}\frac{\text{NC}}{\text{CP}} = \sin^{-1}\frac{p}{r}$$
$$\psi = \text{ATP} = \text{CSP} + \text{SPT} = \text{CSP} - \text{TPS} = \theta - (\phi - \phi^1)$$
$$\therefore\ \psi = \sin^{-1}\frac{\mu p}{a} - \sin^{-1}\frac{\mu p}{r} + \sin^{-1}\frac{p}{r}.$$

Differentiating with regard to ψ and writing p^1 for $\frac{dp}{d\psi}$—

$$1 = \frac{\mu}{a}\frac{p^1}{\sqrt{1-\frac{\mu^2p^2}{a^2}}} - \frac{\mu}{r}\frac{p^1}{\sqrt{1-\frac{\mu^2p^2}{r^2}^2}} + \frac{1}{r}\frac{p^1}{\sqrt{1-\frac{p^2}{r^2}}}$$

or—
$$\frac{1}{p^1} = \frac{\mu}{a}\sec\theta - \frac{\mu}{r}\sec\phi + \frac{1}{r}\sec\phi^1.$$

Differentiating again with regard to ψ—

$$\frac{-p''}{(p^1)^2} = \frac{\mu}{a}\sec^2\theta\sin\theta\frac{d\theta}{d\psi} - \frac{\mu}{r}\sec^2\phi\sin\phi\frac{d\phi}{d\psi} + \frac{1}{r}\sec^2\phi^1\sin\phi^1\frac{d\phi^1}{d\psi}$$
$$= \frac{\mu^2}{a^2}\sec^3\theta\sin\theta\,p^1 - \frac{\mu^2}{r^2}\sec^3\phi\sin\phi\,p^1 + \frac{1}{r^2}\sec^3\phi^1\sin\phi^1\,p^1.$$

Multiplying both sides by $-\frac{(p^1)^2}{p}$, and remembering that—

$$p = \frac{a}{\mu}\sin\theta = \frac{r}{\mu}\sin\phi = r\sin\phi^1, \text{ we get—}$$
$$\frac{p''}{p} = -(p^1)^3\left\{\frac{\mu^3}{a^3}\sec^3\theta - \frac{\mu^3}{r^3}\sec^3\phi + \frac{1}{r^3}\sec^3\phi^1\right\}.$$

But by a well-known lemma the radius of curvature $\rho = p\left(1 + \frac{p''}{p}\right)$.

$$\therefore\ \rho = p\left[1 - (p^1)^3\left\{\frac{\mu^3r^3\sec^3\theta - \mu^3a^3\sec^3\phi + a^3\sec^3\phi^1}{a^3r^3}\right\}\right]$$
$$= p\left\{1 - \frac{\mu^3r^3\sec^3\theta - \mu^3a^3\sec^3\phi + a^3\sec^3\phi^1}{(\mu r\sec\theta - \mu a\sec\phi + a\sec\phi^1)^3}\right\}.$$

If we remember that p is the perpendicular dropped from C to the refracted ray at N, and that p^1 is measured from N along the refracted ray at right angles to CN,

the end of the line p^1 marks a point on the Caustic Curve, while the value of ρ gives the radius of curvature at that point; there is a cusp when $\rho = 0$.

The adjoining table summarizes the results that we have just found. It will be noticed that if the incident wave-front is plane, *i. e.*, if the source of light be at an infinite distance, the angle θ disappears from the previous calculation, and so it does not appear in the final result. We should be wrong if we assigned the value 0 to θ; this is one of the many slips that may be made in such mathematical calculations. It is a common error to think that the absence of a function is the same thing as assigning a zero value to it. Clearly if θ were equal to 0, $\sec \theta = 1$, but when θ is absent $\sec \theta$ is also absent.

FORMULÆ FOR TRACING CAUSTIC CURVES

$$p = r \sin \phi^1$$

Refraction.	Reflection. $\mu = -1 \qquad \phi = -\phi^1$
$\psi = \theta - \phi + \phi^1$ $p^1 = \dfrac{ar}{\mu r \sec\theta - \mu a \sec\phi + a \sec\phi^1}$ $\rho = p\left\{1 - \dfrac{\mu^3 r^3 \sec^3\theta - \mu^3 a^3 \sec^3\phi + a^3 \sec^3\phi^1}{(\mu r \sec\theta - \mu a \sec\phi + a \sec\phi^1)^3}\right\}$	$\psi = \theta + 2\phi^1$, $p^1 = \dfrac{ar}{2a \sec\phi^1 - r \sec\theta}$, $\rho = p\left\{1 - \dfrac{2a^3 \sec^3\phi^1 - r^3 \sec^3\theta}{(2a \sec\phi^1 - r \sec\theta)^3}\right\}$.
If $a = \infty$, θ is absent, $\psi = \phi^1 - \phi$, $p^1 = \dfrac{r}{\sec\phi^1 - \mu \sec\phi}$ $\rho = p\left\{1 - \dfrac{\sec^3\phi^1 - \mu^3 \sec^3\phi}{(\sec\phi^1 - \mu \sec\phi)^3}\right\}$.	If $a = \infty$, θ is absent, $\psi = 2\phi^1$ $p^1 = \dfrac{r}{2 \sec\phi^1}$ $\rho = p\left\{1 - \dfrac{2 \sec^3\phi^1}{(2 \sec\phi^1)^3}\right\} = \dfrac{3}{4}p$.

The above calculation is of interest in another way, as it shows the extreme importance of attending to the signs of angles and measurements.

Here we are regarding the left to right as the positive direction, and the counter-clockwise rotation as the positive rotation. Consequently, as θ is counter-clockwise, $\sin\theta$ must be positive, *i. e.* either $\frac{CM}{SC}$ or $\frac{MC}{CS}$, but as ϕ is here clockwise, $\sin\phi$ is negative, and therefore $\frac{MC}{CP}$. Almost all the books on Optics are slovenly in this respect, a sine is regarded as a ratio of perpendicular to hypothenuse without regard to direction. It is not a sufficient test to find that one's formula gives numerically right answers, one must also get numerically correct answers when the formula is converted into one for reflection by the simple device of replacing μ by -1, and ϕ^1

by $-\phi^1$. I will now show on the blackboard how exceedingly quickly one can plot out the Caustic for reflection at a concave spherical surface ($r = 10$) for an incident plane wave-front by this formula. First, make a little table of values for the conditions when ϕ^1 is 0, 30°, 60° and 90°.

ϕ^1	$\frac{p}{r \sin \phi^1}$	$\frac{p^1}{\frac{r}{2 \sec \phi^1}}$	$\frac{\rho}{\frac{3}{4}p}$
0	0	$\frac{10}{2} = 5$	0
30	$\frac{10}{2}$	$\frac{10\sqrt{3}}{4} = 4{\cdot}33$	$\frac{15}{4} = 3{\cdot}75$
60	$\frac{10\sqrt{3}}{2} = 8{\cdot}66$	$\frac{10}{4} = 2{\cdot}5$	6·495
90	10	0	7·5

Remember from what has been said that when ϕ^1 is positive, sin ϕ^1 is $\frac{CN}{CP'}$, as the radius is also positive.

Draw the radius CP at 30° with the axis, and make CPN also 30°, as the reflected ray. Draw CN perpendicular to it and from N measure Nf 4·33 (or p^1) upwards in the positive direction; f is a point on the Caustic. Similarly find the points indicated by p^1 for the other values of ϕ^1. The corresponding points below the axis can be marked out symmetrically. It will be found in practice that the numerical values of p are of little use in finding definite points on the curve, as the point N is more accurately found by letting the perpendicular CN drop on the reflected or refracted ray.

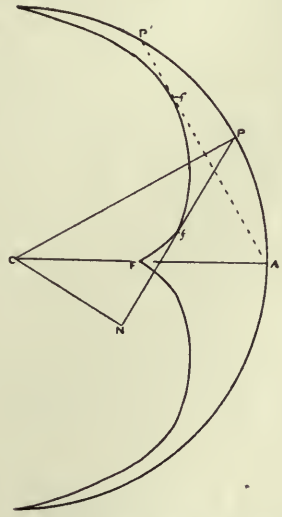


Fig. 2.—Reflection of plane wave-front at a concave spherical surface.

We will now take the case of a plane wave-front refracted at a convex spherical, surface of radius -5 ($\mu = \frac{3}{2}$). As before we first make out a little table for some of the values of the angle ϕ.

ϕ	ϕ^1	$p = r \sin \phi^1$	$p^1 = \frac{r}{\sec \phi^1 - \mu \sec \phi}$	$\rho = p\left\{1 - \frac{\sec^3 \phi^1 - \mu^3 \sec^3 \phi}{(\sec \phi^1 - \mu \sec \phi)^3}\right\}$
0	0	0	$\frac{r}{1-\mu} = 10$	0
30°	19° 28′	$\frac{-5}{2\mu} = -1{\cdot}6$	$\frac{-5}{1{\cdot}0606 - \sqrt{3}} = 7{\cdot}4416$	20·37
45°	28° 7′	$\frac{-5}{\mu\sqrt{2}} = -2{\cdot}357$	$\frac{-5}{1{\cdot}1338 - \mu\sqrt{2}} = 5{\cdot}0633$	$-2{\cdot}357\left(1 = \frac{8{\cdot}088}{{\cdot}9629}\right) = 17{\cdot}44$
60°	35° 16′	$\frac{-5}{\mu}\frac{\sqrt{3}}{2} = -2{\cdot}8867$	$\frac{-5}{1{\cdot}22477 - 3}$ 2·8165	10·10
90°	41° 48′	$\frac{-5}{\mu} = -3{\cdot}3$	0	

For instance, when $\phi = 0$, p or $r \sin \phi^1 = 0$ and $p = +10$; we therefore mark the point F″ at a distance of 10 from C, and we notice that at this point $\rho = 0$, so there is a cusp at F″. Taking the case where $\phi = 60°$ we draw CE at 60° with axis, and from E draw EN at the angle 35° 16′ with CE; the point f' on EN produced distant 2·8165 from N is the corresponding point on the Caustic.

If the Caustic for a planoconvex lens with its plane surface facing the incident plane wave-front is required, it will be as easily drawn as the last; all that we have to remember is that the value of μ is now $\frac{2}{3}$ instead of $\frac{3}{2}$, and of course in this case the angle of incidence must be less than the critical angle (41° 48′). (Fig. 4.)

ϕ	ϕ^1	$p = r \sin \phi^1$	$p^1 = \frac{\sec \phi^1 - \mu \sec \phi}{r}$	$\rho = p\left\{1 - \frac{\sec^3 \phi^1 - \mu^3 \sec^3 \phi}{(\sec \phi^1 - \mu \sec \phi)^3}\right\}$
0	0	0	$\frac{r}{1-\mu} = 15$	0
−30°	−48° 36′	$-\frac{r}{2}\frac{3}{2} = -3{\cdot}75$	$\frac{r}{1{\cdot}5121 - {\cdot}7698} = 6{\cdot}735$	$-3{\cdot}75\left\{1 - \frac{3{\cdot}0015}{{\cdot}40909}\right\} = 23{\cdot}76..$
−40°	−74° 37′	$-{\cdot}964..r = -4{\cdot}82$	$\frac{r}{3{\cdot}769 - {\cdot}876} = 1{\cdot}7245$	$-3{\cdot}75 \times -51 = 91{\cdot}25$

$$\tan \alpha = \frac{r \sin \phi}{H'' F''} = \frac{5 \times \sin 40}{10} = {\cdot}3212625. \qquad \therefore \quad \alpha = 17° 48′ 37″$$

When we have to consider a lens bounded by two spherical surfaces, the matter becomes more complicated. In Fig. 5, for instance, we have drawn a spherical

surface of radius -5 facing the incident light, while the other surface has a radius of 30; in fact, this is a representation of the familiar "crossed" lens. As I wish to compare the Caustic of this lens with that of a symmetrical biconvex of similar focal length and similar aperture, I must consider 54° 56′, as the extreme angle of inci-

FIG. 3.—Plane wave-front incident on convex spherical surface ($r = -5$, $\mu = \frac{3}{2}$).

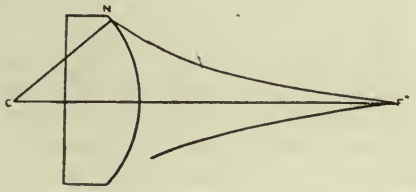


FIG. 4.—Planoconvex. Plane wave-front incident on plane surface.

dence. By the method just given we find that the position of the first focal line due to the first refraction of this thin pencil, incident at an angle of 54° 56′, is at S.

This point S not being on the axis of the lens cannot be dealt with by our previous method without complicating the figure by drawing a new axis through C and S, and then involving us in formidable mathematical complexities. But there is a very simple formula for finding the primary focus of incident eccentric pencils—

$$\frac{\mu \cos^2 \phi_2^1}{V_1} - \frac{\cos^2 \phi_2}{U} = \frac{\sin (\phi_2 - \phi_2^1)}{r_2 \sin \phi_2^1} \quad \ldots ( )$$

In the books the expression is said quite correctly to be equal to $\frac{\mu}{V_2} - \frac{1}{U}$, but this expression for the second focal line is only true if the source of light is on the axis. Fortunately we are now only considering primary foci, so that we can use the expression (a) freely, and obtain thereby accurate results. There are, however, two points that we must ascertain before using this formula—

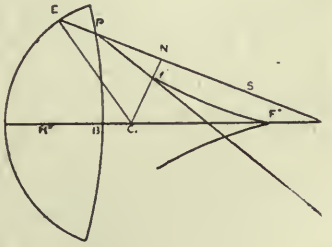


FIG. 5.—Crossed lens. Plane wave-front incident on surface of greatest curvature. Semi-aperture 20° 40′.

(1) The angle of incidence ϕ_2 at the second surface.

(2) The distance U or SP, *i. e.* the distance of the second surface from S.

Now—

$$SP = SN + NE + EP = -p' + r, \cos \phi^1 + EP.$$

We must therefore find an expression for ϕ_2 and EP.

I will not now weary you with the proof which is given in full in the

Appendix. I will merely give the formulæ; these are universally true for any bispherical lens, provided that due attention is paid to the signs of the r_1 and r_2 as well as to the signs of the angles considered. For an incident plane wave-front on the first surface—

$$\sin \phi_2 = \frac{r_1 \sin \phi^1 + (r_1 + t - r_2) \sin (\phi - \phi^1)}{r_2} \quad \ldots\ldots\ldots (1)$$

$$\mathrm{EP} = \frac{r_2 \sin (\phi - \phi^1 + \phi_2) - r_1 \sin \phi}{\sin (\phi - \phi^1)} \quad \ldots\ldots\ldots (2)$$

We have, then, all the data that are required, and we can return to the "crossed" lens of Fig. 5, the thickness of which, AB or t, is 4. By the formula (1) we find $\sin \phi_2$, and hence $\sin \phi_2{}^1$. Both ϕ_2 and $\phi_2{}^1$ are negative, but $\phi_2 - \phi_2{}^1$ is positive ($17° 7\frac{1}{2}'$) as $\phi_2{}^1 > \phi_2$. The centre for the second surface is beyond the limits of the diagram, but by making $f'\mathrm{PN}$ $17° 7\frac{1}{2}'$ we indicate the final emergent ray.

EP is found by (2) and hence we can find U or $-p^1 + r$, $\cos p^1 + \mathrm{EP} = -5{\cdot}9$. On applying formula ($\alpha$), we find that—

$$\frac{\sin (\phi_2 - \phi_2{}^1)}{r_2 \sin \phi_2{}^1} + \frac{\cos^2 \phi_2}{\mathrm{U}} = -{\cdot}14466,$$

and consequently —

$$\mathrm{V}_1 = \frac{\mu \cos^2 \phi_2{}^1}{-{\cdot}14466} = -2{\cdot}8465.$$

The point f' in the diagram indicates the position of the primary focus due to this oblique pencil, for $f'\mathrm{P} = -2{\cdot}84$. . . .

Similarly any other points that may be required can be found on the caustic. The principal focus may be found in several ways; as the source of this point is the cusp of the Caustic for the first surface and is, therefore, on the axis, we may use the formula given in the Table (p. 225), where ϕ_2 and $\phi_2{}^1$ have zero values, and where θ is 0.

$$p_2 = 0, \quad p^1{}_2 = \frac{a^1 r_2}{\mu r_2 + 1 - \mu a^1}.$$

Now if S be regarded as the cusp of the Caustic due to the first surface—

$$a^1 = \mathrm{C_2S} = \mathrm{C_2C_1} + \mathrm{C_1S} = r_2 - t - r_1 + p^1 = 31 + 10$$

$$\therefore \quad p_2{}^1 = \frac{41 \times 30}{20 + \frac{41}{3}} = \frac{41 + 90}{101} = 36{\cdot}53466.$$

$$\therefore \quad \mathrm{C_2F''} = 36{\cdot}53466 \text{ and } \mathrm{BF''} = 6{\cdot}53466.$$

We can check this result by the ordinary method of Cardinal points, whence we find that $\mathrm{F''H''} = -8{\cdot}9108$ and $\mathrm{H''B} = 2{\cdot}3762$, so the results are identical, as they should be. But by ascertaining the position of the second principal point, we can determine the semi-aperture of the lens, for the aperture is the angle subtended by F'' at the second principal plane. We find then that this lens has a semi-aperture of $24° 40'$.

With this aperture it is seen that the praised "crossed" lens is very defective, but we shall find that the symmetrical biconvex is worse (Fig. 6). This lens has the same focal length and the same aperture as the crossed lens. I want you to notice that not only is the Caustic Curve more extensive, but that it is of a different shape.

The difference of shape is still more marked in Figs. 7 and 8. When the

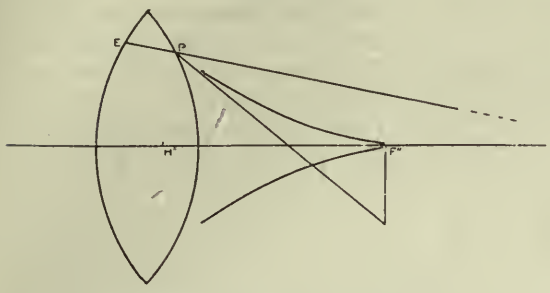


FIG. 6.—Symmetrical biconvex. Plane wave-front incident. Semi-aperture 20° 40′.

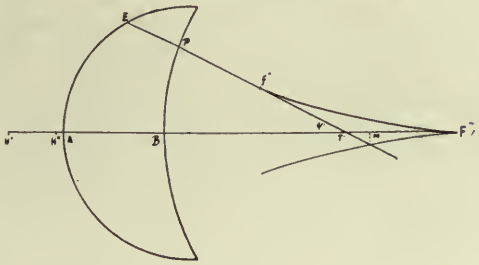


FIG. 7.—Plane wave-front incident on convex surface of meniscus. Semi-aperture 15° 20′.

convex surface of the meniscus faces an incident plane wave-front, the Caustic is that shown from f' to F, and this when the semi-aperture of the lens $a = 15° 20$.

When the concave surface of the same meniscus faces the light, the Caustic extends even beyond the principal planes (at H′ and H″), although the semi-aperture is now only 11° 39′. The angle of incidence ϕ must be less than 20° in this case, or the angle of incidence ϕ_2 at the second surface will exceed the critical angle.

In Fig. 7 the longitudinal aberration $a = 4{\cdot}4572$, while in Fig. 8 $a = 12{\cdot}8431$.

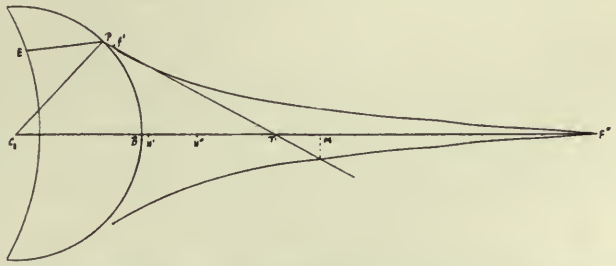


FIG. 8.—Plane wave-front incident on concave surface of same meniscus. Semi-aperture 11° 39′.

Now, in the books a very lengthy dissertation on longitudinal aberration is given; it may be a good academic exercise for the student of differentiation, but it has not the slightest bearing on practice. The formula finally given is only true to the second order of approximations. Of what earthly use is this when we have to deal with lenses of a semi-aperture of 5° or more? Then this formula is still further simplified by neglecting the thickness of the lens. As a matter of fact an accurate determination of the longitudinal aberration can

be obtained of a thin lens of any aperture in two or three minutes, and if the thickness of the lens has to be taken into account, another three minutes will give a correct result by the use of formula (1).

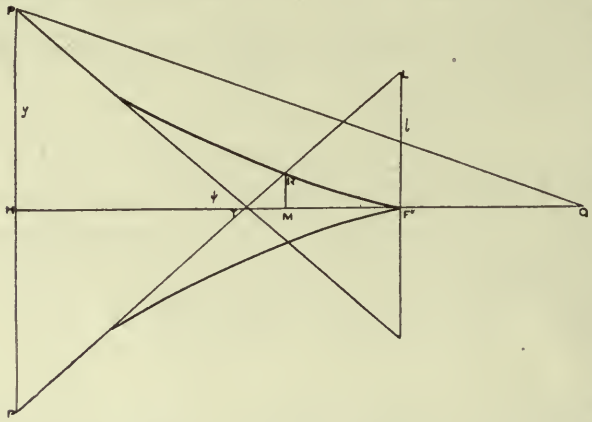


FIG. 9.—Caustic. Longitudinal and lateral aberration. Circle of least confusion.

Fig. 9 shows the procedure for determining both the longitudinal and the lateral aberrations. Let HP or y denote half the aperture of the lens and let a parallel pencil of light be incident at P and at the first surface be refracted to Q and at the second surface of the lens be refracted to T. Then HTP or ψ is the sum of the two deviations, *i. e.* $\phi_2^1 - \phi + \phi^1 - \phi$, so ψ is known. Then the longitudinal aberration TF″ or $a = HF'' - y \cot \psi$ in thin lenses, while the lateral aberration $l = a \tan \psi$. [In thick lenses and menisci, $a = BF'' - BT = H''F'' - H''B + r_2 (\sin \overline{\psi - \phi_2^1} \cot \psi + \text{vers } \overline{\psi - \phi^1_2})$, if due attention be paid to the signs of the angles.]

Now for the size and position of the circle of least confusion, the rules given in the books are absolutely misleading. They are based upon a totally illegitimate approximation to the shape of a Caustic in the neighbourhood of its cusp, but they are apparently given to encourage the student. If M be the position of this "least circle" and if R be its radius, we are told that $MF'' = \frac{3}{4}a$ and that $R = \frac{1}{4}l$.

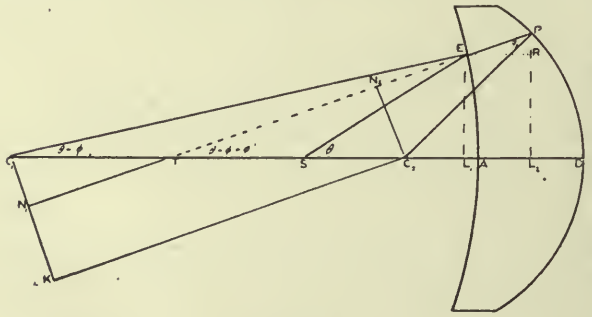


FIG. 10.

Unfortunately the only accurate method of determining the position of the "least circle is by tracing the Caustic, and for this purpose I hope this communication will be of use. The position M is determined by the intersection of the extreme ray PTL with the Caustic, and clearly the radius $R = TM \tan \psi$. The value given in the books is only a rough approximation.

Some Caustics are short, while others are long and thin, when lenses of the same

aperture are compared, but it will always be found that the shortest caustic gives the smallest "least circle." Photographers usually require what they call "depth of focus;" this means that the size of the "least circle" increases only slightly for slight variations of the position of M. A glance at the diagram will show that the rate of increase of R for an alteration of the distance of M is tan ψ. Indeed the evaluation of ψ is all-important. Of course, when $\psi = \alpha$, the angular semi-aperture, there is no aberration and no circle of least confusion.

APPENDIX

To find the angle of incidence ϕ_2 at the second surface and the length (EP) of the path of light within the glass.

Let S be the source of light, and consider the ray SE, making an angle ASE or θ with the axis, incident at E on the first surface and refracted there in the direction EP.

Then $C_1ES = \phi$, $C_1ET = \phi^1$, both being positive angles as they are measured counter-clockwise, while the angle of incidence at the second surface C_2PT or ϕ_2 is negative, being measured in the reverse direction.

We have first to find the value of ϕ_2, in terms of t, the thickness of the lens, θ, ϕ, ϕ^1, r_1 and r_2 which are known.

Produce PT to N_1, and draw C_2K parallel to PTN_1. From C_1 and C_2 drop the perpendiculars C_1N_1 and C_2N_2 on to PTN_1, producing C_1N_1 to K.

Now ATE (the ψ in Fig. 1), is as before $\theta - \phi + \phi^1$, and clearly $N_1K = C_1K - C_1N_1$, or $N_2C_2 = N_1C_1 + C_1K$, *i. e.* $r_2 \sin \phi_2 = r_1 \sin \phi^1 + C_1C_2 \sin KC_2C_1$. But $KC_2C_1 = ETA = -\theta + \phi - \phi^1$ and $C_1C_2 = C_1A + AB + BC_2 = r_1 + t - r_2$.

$$\therefore \quad \sin \phi_2 = \frac{r_1 \sin \phi^1 + (r_1 + t - r_2) \sin (\phi - \phi^1 - \theta)}{r_2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In order to find the length EP, the path of the refracted ray in the lens, drop the perpendiculars EL_1, PL_2, and draw ER parallel to the axis.

Now—

$$REP = ATE = \theta - \phi + \phi^1.$$

and—

$$L_2C_2P = C_2TP + TPC_2 = C_2TP - C_2PT = \theta - \phi + \phi^1 - \phi_2$$

and clearly—

$$RP = L_2P - L_1E = C_2P \sin L_2C_2P - C_1E \sin L_1C_1E,$$

$$\therefore \quad EP \sin REP \text{ or } EP \sin (\theta - \phi + \phi^1) = r_2 \sin (\theta - \phi + \phi^1 - \phi_2) - r_1 \sin (\theta - \phi)$$

or—

$$EP = \frac{r_2 \sin (\theta - \phi + \phi^1 - \phi_2) - r_1 \sin (\theta - \phi)}{\sin (\theta - \phi + \phi^1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When the incident wave-front is plane, θ disappears from the expression. When one of the surfaces is plane, r for that surface becomes infinite, and hence the formulæ (1) and (2) require modification.

1. When the second surface is plane, a glance at Fig. 1 will show that—

$$\phi_2 = \psi \text{ or } \theta - \phi + \phi^1$$

and that EP $\cos \phi_2 = t + r_1 (1 - \cos \overline{\theta - \phi})$, or $t + r_1$ vers $\overline{\theta - \phi}$.

Knowing EP, U can be found as before, $U = -p^1 - r$, $\cos \phi^1 + EP$, and formula (α) becomes the well-known—

$$V_1 = \mu U \frac{\cos^2 \phi^1}{\cos^2 \phi}.$$

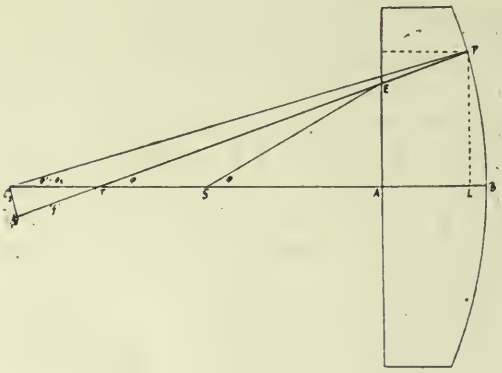


Fig. 11.

2. When the first surface is plane, r_1 is infinite. If the incident wave-front be plane, there is no difficulty, as the method used is that illustrated in Fig. 4.

If, however, the source of light be a point S, distant SA or b from the plane surface, modifications must be made in formula (1) and (2). (Fig. 11.)

The distance $f'E$ due to refraction at the plane surface or—

$$V_1 = \mu U \frac{\cos^2 \phi^1}{\cos^2 \phi} \text{ or } \mu b \frac{\cos^2 \phi^1}{\cos^3 \phi}.$$

Referring to Fig. 11, we that see ASE or θ becomes ϕ, and ATE or ψ becomes ϕ^1, and that $NC_2 = r_2 \sin \phi_2 = C_2T \sin \phi^1$.

and noting that—

$$C_2T = C_2B + BA + AT \text{ or } r_2 - t - AE \cot \phi'$$

and that—

$$AE \cot \phi^1 = b \tan \phi \cot \phi^1 = \mu b \frac{\cos \phi^1}{\cos \phi}$$

we find—

$$\sin \phi_2 = \frac{\sin \phi_1}{r_2} \left(r_2 - t - \mu b \frac{\cos \phi^1}{\cos \phi} \right)$$

and as—

$$BC_2P = \phi^1 - \phi_2$$

$$EP = \frac{t - r_2(1 - \cos \overline{\phi^1 - \phi_2})}{\cos \phi^1} = \frac{t - r_2 \text{ vers } \overline{\phi^1 - \phi_2}}{\cos \phi^1}.$$

Prof. S. P. Thompson remarked that graphical methods had far too much been neglected; all such attempts were worthy of attention. He had not been able to follow the author, however, and to understand where his origin was.

Mr. S. D. Chalmers said that many of them would like to discuss the paper when it was printed.

A TELESCOPE OBJECTIVE ACHROMATIZED WITH WATER

By Thomas H. Blakesley.

(*Monday, June 24th,* 1912.)

The ends in view in seeking a practical solution of the problem of achromatizing by means of water placed a variety of restrictions upon the disposable quantities.

The glass was to be employed as the objective of the telescope, as well as the lens of the collimator, of a spectroscope of which the prisms, giving spectra in opposite directions, were of fused quartz, and were to serve as an optical pyrometer in such situations as a fine annealing furnace for optical glass.

The distance between the *two* images of some bright spectrum-line, as that of mercury, moving as temperature might change, in different directions, as explained in a paper upon Constant Deviation Spectroscopes read at the Optical Convention of 1905, was to be the object observed.

Under these circumstances the lenses of the collimator and telescope would be subject to a large amount of heat radiation, which, unless some means were found to prevent it, would badly influence the optical working of both collimator and telescope.

It was thought necessary, therefore, to " irrigate " these lenses by a continuous supply of cold water flowing between them and the furnace. The water must be held up by a *plate*, for anything of the form of a lens put in its place, *i. e.* next the source of radiation, might itself produce the irregularities in refraction which it was the object to avoid.

Hence at once come in the conditions that the water lens must be the first encountered by the light, that its first surface must be plane, and its second surface coincident with that of the glass lens in contact with it.

Again: I have not found any glass which alone can be achromatized by water under the above conditions, and I very much doubt if it be possible to do so, and at the same time retain the necessary qualities of a telescope objective. A lens compounded of two different glasses has been considered necessary, but here again, to guard against the possibility of any water creeping in between the two glasses it has been thought well to make the contiguous surfaces of one curvature, and to cement them together.

Under these restrictions I have contrived to work out the details of a lens, which I have to show you, materialized in a small telescope. The glasses employed are those from the Jena List—O 919, a rather heavy silicate flint employed in the form of a meniscus in front, and O 57, a light silicate crown, in the form of a double convex behind. The optical details and the more important aberrations derived by

trigonometrical calculation I have embodied in a table which is appended. It contains results for an aperture of $\frac{1}{15}$ and $\frac{1}{10}$.

				APERTURE $\frac{1}{15}$				APERTURE $\frac{1}{10}$			
Ray.	Central Focal Length F.	Internal Central Focal Distance V_c.	External Central Focal Distance U_c.	Marginal Focal Distance V_m.	Spherical Aberration $V_c - V_m$.	Marginal Intercept f.	f – F.	Marginal Focal Distance V_m.	Spherical Aberration $V_c - V_m$.	Marginal Intercept f.	f – F.
C	1·0001078	·9911014	·9982045	·9907852	·0003162	1·0001709	·0000631	·9910224	·0000790	1·0009337	·0008259
D	1·0000000	·9909757	·9981612	·9908140	·0001617	1·0002321	·0002321	·9912567	– ·0002810	1·0012213	·0012213
F	1·0002427	·9911620	·9985149	·9914067	– ·0002447	1·0009205	·0006778	·9923590	– ·0011970	1·0025019	·0022592
				Range of V, ·0006215.				Range of V, ·0015738.			

It is evident from the construction that the telescope should be capable of being used with its objective under water, for viewing distant objects immersed in the water, so far as absorption of light allowed.

A periscope of a submarine constructed in this way even when submerged would not go out of action optically, and would probably serve to detect a strong submerged light at some considerable distance.

I have, with difficulty in the arrangement, but with success in the result, tested this for a short distance in the long clear sea-water pools at Bude, and the kindness and attention of Prof. Silvanus Thompson and Prof. Coker have enabled me to use the long water channel at the City and Guilds College at Finsbury. On that occasion with this small instrument having an aperture of only $1\frac{3}{16}$ inches, I was able to see an ordinary carbon incandescent lamp of low candle-power through $10\frac{1}{2}$ fathoms of water in perfect focus. The water in this case was undoubtedly very clear to ordinary sight, but the apparent colour of the filament was bright green. It would, therefore, appear likely that for lenses in such employment it would be better to employ the variables to obtain as small a spherical aberration, and as complete a fulfilment of the sine condition, as are possible, at some point in the green, say E, than at a point of longer wave-length, at the cost of very minute exactitude in equalizing the focal lengths over an extended region of the spectrum. Of course in such an employment there is no occasion to have the cell proper to the instrument when used in air, attached. The water will itself furnish the water lens.

As a minor matter the cell is extremely convenient to hold an aqueous solution of any light absorbing medium, with a view to observing the absorption bands produced in a spectrum, and it is possible that thus it might find a useful application in photography in colour.

ON SPECIFIC-GRAVITY BALLS

By Thomas H. Blakesley

(*Monday, June 24th*, 1912.)

Of the many quantitative scientific instruments which it is the province of the optician to supply, the simplest, and at the same time when properly used the most exact, is the Specific-Gravity Ball.

My experience of its employment extends to a quarter of a century, during which time I have made many hundreds of observations and determinations with it. It has, however, been generally neglected, and very badly described even by those who have condescended to mention it.

Mr. George Fownes, F.R.S., in his manual of Chemistry, theoretical and practical, says—

"In this manner little glass beads of known specific gravities are sometimes employed in the arts to ascertain in a rude manner the specific gravity of liquids." The figure which accompanies this passage shows the mark 830, *i. e.* the accuracy is one of three significant digits only, beyond the decimal place.

Again, Mr. Balfour Stewart, F.R.S., and Mr. Gee, in their book on Practical Physics state in writing upon the same subject:—

"They may be assorted by trial in liquids of known specific gravity, and finally adjusted by grinding. They are exceedingly delicate and will readily show a difference in the third place in the density of a liquid."

Here also an accompanying figure shows a ball marked 1·002 upon its surface.

The inference from these works would seem to be that beyond the third place of decimals, specific-gravity balls are not trustworthy means of determining densities. I shall take leave to show how false such an inference actually is.

The density of distilled water at 14·8° C. is ·999202; at 17·8° it is ·998705.

Here the temperature difference reaches 3° C., and the density difference is ·000497, or just half a unit in the third decimal place. If, therefore, the above inference were just, a ball of specific gravity ·998953—*i. e.* half way between the densities of the water at 14·8° and 17·8°—would show a doubtful behaviour in the water between the temperatures mentioned, *i. e.* over a range of 3° C. I assert that even in the hands of a tyro the range of doubt would not exceed $\frac{1}{10}$ of one degree, and with a more skilled operator $\frac{1}{30}$ of one degree.

At 15° C. a change of $\frac{1}{10}$ of a degree implies a change of density in distilled water of ·000015, and $\frac{1}{30}$ of a degree a change of ·000005. It follows that five parts in one million, or 5 in the sixth decimal place may be considered as the degree of accuracy of these instruments.

A ball, whose specific gravity at 18° C. was ·999018, was placed in distilled water

at that temperature. Hence the water density was ·998667, differing by only ·000351 from that of the ball. The ball sank at the steady rate of about one inch in five seconds, a speed greatly in excess of what would constitute a "ready indication" of the ball exceeding the liquid in density.[1] That ball was in equilibrium with the same water at 15·58°.

The vessel best suited for making accurate determinations is a vertical glass cylinder from 7 to 8 ins. in height, and from $1\frac{3}{4}$ to 2 ins. in internal diameter, capable of containing a depth of from 6 to 7 ins. of liquid, closed at the top by a perforated plate through which a thermometer can be inserted, having its elongated bulb about 3 ins. below the surface. In such a vessel the convection currents are probably very small, as the area through which heat passes to or from the atmosphere is constant for the same quantity of liquid. Any motions of the liquid due to accidental disturbances are also quickly suppressed. The ball immersed in the liquid will necessarily pass the bulb in close proximity to it, so that when the velocity is small, their temperatures are likely to be closely identical.

When the liquid has been rendered uniform in temperature by gentle stirring, and a suitable specific-gravity ball is immersed in it, the thermometer being in its place, the ball will be either "up" or "down." Suppose it is "up": the vessel is then slowly and steadily warmed. Bringing it into a room from the outside air is often sufficient.

After a time the ball will begin to fall quite gently, and in due course will pass the bulb of the thermometer, which is read at that moment. After a few minutes the process may be reversed by cooling, the mercury being made to fall and the ball to rise. The temperature of the upward passage of the ball is noted. The mean of the two temperatures is taken as the point of temperature at which the specific gravities of the ball and the liquid are identical.

The process described can be repeated as often as is thought desirable; and certainly should be so repeated if the "up" and "down" temperatures differ by ·08° C. A wide difference would be caused by hurry in heating and cooling.

When such an experiment is performed in distilled water it settles the density of the ball at the temperature of passage. Let these be A_o and t_o. The density of the ball at any other temperature t will be:—

$$A_o - (t - t_o)b$$

where b is the coefficient of expansion in volume of the glass of which the ball is made.

If, therefore, t is the temperature of passage in another liquid, the density of that liquid at this temperature is expressed by the form:—

$$A_o - (t - t_o)b.$$

[1] In the apparatus employed by the writer, the law connecting the steady velocity with the difference of density may be represented, with sufficient accuracy, by the formula $T = \frac{200}{d^{\cdot 63}}$, where T is the time in seconds of describing one inch, and d is the density difference multiplied by a million.

Further, if A_1 is the density of distilled water at this temperature—

$$(A_o - A_1) - (t - t_o)b$$

expresses the excess of density of the second liquid over that of distilled water at the temperature *t*.

For ordinary drinking waters the number will generally be expressed by significant digits in the fourth, fifth and sixth place of decimals. In mineral waters the third place commonly contains a significant digit.

It is convenient to quote this excess as parts in a million, *e. g.* a specimen of London water was found to have a density excess of 290 at 17° C., meaning that its density at 17° exceeded that of distilled water at the same temperature by ·000290.

This number is constant within the limits of error of observation for a few degrees of temperature, so that if four or five specific-gravity balls are employed at the same time, the density excess derived from each may be meaned, and the mean quoted at the mean temperature.

The coefficient of expansion of the glass can be easily found by making a weight thermometer of the material and employing it filled with some standard liquid as mercury in the usual way. Coefficients of glass have their first significant digit in the fifth place. Hence if their third significant digits are correct to one or two units this error will not affect the sixth place of the result if the range of temperature ($t - t°$) is small, as it usually is in testing for hardness, or for the effect of softening processes etc.

The following table gives some actual results in the case of a softening process—

October 15, 1890.

No. of Ball.	Equilibrium Temperature in Original Water.	Density Excess.	Equilibrium Temperature in Softened Water.	Density Excess.
1	12·52° C.	300	11·60° C.	219
2	12·52°	296	11·60°	215
3	12·75°	298	11·90°	220
4	13·50°	292·5	12·70°	212
5	15·55°	296	14·90°	212
6	16·55°	295	16·00°	221
		Mean 296·25		Mean 216·5

Result.—The process removed 80 parts in 296.

When a ball has too high a density to be standardized in distilled water, its density must be determined in some liquid which has been itself tested by specific-gravity flasks.

The accuracy will never be so high in such a case. But the ball will be just as

sensitive to *differences* of density, such as might occur in oceanic water when diluted with fresh water derived from dissolving icebergs.

At the time when Captain Scott's first Antarctic expedition was in contemplation, I endeavoured to bring this subject to the minds of those engaged in the scientific equipment of the *Discovery*. If icebergs consist of frozen fresh water their dissolution will take place in sea-water, even if the latter is below the zero point of the centigrade scale; and must result in the diffusion over the surface of the sea of a lighter material than the undiluted sea-water. How far from icebergs this effect is capable of being detected is a proper subject of scientific investigation, which in view of the disaster to the *Titanic* may be now recognized as important.

What is wanted is that a careful collection of specimens of surface water should be made and brought home for examination, the distances from visible icebergs being noted or otherwise given by the rate of steaming, other matters such as winds and currents being also noted as nearly as possible. The information afforded by the examination of such collections would be of the first importance in settling the *details* of an apparatus for *quickly* detecting danger from icebergs. But the principles of such an apparatus are neither difficult nor obscure. The thermometer must be discarded as too slow in operation, but instead, the specimens, already near the zero point, must always be brought as close as possible to one temperature. This is most conveniently accomplished by a zero bath of granulated ice and water well stirred and drained of water at a fixed rate, the granulated ice being constantly replenished. If into such a bath a series of metal vessels, of the shape and size of a test-tube, containing sea-water specimens, is plunged, after a very short interval of time the temperature inside the tubes will not differ from zero by more than a quarter of a degree C., which, with sea-water, would mean a difference in density of about ·000012.

Each tube has a window of glass about 1½ ins. from top to bottom. Inside, at the levels of the top and bottom of the window, are two grids of very open structure, so that the tube may be quickly emptied and refilled. The specific-gravity ball is in the chamber between these grids, and, therefore, cannot go out of sight, up or down, nor be lost in the emptying of the tube. The specific-gravity balls are in a series differing by ·000020. Suppose a dozen of them and that the tenth is just heavier than normal sea-water. Then with a charge of normal sea-water nine of the balls would be " up " and the three heaviest " down."

With a charge of sea-water lighter by 20 parts in 1,000,000 than normal sea-water, eight balls would be " up " and four " down." With a charge lighter than normal by 40 parts in 1,000,000, seven balls would be " up " and five " down," and so on for the rest.

As the density of oceanic water differs in various parts of the world no one ball of the series can be regarded as representing the normal sea-water over more than a limited space in a ship's voyage. The operator must determine by experiments made before sundown and when the horizon is clear of ice, which particular ball is to be considered as representative of normal sea-water during the ensuing night; or, in the case of fog, until the next clear view of the horizon is obtained.

All the tubes would be removed from the bath, emptied, refilled, and replaced at once and together by the same mechanical motion. It is improbable that this portion of the operation would take more than half a minute. The interval of rest in the zero bath, during which the motions of the water inside the tubes would be suppressed and the required approximation to zero be attained, is open to determination, and can easily be determined in any properly supplied technical institution. It may, perhaps, be a minute and a half, and if so it would be possible to make a determination every two minutes, or once in every mile for a vessel running thirty miles an hour.

The first part of this paper indicates how the unrecognized sensibility of specific-gravity balls may be employed by the " slow " method to give extraordinarily accurate results, with no great difficulty, in density determinations; and if this extreme sensibility can be utilized, by the " quick " method described in the latter paragraphs, to give indications of danger from ice, something it is hoped will have been attained in diminishing the insecurity of North Atlantic navigation.

A NEW OPTICAL METHOD OF SCALE-RECORDING FOR OPHTHALMOLOGICAL INSTRUMENTS

By John H. Sutcliffe (Secretary of the British Optical Association)

(*Monday, June 24th*, 1912.)

There are many optical instruments where some mechanical movement of the optical system causes a corresponding movement of the image seen through the apparatus. It is then possible to arrive at certain measurements by noting on a scale the altered position of the image. Unfortunately this alteration is rarely regular. An equal movement of some part of the optical system does not necessarily produce a correspondingly regular alteration of the image-position, thus necessitating an unequally divided scale.

The use of an unequally divided scale is satisfactory when it is possible for the maker to mark off each scale separately according to the optical variations of each instrument, and when having done so, one dimension only of measurement is required.

When it is not possible to ensure individual scaling, even where only one dimension is required, or where, as in the case of two dimensions, a " differential " scale is necessary, then an irregular or unequally divided scale is useless.

A differential scale is one showing the difference between the readings of two scales or two dimensions. Thus readings of 49 and 50 or of 51 and 50 would show on such a scale as readings of − 1 and + 1 respectively.

For the purposes of illustration, I show the application of the new system to such instruments as ophthalmometers.

In Fig. 1, A is an unequally divided scale rack, operated on by some movable optical system, and is so divided because the optical power of the system varies according to its position.

To produce an equal optical movement that shall appear equal to the observer, the rack travels unequally past the fixed point, G.

C is a round cogged-edge disc which rotates when A travels past the point G. Obviously the circumferential edge of C travels at exactly the same rate as A; B is a facsimile of A; D is a fascimile of C, but has a pointer instead of a scale marked edge. D is affected by B, as C is by A. D has the same diameter as C, and being behind C cannot be seen in the drawing. The rack A measures one dimension, the rack B measures the dimension at right angles.

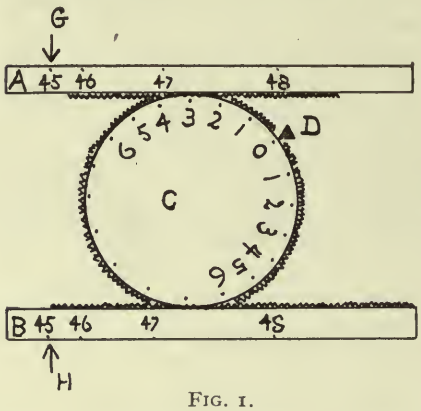


Fig. 1.

If the whole is so adjusted that when two similar figures, say 45, are opposite the fixed pointers, G and H, the index on D points to O on the disc C, then any figures opposite G and H, provided that they are similar will cause the discs to rotate equally and the index on D will continue to mark O on C, showing that there is no difference. The spacing on the disc, C, might be so arranged that when one rack was at 45, and the other at 46, the difference would show as 1, but it will be seen that the differential disc, C, would not record truly the difference between 50 and 51 on account of the unequal spacing.

A theoretical solution of these difficulties would lie apparently, instead of using rack or screw mechanism, in the employment of some cam which could compensate for any irregularities. A practical and unfortunate experience, however, with a long use of cam systems has quite convinced me that, from a manufacturing point of view, they are quite unreliable when used in delicate optical instruments; as ophthalmometric mechanical movements generally consist of a travel of 1 mm. for each dioptre, the least inaccuracy in a cam is followed by fatal results.

With all the various ophthalmometric optical systems there are certain irregularities. These irregularities are due either to—

(*a*) The optical laws governing the movement of lens systems; or—
(*b*) To the difficulty of securing the exact duplication in a second instrument of the original one on which the scale was founded; or—
(*c*) To actual imperfections in the commercial production of the lens system.

They may be due to any or all of these. Our object, therefore, is to find some

method whereby with no matter how irregular an optical movement, we can secure an equally divided scale, and that without having recourse to any mechanical contrivance such as a cam, which is unsatisfactory for the reason I have given.

My suggestion is that all the present mechanical scales shall be dispensed with, and in their place be substituted the suggested new system. Upon this point I wish to make myself quite clear, as it is possible confusion may arise as to why the introduction of another system should be necessary. The new system is not an auxiliary but dispenses entirely with all outwardly visible scales, and depends absolutely for its readings upon what is seen through the eye-piece. The optical system may be regular or irregular in its progression, there may be imperfections of lens workmanship, or it may possess mechanical faults such as rack-work, loss-of-time, etc.; but whatever the result is, so long as that final result can be seen by the observer, it becomes possible to have a correct record on an equally divided scale. An observer, therefore, when examining a patient arrives at his results and readings without removing his eye from the eye-piece.

The system also has this advantage that makes it possible to convert into one-position keratometers all ophthalmometers which before have been used only as double-position instruments.

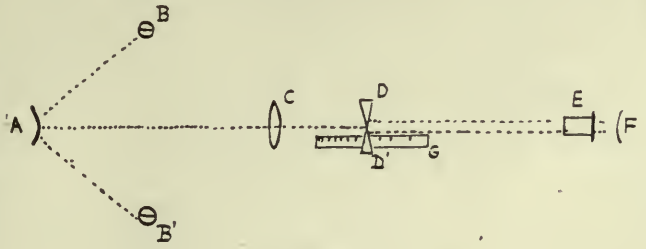


FIG. 2.

The system consists solely of—

(*a*) A fixed illuminated vernier;
(*b*) A pair of right-angled prisms set at an angle of $22\frac{1}{2}°$;
(*c*) An equal-graded micrometer scale in the eye-piece.

Let us take for application the Landolt-Chambers-Inskeep pattern. Fig. 2 shows how a corneal image, A, of the two mires, B and B', is viewed through a telescope objective, C; a pair of prisms bases or apices together, D and D'; and an eyepiece, E, producing a double image of the original corneal image in the eye of the observer at F. The result is read on a scale (simplified) at G. The essential feature of this optical system is that these Rochon or Fresnel prisms shall be in non-parallel light, otherwise their movement would have no effect on the separation and approximation of the images. The distance of the prisms from the objective regulates the approximation of the images. For any two approximations of the images, however, the travel is unequal, and when to that is added the difficulty of calibration of the prisms, it becomes quite impossible to secure an equally spaced travel and consequently differential scale. Suppose, however, we dispense with the scale at G,

substituting instead the following system. In Fig. 3, K is a fixed target or index-finger illuminated by the mire lamp, B; C is an objective; L a pair of right-angled prisms set at 22½°; and M a transparent scale set at the stop of the Huyghenian eye-piece. The eye-piece is focused to suit the refraction of the observer by unscrewing in and out the eye-lens. The prisms are fixed on a swinging joint so that when in position, they obstruct the view of the cornea and the index K is seen by the operator. When swung out of position, the image on the cornea of the patient only can be seen.

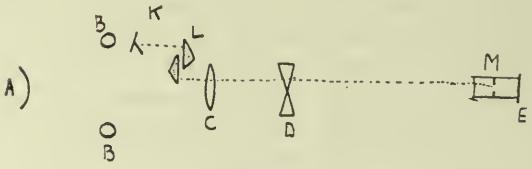


Fig. 3.

Fig. 7 gives the verniers or index-fingers; the central line is a differential mark, and shows the amount of astigmatism by travelling across the fixed transparent micrometer-scale in the eye-piece; the other two lines at right angles to each

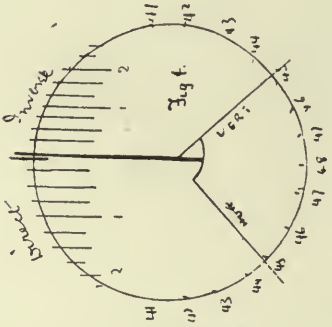


Fig. 4.

Fig. 7.

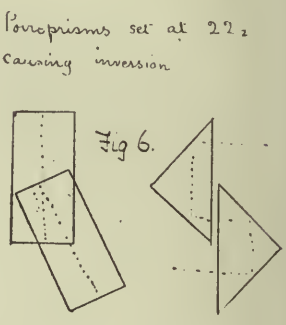


Fig. 6.

other mark the corneal curvatures at the two axes. This central index is set fixed at 45° or diagonally, so that of the other two lines one marks the vertical curvature travelling vertically with its own directing prisms, and the other notes the horizontal curvature.

Fig. 6 shows two right-angled prisms set at 22½°; this has the effect of causing all vertical and horizontal lines to appear as if at a diagonal axis, and inversely with a diagonal line. The same effect applies to all movements seen through the right-angled prisms. When the deviating approximating prisms cause a vertical or horizontal movement, the result is an apparently diagonal movement. The effect, therefore, is that in an instrument such as a keratometer, where deviating prisms are used to secure movement simultaneously in two axes, the diagonal differential line appears to travel horizontally across the field, and the operator is looking at and measuring with what is apparently a vertical line instead of a diagonal one. The

reason for setting the right-angled prisms at this particular angle to secure this effect is that I have found that it is an extremely difficult matter for the majority of observers to take accurate measurements by means of a diagonal scale, or with other than a vertical or horizontal ruling.

This will be readily apparent to any microscopist using an eye-piece at any but a vertical or a horizontal axis. The general tendency is to twist the head round and so very often impair the results.

The cause I put down to the general and almost universal slight uncorrected astigmatism; in those rare cases where the observer has absolutely no astigmatism, to the natural desire for the vertical and horizontal line as opposed to the diagonal. It may seem a small matter to any one who has not given it much attention, very possibly because it is not the custom to have diagonal movements or scales in optical instruments.

In a differential keratometer, however, a diagonal result and movement is unavoidable, hence the above method of overcoming it.

Viewed through the two $22\frac{1}{2}°$ prisms, the scale in Fig. 7 appears as at Fig. 5.

Fig. 4 is the micrometer scale in the eye-piece stop. The figures on the periphery of the circle represent the corneal curvatures in dioptres, the vertical rulings show the difference "with or against the rule" of the two curvatures. It will be seen that if the right and left hand index marks pointed to similar figures on opposite sides of the circle the centre line would point to O, showing that as the curvature was equal obviously there was no astigmatism.

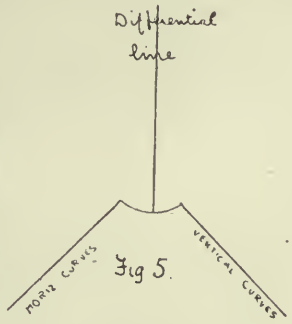


Fig. 5.

If now, this optical system shown in Fig. 3 be fitted to such an instrument as seen in Fig. 2, with of course all outward scales such as G removed, the effect is that when the $22\frac{1}{2}°$ prisms are swung out of view, the operator takes his measurements in the ordinary way by racking in or out the longitudinal prisms at D, with this difference, that he is now able to have an extra pair of prisms at the opposite axis, thus converting the instrument into a keratometer.

Having moved his doubling prisms so that his images are in contact in both meridians, instead of seeking his results by examining any outward scales or dials, he swings into position the $22\frac{1}{2}°$ prisms. The original corneal images disappear, leaving in place an image of the fixed index, K.

Obviously whatever movement or alteration in position may have been effected upon the corneal image by the deviating prisms, must take place also with the index line, inasmuch as it is also acted upon by the deviating prisms. If, therefore, the deviating prisms are irregular either in their manufacture or in their optical progression, an exactly similar compensating effect takes place with the position of

the index finger on the micrometer scale in the eye-piece. This method can be applied to all systems of ophthalmometry.

In the Sutcliffe split-cylinder No. 4 model shown to you to-day new methods of lens manufacture have made equality quite feasible within reasonable limits. Up to certain degrees of astigmatism, the split-cylinder system allows of sufficiently regular travel so as to produce a regular spacing on the differential scale. The application of the suggested optical recording to this type of instrument is scarcely necessary except for laboratory work, or as a matter of economy, and where it is more convenient to take one's readings without removing the observer's eye from the eye-piece.

It might be adapted to the Javal-Schiotz pattern for similar reasons. In this case, the travelling mire would act as the index; whatever position the index took up would be recorded in the eye-piece without passing through the Wollaston prisms.

Its utility will be most apparent in instruments of the Landolt type, a class of instrument undoubtedly having certain optical advantages, notably of clearness of image and alignment of axis.

The advantages of the system are chiefly the convenience of reading one's results without removing the eye, the result being *in situ* even after the patient has moved, the independence of eccentricities in the optical system, and an equally divided scale, making a differential scale possible.

Mr. S. D. Chalmers called attention to the possibility of an optical illusion, in judging whether a mire of the type shown by Mr. Sutcliffe was quite circular.

Mr. Tomlinson inquired whether the correction would be used to calibrate the mechanical scale, or whether it was used for each reading.

Mr. Sutcliffe replied that this correction was applied automatically in the reading.

THE OPHTHALMIC SIGNIFICANCE OF FACIAL ASYMMETRY

By Harry L. Taylor

(*Monday, June 24th,* 1912.)

My object in reading a paper upon the subject of Facial Asymmetry is twofold: firstly, to put before you some observations which I have from time to time made; and secondly, to ask you as opticians to give a little more attention to the subject. It is within my knowledge that there are many who have thousands of cases annually where various facial measurements are taken. These details are doubtless down in their record books, and yet one feels that we have no collated information upon such matters as correlation and correlated variation.

These terms bring me at once to a part of my subject. They are used by physiologists to denote the general coexistence of two attributes in an animal, for which no good reason can be given. For instance, it is curious that nearly all white male cats with blue eyes are deaf, and that people who have strongly projecting supraorbital ridges have usually great muscular development. Whether in each case the one characteristic governs the other, or whether both are governed by a common factor we cannot at present say.

To the uninitiated the coexistence of the pink eye of the albino with his almost colourless hair would be a problem of this kind. It is to us one of the solved problems of correlation.

Javal has given us several cases of correlation which are interesting. He notes, for instance, that in the Jewish race astigmatism against the rule is more frequent than with the rule. Again, he says that in most people the position of the meridian of minimum curvature coincides, in cases of high astigmatism, with that of the palpebral slit; it runs upwards to the temporal side in eyes like those of the Chinese, and downwards in cases where the large angle of the lids is lower than the caruncle. We do not yet know why astigmatism against the rule should exist in certain races, but there does seem a reason for the other examples given.

Is facial asymmetry correlated with ocular defects, and if so what is the significance in each case for the optician?

I purpose dealing with asymmetry of two quite different kinds. The one is caused by a displacement due to growth, and which is, in this country and amongst educated people, fairly general. It is the unequal position of the two eyes measured from the median line of the nose. The other form varies greatly, and is caused by the special exercise of certain groups of muscles, being evidenced by wrinkles or folds in various parts of the skin about the orbits.

That the eyes may move asymmetrically during growth, in obedience to the demands of the organism, is evidenced clearly in some of the lower animals, noticeably in the pleuronectidæ, or flat fishes. When young these animals are quite symmetrical, and have one eye on each side of the head. They swim in a vertical position, but get a habit of lying upon either one side or the other. Whichever is undermost has the corresponding eye downwards, and the organ gradually forces its way through the head, appearing in the adult alongside its fellow.

My experience is that quite eighty per cent. of educated people have the right eye further removed from the median line than the left, and consequently the right visual axis is always more outwards than its fellow. I do not think that this obtains to a like extent amongst the rural population, but here we require further information.

It is not a difficult matter to verify this in taking interpupillary measurements. One of the simplest demonstrations is to place a spectacle frame upon the face, noting that the bridge fits well, and then direct the subject to gaze into the observer's eyes, each keeping only the opposed eyes open, right to left, and left to right in turn. The differences in the positions occupied by the pupils within the eye of the spectacle frame are quite sufficient to be noticeable in most cases.

For actual measurement other and more accurate methods must be used. I will describe two of these—the one objective, and the other mainly subjective. For the first a type of frame very well known is employed, in which the nose is fitted by a bridge over a part which is very convenient for the purpose, viz., that where the bridge of a spectacle usually rests. The arms of the frame are furnished with pointers capable of movement by a screw, so that knobs upon the ends may be brought in front of the subject's pupils when viewed in turn by the opposed eye of the observer. A scale upon the arms records the interpupillary distance, and as both pointers travel concurrently, then, by taking observations by each eye in turn the semi-interpupillary distances may be estimated.

There are several sources of error with this device. The principal one is due to the size of the travelling knobs and the variations in size of the pupil, and there are also minor ones, such as the probable differences in centres of observer and observed.

By the second method which I have referred to we obtain much greater accuracy, with considerably more information of service to the optician, although, as it is partly subjective, there is often some difficulty in obtaining satisfactory results with many people. In principle it is the use of binocular pinhole discs, and in practice it is an adaptation of the Dixey interpupillary measure, or as I would prefer to call it, the interaxial measure, because in reality it measures the distance between the visual axes of the eyes, for any given convergence and in the plane in which the measure is situated.

The particular apparatus consists of a series of black celluloid discs, each pierced by two pinholes at certain distances, say from 53 to 64 mms. Midway between the pinholes is a pointed aperture, so arranged that a line uniting the holes would pass about half way through its length, the opening being large enough to allow each scale to be placed over the nose, so that the eyes may have vision through the pinholes simultaneously when the particular scale suitable for the interaxial distance is used. Here is where trouble generally arises, for both pinholes must appear as one when used before the two eyes, and care is necessary in selecting the particular scale to give this effect.

When the eye looks through the pinhole the circle of light seen is in reality an image of the margin of the pupil, as is well known, and observation will show variations in size as the iritic aperture enlarges and contrasts. To ensure that both eyes have vision through the pinholes they should be alternately closed, or the scale may be tilted a trifle when one disc of light will appear above the other. This latter method enables great accuracy to be obtained, because, unless the holes correspond exactly to the visual axes there will be a slight lateral displacement of the one disc.

Presuming that the correct scale has been chosen the subject places it in front of his eyes, and views an object at a considerable distance, the observer placing his head below and in front, in order to see what position the pointed apex of the nose aperture occupies. This will generally be to the right of the middle of the nasal crest.

FIG. 1.—Illustration showing complete Asymmetry of the two sides of the face, in this instance the left eye being horizontally displaced.

FIG. 3.—Examples of Asymmetry caused by the unequal use of groups of muscles upon the two sides of the face.

[*To face page* 248.

It is well for the subject to select also a scale adapted for the eyes when used at the working or reading distance, so that the difference in the inter-visual-axial measurements may be estimated as between "far" and "near" vision, and it will be found that the amount allowed in frame fitting or lens decentration is insufficient as usually advised.

The amount of ocular asymmetry as shown by using the interaxial scales is considerable, in the majority of adults being from 2 to 3 mms.; and, as I have already stated, consisting in a removal of the right eye by that much in excess of the left. With children use of the device is almost impossible, and the less accurate methods have to be employed; but, from what I have observed, asymmetry in position is somewhat rare, and it appears to develop during school life.

In connection with these conclusions we have the existence of inequality in the facility with which we use the two eyes, causing the one to become the directing eye and predominant partner in vision. In the majority of people this will also be the right eye, several experiments enabling this to be determined with certainty. Perhaps the most conclusive consists in stretching the arm fully out in front of the head and pointing with the finger at an object, keeping both eyes open, just as if aim were being taken. The eyes are now closed in turn, and it will be found, of course, that the finger is only pointing for one eye, a decided movement being needed if it is to point for the other.

Again, if we hold a sheet of coloured paper or book in front of the face, at about twelve to fourteen inches' distance, so that it may cover each eye in turn when a distant object is viewed only by the uncovered eye, we shall have little difficulty in suppressing the image of the book when the directing eye is viewing the distant object, but considerably more trouble when the directing eye is covered and the following eye is gazing at the object. Indeed, in the one case the only effect is to tint a portion of the field corresponding to the outline of the cover as seen by the following eye, and even this may be unnoticed unless the colour is bright or the experiment is made soon after waking in the morning. Many similar experiments show a like result.

There seems to be little doubt but what the displacement of the right eye bodily in the majority of cases is due mainly to the greater use of the organ, and it is worthy of note to what an extent work, books, manuscripts, etc., are placed before the body upon the right-hand side. Sometimes this displacement is shared by the whole of that side of the face, which seems to be broadened at the expense of length, the two halves appearing quite different when compared (Fig. 1).

How are these cases of asymmetry to be fitted so that the lenses shall give the best results? As the right eye is generally the directing eye it is very essential that the visual axis shall pass through the optical centre of the lens placed in front of it, but the provision of the usual symmetrical frame giving this result means that the left eye-wire seems to project too far upon the left side; and generally the optician notices this more than if the reverse were the case, in which the left is correctly centred and the right has not its full allowance. The method of apposition and

alternate viewing of the two eyes will prove how many persons fitted symmetrically are not so well fitted as one could wish. The remedy is obviously to put a longer shank to the right side of a spectacle bridge than that on the left, and in case the client objects to this procedure then fairly large eye-wires should be put upon a symmetrical frame, and the lenses decentred accordingly. Similarly *pince-nez* and clips may be adjusted, so as to give greater satisfaction in the correction of vision.

There are many cases which are more important than where only spherical lenses are worn, notably those of astigmatism with oblique axes of cylinders, even the direction of the obliquity having an influence upon the arrangement of the frame, because when convergence comes into play a cylinder with axis down and inwards towards the nose is more convenient than one which runs down and outward away from it.

Bifocal lenses also call for attention so far as the position of the reading segment is concerned, for with a symmetrical frame upon an asymmetrical face the right eye is usually very inefficiently provided for, especially when the wafers or segments are small.

My experience is that even when the left eye has a slightly better acuity than the right it yields to it in predominance, but it seems quite possible that when equal the inherited lead assumed by the latter usually gives it better acuity by use, and also perhaps because the left eye may be subject to greater detrimental influences, from having to follow such lead.

Turning now to the second forms of asymmetry caused by the unequal use of certain muscles, or groups of muscles, upon the two sides of the face, it is necessary to have some slight knowledge of these in order to understand the manifestations of their action. Such muscles are very intimately connected with the skin, and their contraction causes wrinkles or folds in the integument which give to some features their individuality, and which always convey to us certain ideas of expression, such as surprise, annoyance and the like.

Running perpendicularly over the whole of the forehead is a broad band known as the occipito-frontalis or frontal muscle (F), and contraction of this naturally causes the appearance of the long lines of more or less horizontal wrinkles so often seen. Almost at right angles to this, from above the nose on each side runs a lateral band over the eyebrows called the corrugator supercilii (C), a muscle which is intimately concerned in frowning, and which aids in producing what we are accustomed to term a " supercilious " appearance (Fig. 2).

These two muscles are antagonistic, inasmuch as when exercised separately the wrinkles caused by one disappear, so that in frowning we get a series of vertical wrinkles above the nose, caused by contraction of the corrugators, and a lessening or disappearance of the horizontal wrinkles due to the frontal muscle.

A little lower down we find a triangular band of muscle passing downwards over the crest of the nose, called the pyramidalis nasi (P), which, when it contracts, pulls the eyebrows and forehead still lower than the frown due to the corrugator, and causes the appearance of short transverse wrinkles across the top of the nose.

Then we have the ring-like sphincter, called the orbicularis palpebrarum (O), which quite surrounds the eye, and by the agency of which the lids are closed. In frowning it plays a part with the corrugator, and appears to act more strongly in closing the eye when it follows a contraction of the corrugator.

Below the eye and running under the skin of the cheek are three bands of muscle which connect this with the upper lip. The one starting from near the nose is the levator labii superioris (LS); the second, coursing under the middle of the cheek, is the levator labii proprius (LP); and the end one, starting from the outer corner of the eye, is called the zygomatic (Z). They all act in raising the upper lip, and their connection with the present subject is seen when we consider how often some ocular defect is associated with more or less screwing up of the lids and a partial opening of the mouth, which is often more or less asymmetrical.

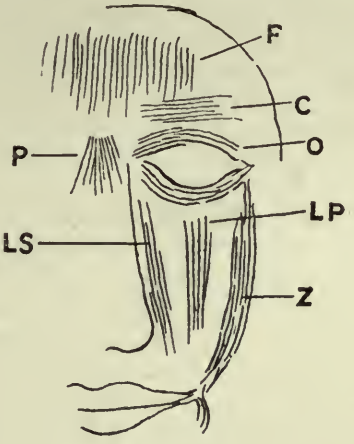


FIG. 2.

As these muscles are so largely used in expression of the features there is always the probability of asymmetry of the face caused by their action, indicating some difference between the condition of the two eyes, because we know that it is quite easy by practice to use certain groups upon the one side in excess of, or independently of, those of the other. If evidence of this is necessary one only needs to consider the cases which are often seen where some disfigurement, due to squint, etc., is partially concealed by a habit of closing the eye which has the trouble referred to. The difficulty in these cases is to dissociate the corresponding groups upon the two sides, because there is a decided and combined action in the group which closes each eye; and even during strong closure this may be involuntary, for Donders found that during violent respiratory efforts, such as screaming, coughing, sneezing, etc., there is a definite purpose in external pressure upon the globe, in order that the pressure of blood in the vessels shall not damage the fine capillaries of the retina and other membranes.

The greatest caution, however, is necessary in drawing conclusions from the vagaries of expression, and also in much that follows, because there are other causes which operate in causing asymmetry. The greatest of these is probably anatomical, the muscles not corresponding in their insertion, or the directions of their pull being dissimilar. Again, habit induced by quite different causes may be responsible, and this is why some of the matters which I put before you must be regarded as suggestions which require the evidence of a great number of careful observations before they become conclusions.

There is one thing very noticeable in asymmetry of this nature. It is rarely found in young children, but seems to develop during school life. There is, on the

other hand, nearly always some when man passes the prime of life. Dividing the population socially we can easily see that the lower classes and ignorant persons will show it more readily than the more favoured in position and bearing, because of the freer play given to expression.

A common form of muscular action causing various degrees of lid closure and pressure, which may be asymmetrical, is made particularly evident by downward movements of the eyebrows. There are three purposes to be served by such. Where the brows are placed in a position simulating frowning, it is often strong light which induces the movement, the eyebrows being lowered in an endeavour to cut off light which comes from above, and which blurrs the retinal image. Such contraction causes more or less vertical furrows above the nose, and it would appear that asymmetry of position of the eyes often influences the position of these.

Light which causes partial closure of the lids is from the excessive illumination of objects generally, and the state passes off when the glare is reduced, eyes which have more pigment in their irides faring much better than those of lighter hue, such as the blues and greys.

The production of a stenopaic aperture by the lids under all conditions is, as doubtless you are aware, often symptomatic of astigmatism, but this is sometimes accompanied by greater pressure upon the eyeball, as shown by one or other upcurved extremity of the partly opened mouth. This pressure may be demonstrated by first tightly closing the mouth, contracting the muscles around the eyes and then relaxing the lips. Pressure upon the eyeballs is immediately increased, and it is possible that the grin commonly seen with some myopes may be due to an effort to obtain better vision by a slight modification in the form of the globe.

Anisometropia is a very common cause of asymmetry, one particular class nearly always leaving some tell-tale marks upon the features. I refer to those who are myopic in one eye to such an extent that they can always read with it without the aid of a lens, holding the book at a distance indicated by the amount of their myopia. The other eye is used for distant vision, and although rarely possessing normal acuity, satisfies the individual. Position of the head to one side during close work, and the ordinary partial closure of the lids in an endeavour to get a stenopaic effect, always causes considerable asymmetry.

There are many cases of asymmetry, manifested by tell-tale wrinkles and position of the eyebrows, which are caused by the carriage of the head or by a posture assumed as a result of ametropia, and more especially caused by astigmatism or anisometropia. One of the most common of these is seen in elderly people who have their presbyopia under-corrected, and who in an endeavour to hold the book or paper at a distance which will allow them to read distinctly, throw the head back, a contraction of the frontal muscle arching the eyebrows and forcing the forehead into longitudinal folds, while the orbicularis partially closes one or both eyes in an effort to correct the astigmatism which is often present, the eyes being directed downwards upon the type. The natural fatigue of the effort causes the reader to entirely close the lids at times, when

the frontal muscle relaxes, but in nearly every case a habit is formed in which it is partially contracted, and the signs remain.

A somewhat similar carriage of the head is often seen in myopes who walk with the face tilted upwards and often a little aside, a habit which grows with the development of the myopia, and an endeavour to obtain better definition of the vertical components of objects seen by tilting the head sideways, an action caused by astigmatism. An alteration of the position of the visual axis by myopic development of the eye has been put forward to account for the backward inclination of the head.

Savage's experiment of the double prism before one eye, when both are viewing a horizontal arrow, showing the presence of torsion in some eyes, may also account for asymmetry of the face caused by the strain which must be developed if perfect single vision is maintained. This experiment is readily performed by putting a double prism before one eye, so that a horizontal arrow is seen doubled, the other eye seeing it singly between the two images. If the middle arrow is not parallel with the others then torsion exists.

In conclusion, I would ask you when making records to give particular attention to facial characteristics accompanying four types of ametropia :—

I.—General ametropia, with considerable displacement of one eye.

II.—Anisometropia, simple or combined with astigmatism.

III.—Astigmatism, where the axes of cylinders before the two eyes are at right angles, V and H respectively.

IV.—Astigmatism with oblique axes of cylinders.

ERRORS OF OBSERVATION

By T. Y. Baker, Naval Instructor, R.N., and G. B. Bryan, D.Sc.

(*Tuesday, June 25th*, 1912.)

The object of the authors of this paper has been to determine, by direct experiment, the accuracy of which the eye is capable when making the adjustments necessary during the use of certain instruments of precision. For example, when using a sextant with an artificial horizon the observer has to make contact between the edges of the two images of the sun as seen in the telescope. An error of some sort will necessarily be introduced, as the observer will not be able to do this with absolute accuracy; either the two suns will overlap at the instant of supposed contact or will

be separated, and the error can be expressed most easily as an angular value as seen by the observer's eye. All other measuring instruments will be subject in their measured results to errors of the same description, and in every case it is convenient to denote this error as an angle.

As far as we have been able to ascertain, no systematic set of tests of ocular accuracy has been carried out; and there seems to be a wide diversity of opinion among writers on optical subjects, as to what the approximate value of such accuracy is in a normal-sighted person, the value being given variously as from three minutes to a few seconds.

It is hardly to be expected that the error will be the same for all types of adjustment, and consequently we have endeavoured to obtain results by making tests of a number of different observers, at adjustments that can be divided into three separate classes.

(i) *Coincidence Adjustments*, in which the object of the observer is to set the parts of a broken line so as to form a continuous line. This type of adjustment occurs mainly in range-finders and also setting a vernier to a particular reading.

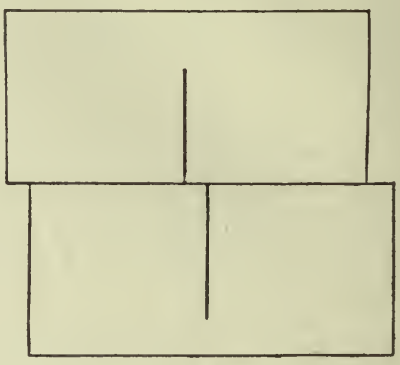


Fig. 1.

(ii) *Contact Adjustments*, in which the object to be attained is the setting of one object in exact contact with another, as in the case of the sextant quoted previously.

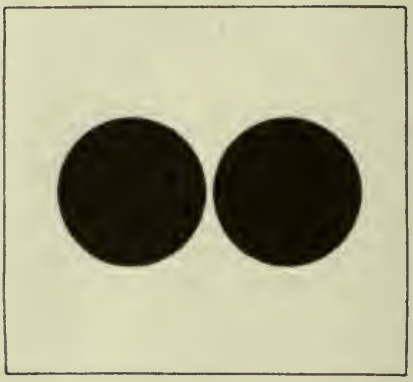


Fig. 2.

(iii) *Bisecting Adjustments*, necessitating the setting of one object, usually a line, exactly half way between two other lines.

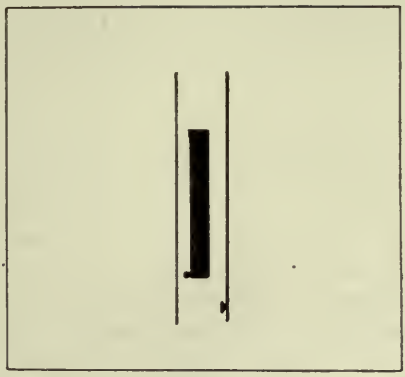


Fig. 3.

Description of the Apparatus.—In making all these tests the observer has been placed some distance from the apparatus carrying the objects that have to be placed in adjustment, and he has endeavoured to set the adjustment as accurately as he could, after which the amount of the error was measured on the target and the result reduced to the angular value subtended at the observer's eye. The apparatus for carrying the targets consists of a rectangular framework, to the top horizontal bar of which one target can be fixed, and to the lower one the counterpart of the target.

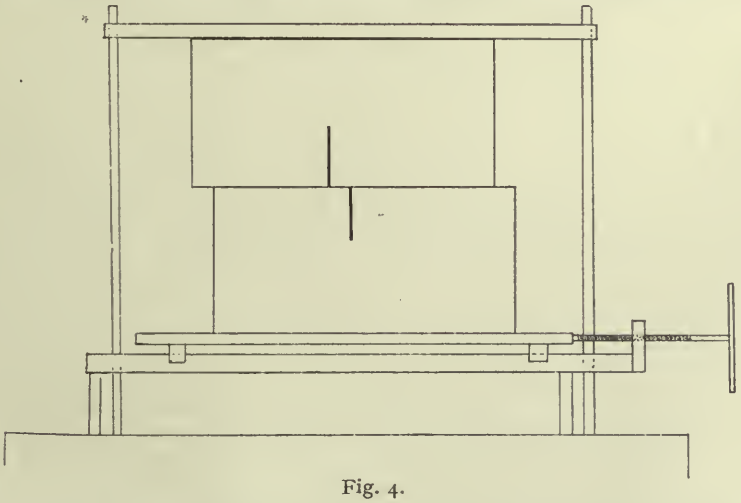


Fig. 4.

The lower bar can be traversed to the right or left by means of a micrometer screw worked by a small motor, which is under the control of the observer. The upper part of the target being fixed, and the lower part movable, the observer has to run the lower one into what he considers to be correct adjustment by means of the motor, and the reading of the apparatus can be made from the micrometer screw. The

zero or true adjustment is made before and after the experiment, setting the marks in adjustment under a fairly high magnifying power. The zero error varied according to the nature of the target; with the coincidence adjustments it was (at the distance of observation, 33 feet) less than half a second, with some of the others it was about one second.

The targets in use were of various descriptions, according to the nature of the adjustment that was being made, some of them white cards marked in black, some photographic plates, and some models of various descriptions.

Considerable difficulty was experienced in the lighting of the targets. We were endeavouring in all cases to simulate the adjustment that has to be made in some particular instrument, and it was found by no means easy to reproduce the exact appearance when the objects to be brought into adjustment were drawn on white cardboard, instead of being seen in the field of view of a telescope. We cannot, consequently, be certain that this difference in appearance between the artificial objects we used, and those that occur in the various instruments themselves, may not make our results slightly inaccurate. All the persons tested stated that the adjustment made on our targets were more difficult to make than in the instruments themselves; so that, on the whole, one may assume that the errors we have obtained are, if anything, larger than would be obtained with the instruments.

Details of Tests

List of Observers.—In all, observations were made by thirty-eight different persons. Numbers O. 1 to O. 9 are members of the Staff of the R. N. College, Greenwich, and of them O. 1, O. 2, O. 3, O. 4, O. 6, O. 7 have had considerable experience in making observations of the types included in the experiments. The others have no special experience in this respect.

O. 10 to O. 17 are gunnery lieutenants studying at the college. They have all had considerable practice with range-finders and sextants. The same remark applies in a less degree to the torpedo lieutenants—O. 18 to O. 21.

Four officers of the Royal Indian Marine—O. 22 to O. 25—have all been at sea for some years, and are skilled observers with a sextant, but have not had much experience of other types of observations. These officers have done all their sea-service in Indian waters.

O. 26 to O. 32 are second lieutenants of the Royal Marines. These officers had only just entered the service and had had no experience in the use of any instruments of precision. The same remarks apply to O. 33 to O. 36—students of naval architecture; O. 37 and O. 38 are laboratory assistants.

Method of Making Tests.—The observer being tested was seated at a distance of 33 feet from the target and ran the motor one way or the other until the two parts of the target seemed to him to be in correct adjustment. The reading of the micrometer was then taken and the whole process repeated ten times. From the ten errors of observation so measured four separate values have been tabulated: (i) the mean error irrespective of the sign; (ii) the algebraic mean; (3) the maximum error, and (iv) the mean square or probable error.

Results of Tests

Coincidence Adjustments.

1. Black line on each half of the card at right angles to the dividing line—

(*a*) Narrow line about 2 mm. broad; and
(*b*) Broader line about 1 cm. broad.
(*c*) Broader line, using both eyes.

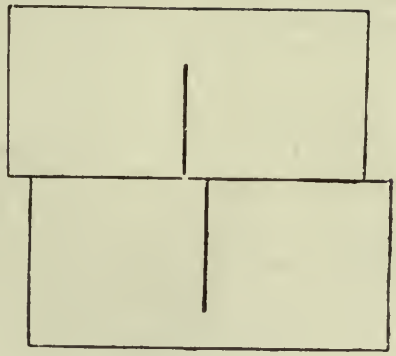


Fig. 5.

1 *a*

Observer.	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2	6·3″	2·4″	17·0″	7·6″
O. 33	30·4″	30·4″	51·0″	32·6″
O. 34	7·3″	6·5″	16·0″	9·1″
O. 37	24·9″	22·6″	53·8″	31·2″
O. 3	11·4″	1·7″	30·7″	15·0″
O. 1	7·1″	3·4″	13·9″	8·4″

1 *b*

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 26	11·1″	11·1″	17·7″	12·3″
O. 27	5·5″	5·4″	8·7″	6·2″
O. 28	4·4″	3·3″	10·5″	5·1″
O. 2	12·4″	9·5″	25·1″	14·7″
O. 2	10·4″	6·5″	23·4″	12·3″

1 *c*

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 33	9·1″	6·3″	15·0″	9·8″
O. 34	5·7″	3·1″	14·0″	7·3″
O. 8	5·7″	5·7″	13·3″	7·3″
O. 29	9·9″	2·7″	17·4″	11·4″
O. 22	10·2″	2·9″	16·0″	10·9″
O. 23	4·3″	2·9″	15·0″	5·3″
O. 4	7·0″	6·2″	14·3″	8·1″
O. 35	3·7″	0·8″	8·7″	4·4″

The zero error with these targets was in all cases less than half a second.
In all these three tests several observers complained that their eyes very quickly

got tired, and it was noticed that the errors immediately increased when this was the case, and that in most cases a subsequent improvement took place. This fatigue of the eye was probably due to the lighting of the target, which was a white card with a black line upon it, the whole being illuminated by reflected light from incandescent lamps placed in front of it.

(1*a*) O. 33 is astigmatic and had considerable difficulty in seeing the line at all. O. 37 and O. 3 complained of eye-strain.

(1 *c*) It will be seen that O. 33 using both eyes obtains much better results.

On the whole the broad line seems to give a more accurate method of setting than a narrow one, and it seems justifiable to assume that if the observer can use two eyes on a suitable mark the coincidence error with a trained observer should be under 10″ for a single observation.

It is doubtful how far these figures could be applied to the accuracy of reading a vernier. The latter possesses an advantage in that the observer is able to compare the appearance of the succession marks with one another, but on the other hand, the lines are seldom so sharply defined as in the case of those used in these experiments.

2. Black line with a short piece broken out of the middle—

(*a*) Narrow line;
(*b*) Broad line;

using one eye only in each case.

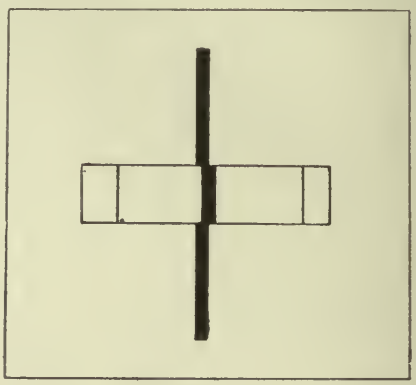


Fig. 6.

2 a

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2	3·2″	1·0″	8·3″	4·0″
O. 1	9·5″	8·0″	17·4″	11·8″
O. 4	8·9″	8·9″	21·0″	10·5″
O. 10	6·7″	3·7″	17·0″	8·1″
O. 11	7·9″	7·2″	11·7″	8·6″
O. 12	6·3″	0·8″	13·4″	7·5″
Q. 13	7·3″	4·2″	16·2″	9·2″
O. 30	7·5″	4·7″	14·3″	8·5″
O. 31	6·6″	4·1″	12·0″	7·3″
O. 32	6·3″	2·4″	16·0″	7·2″
O. 36	16·1″	14·0″	27·6″	18·4″

2 b

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 10	5·9″	5·2″	10·5″	6·9″
O. 11	1·8″	0·3″	4·9″	2·2″
O. 12	7·6″	5·8″	13·3″	8·3″
O. 13	4·9″	4·0″	12·2″	5·9″
O. 30	4·5″	0 ″	11·0″	5·3″
O. 31	8·7″	5·4″	20·5″	11·1″
O. 32	5·7″	5·2″	9·6″	6·4″

These observations furnish better results than the single line coincidence experiments. The same complaint of the difficulty of seeing the target after a few observations was made by O. 10, O. 11 and O. 12, who all made their worst observations at the time of complaint and subsequently improved.

An interesting point in these results is that the four gunnery lieutenants, O. 10, O. 11, O. 12, O. 13, made greater errors than the three second lieutenants Royal Marines, although the latter are quite untrained observers.

The gunnery lieutenants' training, however, has not been with observations of this description.

No experiments were made using both eyes, but it is probable that better results still would have been obtained, and that with a sufficiently broad line a trained observer will consistently make observations with errors of less than 8″.

3. Bright line on black background. Made by each half of the target carrying a vertical slit which was illuminated by transmitted light. This corresponds to a range-finder used at night-time with the stigmatizing lens.

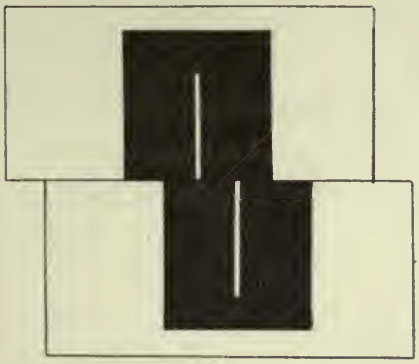


Fig. 7.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2	6·0″	1·8″	14·1″	7·4″
O. 3	7·3″	5·3″	17·1″	8·5″
O. 3	6·0″	2·8″	14·4″	8·0″
O. 1	5·4″	0·6″	18·0″	7·1″
O. 4	4·0″	0·6″	8·0″	5·0″
O. 3	7·2″	1·4″	20·9″	9·2″
O. 11	8·3″	4·9″	21·8″	10·5″
O. 14	5·5″	0 ″	12·6″	7·1″
O. 15	10·5″	10·5″	24·8″	11·2″
O. 16	7·8″	5·6″	24·1″	10·7″
O. 17	10·2″	8·1″	24·8″	13·1″

The ten observations made by O. 4 were carried out rapidly, running the two marks together first from one side and then from the other, and in each case stopping the motor when coincidence seemed to be obtained and making no further adjustment. The ten separate errors of this observer were 3·1, 3·5, 0, 6·6, 7·6, 2·1, 0, 7·3, 8·0, 2·1 seconds.

This list of errors is much better than those obtained by any other observer and seems to indicate that better results are obtained if observations are made quickly.

Contact Adjustments.

1. Contact between two equal circles—

Bright circles on dark background.

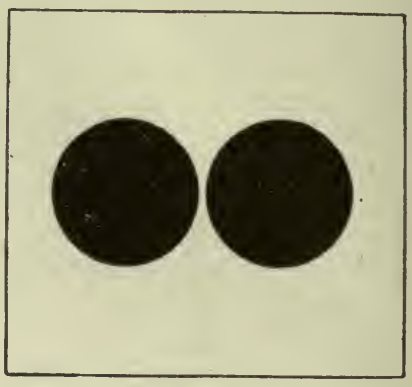


FIG. 8.

(*a*) Two bright circles each subtending an angle of about 40′ at the observer's eye. The observations were made by running the motor from a position for which the two circles overlapped and stopping it at the apparent instant of contact, *i. e.* when the small, extra bright overlapping portion was no longer visible. In this and in certain other experiments of a similar type allowance had to be made for the amount that the motor overran; it being found that this amount could be determined by experiment with about the same degree of accuracy as the zero error. A + sign prefixed to an error signifies that the circles were separated, and a − sign that they overlapped.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 3	4″	+ 2·7″	8·0″	4·6″
O. 7	2·4″	+ 0·9″	4·2″	2·7″
O. 24	12·6″	− 12·5″	24·4″	14·3″
O. 24	7·9″	− 4·9″	16·9″	9·4″
O. 25	8·3″	− 6·2″	17·4″	10·2″
O. 22	13·2″	+ 12·9″	36·6″	23·8″
O. 2	4·9″	− 3·6″	14·8″	6·2″
O. 3	4·5″	− 3·5″	11·0″	5·7″
O. 25	7·8″	− 6·2″	17·4″	9·6″

(*b*) The same targets as before, but observations made when the two circles are first separated and made to close in on one another. The first five observers waited for the bright overlap to become visible before stopping the motor, and the last two stopped it when they thought contact was made.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 24	18·3″	− 18·3″	25·2″	18·7″
O. 25	34·7″	− 34·7″	52·5″	35·7″
O. 3	23·1″	− 23·1″	29·2″	23·7″
O. 23	51·5″	− 51·5″	69·0″	53·7″
O· 2	27·3″	− 27·3″	47·0″	30·8″
O. 25	45·2″	+ 45·2″	65·8″	46·7″
O. 3	11·9″	2·6″	26·7″	13·9″

Comparing the results shown in these two tables it is clearly evident that much greater accuracy can be obtained when the bright circles are made to separate than when they close, and that sextant observations made in the forenoon should consequently be observed in the sun's lower limb, and in the afternoon on the upper limb.

In an actual observation with a sextant, a slight movement of the instrument will cause the two images of the sun to separate in a horizontal direction, so that it would hardly be fair to assume that the accuracy obtained in the first of these two experiments where the two bright circles had not this movement can be expected in sextant work. Possibly in the artificial horizon a single observation will contain an error of not more than thirty to forty seconds when no telescope is used with a corresponding reduction according to the magnification if a telescope is employed.

(*c*) The advantage of using a magnification of 5 diameters is seen in the result obtained in this and the following series of observations, where the bright circles subtended an angle of about $3\frac{1}{2}°$ at the observer's eye. This set was taken in the same manner as (*a*); and in—

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2	2·8″	+ 2·4″	6·6″	3·9″
O. 1	4·7″	− 1·5″	13·2″	6·2″
O. 26	3·7″	+ 3·4″	9·9″	5·3″
O. 27	2·8″	1·7″	6·4″	2·9″
O. 28	1·8″	1·2″	5·9″	2·6″

(*d*) the two circles were adjusted so as to be apparently in contact. It was found in all cases that this adjustment could be made most easily by allowing the circles to overlap and then manipulating the motor until the bright overlap just disappeared. Any attempt to make the adjustment from the other side involved much larger errors.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2	1·9″	1·6″	4·9″	2·3″
O. 1	4·0″	1·5″	5·7″	4·2″
O. 26	3·3″	2·3″	9·7″	4·4″
O. 27	9·9″	9·7″	16·9″	11·4″
O. 28	4·9″	4·9″	8·0″	5·1″

2. Small circle (40′ diameter) and straight line. These observations were made by running and stopping the motor as in 1 *a*. Errors with a + sign are those in which the circle was clear of the line and with a − sign those for which the circle

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 9	3·9″	+ 1·6″	9·8″	4·7″
O. 10	9·7″	− 9·3″	14·8″	10·8″
O. 15	6·1″	− 6·1″	10·8″	7·1″
O. 15	5·5″	− 5·4″	9·7″	6·0″
O. 5	19·1″	− 18·9″	39·6″	21·7″
O. 1	5·0″	− 4·3″	7·7″	5·6″

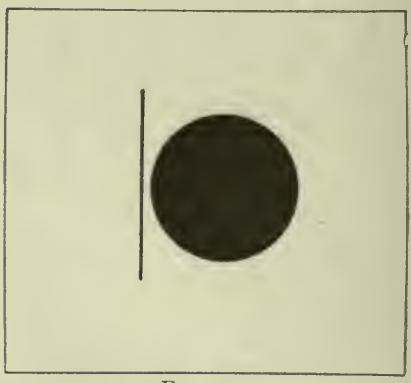


Fig. 9.

overlapped the line. Except in the case of O. 5, who was apparently a bad observer, it will be seen that the probable error of a single observation is somewhere about 7″.

3. The same circle in contact with the edge of a bright rectangle. The error made here is apparently of the same sort of value as in 2.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2.	4·3″	−3·6″	7·3″	4·7″
O. 1.	4·0″	+2·2″	8·4″	4·9″
O. 5.	8·8″	−3·1″	15·7″	9·8″

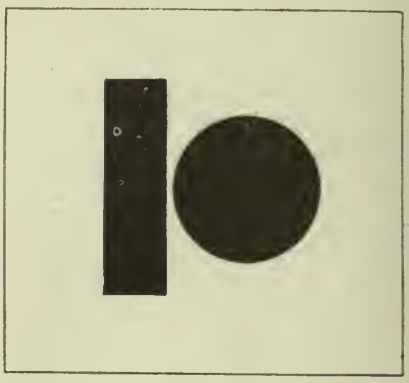


Fig. 10.

4. Contact between the edges of two bright rectangles. These three sets of observations were made in the same manner as in 1 *a*, viz. the two rectangles were placed so as to overlap and the motor run and stopped at the instant when the bright overlap ceased to be visible. A very curious point arose in connection with this class. All three observers judged the bright overlap to have disappeared considerably too soon. O. 6 made ten observations with errors lying between 19·5″ and 14·5″. O. 1 took his between 23·3″ and 15·3″ and O. 2 between 20·4″ and 13·9.″ All the thirty errors were cases of overlapping. Owing to the singular nature of these results

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 6.	18·0″	−18·0″	19·5″	18·1″
O. 1.	20·3″	−20·3″	23·3″	21·7″
O. 2.	16·9″	−16·9″	20·4″	17·1″

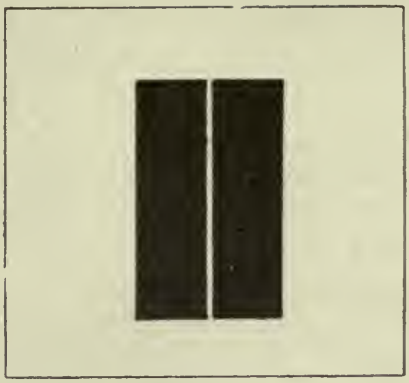


Fig. 11.

very great care was taken in estimating the zero error and the amount of overrun of the motor, and the results given are reliable to something less than one second.

From the nature of these results it would seem that marks of this type are not particularly good except for difference measurements, in which cases the angles obtained could be trusted to four of five seconds.

5. Contact between a diamond-shaped pointer and—

(*a*) A straight line.

(*b*) The edge of a rectangle.

In both cases black objects on a white ground.

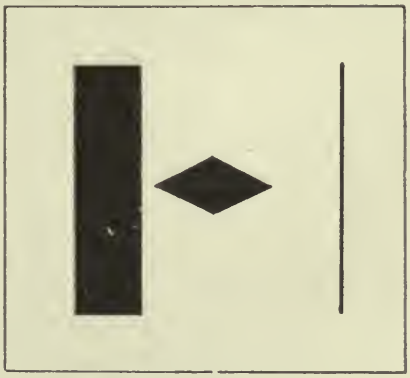


Fig. 12.

(*a*) These observations were made by setting the point of the diamond on to the line. The errors were nearly all cases in which the point did not reach the line, only five out of forty being of the other type. It will be seen that the results are much poorer than in the contact of two bright circles.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 18	17·4″	15·0″	37·8″	20·5″
O. 19.	27·9″	26·2″	45·5″	29·7″
O. 2.	10·2″	8·1″	28·7″	12·8″
O. 1.	15·2″	15·2″	36·0″	17·6″

(*b*) The errors in this case were much more evenly divided as to the type, thirty-four cases of overlapping as against twenty-six cases of separation. It is interesting to notice the bad effects produced on O. 2's powers of observation by a slight bilious attack.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 18.	19·4″	10·0″	30·4″	20·9″
O. 19.	28·7″	28·7″	47·8″	30·4″
O. 20.	18·2″	15·7″	47·4″	26·6″
O. 21.	18·8″	13·9″	40·1″	21·4″
O. 1.	20·1″	9·5″	42·8″	21·8″
O. 2.	56·0″	56·0″	82·2″	60·3″

Bisecting Adjustments.

At the time of the Convention only one set of observations had been made in this class.

The target represented the middle diagram of Fig. 13.

The outer lines had a breadth of about 70″ each and a separation of about 12′. The middle line had a breadth of 5′ as seen by the observer.

It will be seen that the results obtained were exceedingly good, so good in fact that O. 2, thinking that his first set were accidentally good, took another set later in the day and obtained practically the same results. O. 37 and O. 38 are unskilled observers.

It seems probable that by a suitable choice for the widths of the various lines and the intervening spaces, the accuracy of this particular type of setting will equal and possibly be superior to the coincidence adjustment.

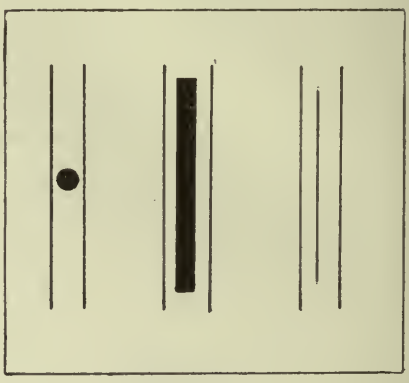


Fig. 13.

	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
O. 2	2·4″	1·9″	6·1″	3·1″
O. 1	7·0″	6·1″	10·5″	7·6″
O. 2	2·6″	0·3″	6·1″	3·3″
O. 4	10·6″	10·6″	18·1″	11·2″
O. 37	7·6″	7·6″	10·3″	8·1″
O. 38	13·2″	13·2″	25·0″	15·3″

Most of the different observers were engaged in a few only of the several tests, so that it is not possible to give an extensive set of results showing the errors made

at the various targets by the same observers, but this list has been compiled in the case of the authors of this paper, O. 1 and O. 2, and will be found below.

	O. 1.				O. 2.			
	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.	Mean Error.	Algeb. Mean.	Max. Error.	Prob. Error.
Coincidence 1 *a*	7·1″	3·4″	13·9″	8·4″	6·3″	2·4″	17·0″	7·6″
,, 1 *b*	—	—	—	—	10·4″	6·5″	23·4″	12·3″
,, 2 *a*	9·5″	8·0″	17·4″	11·8″	3·2″	1·0″	8·3″	4·0″
,, 3	5·4″	0·6″	18·0″	7·1″	6·0″	1·8″	14·1″	7·4″
Contact 1 *a*	—	—	—	—	4·9″	−3·6″	14·8″	6·2″
,, 1 *b*	—	—	—	—	27·3″	−27·3″	47·0″	30·8″
,, 1 *c*	4·7″	−1·5″	13·2″	6·2″	2·8″	+2·4″	6·6″	3·9″
,, 1 *d*	4·0″	1·5″	5·7″	4·2″	1·9″	1·6″	4·9″	2·3″
,, 2	5·0″	−4·3″	7·7″	5·6″	—	—	—	—
,, 3	4·0″	+2·2″	8·4″	4·9″	4·3″	−3·6″	7·3″	4·7″
,, 4	20·3″	−20·3″	23·3″	21·7″	16·9″	−16·9″	20·4″	17·1″
,, 5 *a*	15·2″	15·2″	36·0″	17·6″	10·2″	8·1″	28·7″	12·8″
,, 5 *b*	20·1″	9·5″	42·8″	21·8″	56·0″	56·0″	82·2″	60·3″
Bisection . .	7·0″	6·1″	10·5″	7·6″	2·4″	1·9″	6·1″	3·1″

THE TEACHING OF MATHEMATICS FOR STUDENTS OF ELEMENTARY VISUAL OPTICS

By Dr. Max Coque

(*Tuesday, June 25th*, 1912.)

Mr. President and Gentlemen,—I feel it a great honour to be asked to take a part, humble though it may be, in this Optical Convention of 1912.

I must, however, crave your indulgence for my imperfect knowledge of the English language, while I put before you a few suggestions culled from a teaching experience first in my own country and more recently in this country.

The teaching of mathematics in relation to Optics—what a vexed question this opens up!—How to steer clear of the old school of opticians with their cry, "We never did any mathematics and we never felt the need of them and yet we got together good paying businesses," and the newer school of thought who maintain with equal vigour and energy, that "nothing can or should be done without a solid foundation of mathematics." They continue, "Let us have the theory before the practice—abstract notions before the concrete," and add "Train, but above all train the would-be optician in the principles before you admit him to the practice of his profession." To-day I should like to see if there is no *via media*, no happy medium, reconciling these two schools of thought. But before doing so I must admit that I have a distinct prejudice in favour of a sound mathematical and scientific training. I will not go so far as Prof. Stirling who said that medical training was the best of all training for every purpose, but I do urge upon those in authority the necessity of insistence upon improved mathematical and scientific teaching for those likely to take up optical work as a career. In so far as lies in my power I should wish to help forward the science in which I take the keenest interest, and incidentally to further the increase of optical knowledge in the country, which, though not the country of my birth, has now become the country of my predilection and adoption.

Before asking ourselves how we should teach mathematics, it may be well to consider for a moment why we should teach it at all. There are two conceivable objects in teaching any subject.

Firstly—Because the thing taught is necessary or useful or may be turned to practical account; or—

Secondly—Because the incidental effect of teaching it is to bring into play and to exercise certain powers and capabilities and so to serve a real educational purpose.

Some subjects which are taught in the course of general education are justifiable on the one, and some on the other, ground. And it behoves all teachers, whatever be the subject they teach, to make sure which of these two purposes they are aiming at. For if lessons on any subject are not valuable either for their obvious practical use, or for the disciplinary effect on the general power and capacity of the student, there is no justification for teaching that subject at all.

For the student of optics, we may safely say, at the outset, that mathematics—elementary but practical mathematics—form a subject the importance of which cannot be questioned. To understand clearly the fundamental laws of optics, the theory of lenses and lens-systems, the working of the eye as an optical instrument, the correction of errors of refraction, a slight knowledge of practical mathematics is absolutely necessary. Even in the most elementary part of his work, the student of visual optics can never master the subject of prisms and lenses, nor become conversant with the details of decentration if he has not some sort of mathematical training.

He may, it is true, work out problems in these subjects by rule-of-thumb, but he can never in any real sense know them. Moreover, is not a too faithful adherence to rule-of-thumb the greatest obstacle to progress and scientific advance?

Granted, then, the need of mathematics for the student of visual optics, how much greater must be the requirements of those who intend to devote themselves to the designing of optical instruments. The question which arises now is this: Is the usual mathematical knowledge of the average youth on leaving school sufficient to enable him to begin the study of optics?

During the last ten years, I have interviewed a considerable number of would-be optical students, and the most frequent answer to the inquiry: "Have you any knowledge of elementary mathematics?" is, "Oh, I hate mathematics." Some are slightly ashamed to own it. Others appear to glory in it. I must say that the British student is not worse in this respect than the continental one. I have found the same antipathy to mathematics in many of the students of my old University during my own medical student days; and more especially afterwards, when, as assistant to Prof. Monoyer, of diopter fame, it was part of my duty to initiate students into the mysteries of physiologic optics.

I will not try to explain this peculiar idiosyncrasy of the modern young man, although the explanation may not be difficult to find. The same condition exists in other branches of study, and the Board of Education evidently realized this when they included in their Syllabus of Courses of Instruction in Mathematics and in their mathematics examinations an alternative scheme of mathematical teaching which is described under the title of Practical Mathematics. It is intended to meet the requirements of those students who approach the study of mathematics from a practical point of view, and whose desire in that study is to fit themselves to use mathematics readily in relation to mechanics and engineering practice.

A course on similar lines for the student of optics can easily be sketched, and experience has clearly shown me that the average student, with only the amount of spare time enjoyed by those who have to earn their living, may easily gain, in a very few months, a sound, practical, working knowledge of mathematics which will fit him for his optical studies.

The need for reform in this branch of education is emphasized by Prof. John Perry, who says—

"The average boy receives no education at school, except through his sports. He ought to be acquainted with concrete things before he is asked to reason about them. Quite early, he ought to play at keeping shop; to measure, weigh and compute. A boy's main business at school is to continue the study of observational and experimental science which he began on the day he was born. He is taught many subjects in water-tight compartments, whereas he ought to learn all subjects as if they were one.

"Again, we make two very great mistakes; we painfully impress on a child that an idea is difficult to understand, an idea perfectly familiar to it since it was three years of age; we assume that a child can understand quite readily some grown-up idea that looks to us simple. Thus, we produce stupefaction and the child is mentally shipwrecked: it seizes upon any raft for safety, and the raft ever ready is a formula,

a rule. If schoolmasters studied their pupils as trainers of animals study theirs, they would find the average boy capable of the highest kinds of intellectual work."

The fundamental idea in teaching practical mathematics, is what the Germans call Anschauung, which literally means " looking at " or " beholding." The actual meaning of the word includes not only seeing what can be seen, but also hearing what can be heard, smelling what can be smelled, touching what can be touched, tasting what can be tasted. If a child had never seen fruit of any kind, and you wished to give him an idea of an orange, you would not read to him a description of an orange : you would show him a real orange, and let him smell, handle and taste it. That would be teaching by Anschauung : it would be bringing the thing to bear on the child's senses immediately instead of talking about the thing. And five minutes of this teaching would do more than a month of mere description to give the child the notion of the thing in question, namely, an orange.

There are subjects, and mathematics is one, in the teaching of which, if the teaching is to be really successful, the foundation must be laid in Anschauung. There must be the action of the student's own sense or senses, an action directed or supported by the student's own will; he must experiment on concrete things, instead of reasoning on mere abstractions.

To my mind, there is no doubt that a rapid progress on these lines, with frequent returns to pick up, re-examine and use difficult and at first partially understood operations, is the best method of teaching the subject under consideration.

On this point, let me quote this distinguished mathematician, the late Prof. Sylvester—

" I should rejoice to see Mathematics taught with that life and animation which the presence and example of her young and buoyant sister (viz. Natural and Experimental Science) could not fail to impart: short roads preferred to long ones; morphology introduced into the elements of algebra; projection, correlation, motion accepted as aid to geometry; the mind of the student quickened and elevated, and his faith awakened by early initiation into the ruling ideas of polarity, continuity, infinity and familiarization with the doctrine of the imaginary and inconceivable. It is this living interest in the subject which is so wanting in our traditional and medieval modes of teaching."

I am going now to give you a telescopic rather than a microscopic survey of a course of mathematics as I understand it to suit the requirements of the optical student. I am supposing that the student has the ordinary elementary ideas of numbers, that he knows the four rules and has a slight acquaintance with vulgar fractions.

Training in practical mathematics should begin with arithmetical calculations, the use of decimals being introduced at the very outset through measurements and computations. At present, a schoolboy is not supposed to know what a decimal means before the age of ten : he ought to be familiar with decimals at the age of eight.

A fact which should be impressed upon the student from the first, is that when

calculating from observed quantities, it is dishonest to use more figures than we are sure of.

In the textbooks we used when I was at school, it was stated that the velocity of light was 314,262,944 metres per second! Why not give centimetres and millimeters? In the same way, we were taught that the mean distance from the earth to the sun was 95,142,357 miles. Here again, I wonder feet and inches were left out. The best knowledge we now have is that it is not more than 93 millions and not less than $92\frac{1}{2}$ millions.

Mathematical symbols are merely a very easy form of shorthand; they usually instruct us to perform certain arithmetical operations. Thus, $+ - \times \div$ are well known. Almost everybody knows that $a \times b$ means the same as a,b or ab when letters are used to represent quantities, but that, for obvious reasons, the first of these forms ought to be used for numbers.

Again, $a \div b$ or $a : b$ or $\frac{a}{b}$ or a/b have the same meaning.

A student generally has no difficulty in understanding the meaning of 3^2 which means 3×3, or of a^5 which means $a \times a \times a \times a \times a$.

When he has realized that $a^2 \times a^3 = a^5$ or generally that $a^p \times a^q = a^{p+q}$, and again, that $\frac{a^6}{a^4} = a^2$; and $\frac{a^p}{a^q} = a^{p-q}$, the road is paved for the meaning of negative and fractional powers and it becomes clear to him that $\sqrt{a}$ is the same as $a^{\frac{1}{2}}$; that $a^{\frac{2}{3}} = \sqrt[3]{a^2}$, and again that $a^{-m} = \frac{1}{a^m}$.

The use of logarithms in calculation should be introduced at a very early stage.

Some mathematicians assert that nobody ought to be allowed to use a table of logarithms until he knows how to calculate it. This is a hard saying. It is exactly like saying that nobody ought to wear a watch, or to ride a bicycle until he is able to manufacture a watch or a bicycle.

The solution of simple equations and the expression of arithmetical rules by algebraic formulæ are sure to be a source of interest to the student. He should be shown that a formula includes all particular cases in a general statement, and so, by the use of a single algebraic expression, he is enabled briefly to express a whole class of results in a form at once simple, easily remembered and easily applied.

The use of squared paper and the construction of graphs should also be introduced early. A squared paper record of the rise and fall of the barometer during a week gives a good idea of the graphic method. Everybody understands the meaning of it. Horizontal distances represent time since Saturday midnight; vertical distances represent the height of the barometer.

Again, let us take a table giving the population of England and Wales, Scotland and Ireland respectively, say, in the middle of each specified year from 1800 to 1900. A student will find it easy to plot the different points and to draw the curves showing the movement of the population in the countries considered. He will easily find the

probable population of England and Wales in, say, 1845; the probable population in 1901, the probable rate of increase per annum.

These examples may be multiplied *ad infinitum.* The student ought to practise by himself using his common sense, and in this way he will be able to solve any such problem; and furthermore, he will require no elaborate proofs; things will be so self-evident as to require no proofs.

Proofs, indeed! Some people need proof that they themselves exist.

An orthodox mathematician will tell you that the subject ought to be called analytical geometry or co-ordinate geometry; he will tell you that nobody ought to be allowed to begin it until he has mastered the most elaborate algebraic and trigonometric investigations. Really the graphic method could be used and used wisely and well by an illiterate man.

Numerous exercises on the plotting of curves should be given. The student ought to use squared paper very extensively and with numbers he is interested in.

Let him take *Whitaker's Almanack* and look up, say, the table giving the price of consols every year since 1790. Let him plot the curve and get, in this way, some idea of the decreasing value of money. Or, if this particular subject is too sad, let him plot the traffic receipts or the number of passengers of the railways of the United Kingdom, and show in a curve, how the number of second-class passengers has diminished since 1883 and the number of third-class passengers increased.

These examples, which may be multiplied indefinitely, prepare him to represent the variations of a function by a curve, and show him how an equation may be solved, at least approximately, by the graphic method. They also prepare him for the determination of laws. In most quantitative experiments, we have two varying quantities. We take various values of one of these and observe the corresponding values of the other. If we plot the two sets of values on squared paper, we usually find that the points obtained lie approximately on some continuous curve, the equation of which is the law governing the phenomenon considered.

As soon as the student knows how to evaluate formulæ, he may go through a course in geometry, illustrating his study by practical work. Assuming the truth of certain things from his teacher's statement, or better still from his own observation, he ought to prove the truth of other things which may not be so obviously true.

Here, there is a danger of a long and useless course on practical geometry or a long course of abstract reasoning on the lines of Euclid. What is wanted is a common-sense treatment of the subject such as will interest the average student using the kind of reasoning that every thinking man employs in matters in which he is interested.

Allow me to quote Mr. Oliver Heaviside. In his book *Electromagnetic Theory*, vol. i., p. 148, he says:

" . . . There is a society for the improvement of Geometrical Teaching. I have no knowledge of its work, but as to the need of improvement, there can be no question when the reign of Euclid continues. My own idea of a useful course is to begin

with arithmetic, and then, not Euclid, but algebra. Next, not Euclid, but practical geometry, solid as well as plane; not demonstration, but to make acquaintance. Then, not Euclid, but algebra applied to geometry and trigonometry." A little farther down, Heaviside adds: "Euclid might be an extra course for learned men like Homer, but Euclid for children is barbarous."

Here is a brief syllabus of a course in geometry—

Usual definitions. Measurement of angles in degree. Draw any triangle. Measure the three angles, add them together. Test the accuracy of the graphical work. Bisecting of any angle and of any straight line. Drawing of perpendiculars. Construction of triangles when the three sides are given, or when two sides and the contained angle are given, or when one side and the adjacent angles are given. Drawing of parallel lines.

Measurement of angles in radians. Notion of similarity. Determination, experimentally, of the ratio of a circumference to its diameter. Meaning of trigonometrical ratios, especially sine, cosine, and tangent. The student is made to calculate the sine, cosine and tangent of the angles of a right-angle triangle and to compare them with the values found in tables. He will find no difficulty in working out simple problems on heights and distances, if he is not frightened by the information that this is a new and advanced science called trigonometry.

There are only a few important principles which ought to be proved, illustrated and dwelt upon.

Rules in mensuration should be stated as formulæ and proved (when the proofs are easy) as part of the geometrical work. Surely, it is absurd to maintain the present artificial distinction between geometry and mensuration, when they are both really the same and to scorn arithmetic and algebra, simply because Euclid was quite ignorant of these branches of mathematics.

Were it not that I am afraid of trespassing on your patience any longer, I should have liked to show you how such a subject, usually considered beyond the scope of ordinary students, as the calculus might be approached on similar lines, experimentally, so to speak, and rendered practical by stripping it of all purely academic ornaments.

However, in the limited time at my disposal this is an impossibility.

Such, Mr. President, are briefly the outlines of a syllabus which I should have liked to have been able to develop more fully. It is my intention, however, to complete it in book form, and I should be glad to receive ideas and suggestions from those like myself keenly interested in these matters. Judging from my own experience, there has been a great improvement during the past ten years in the preliminary training of the average optical student, but there is still room for improvement.

May we not, therefore, reasonably hope for a still brighter future for the study of optics when students with a good preliminary training have every encouragement to develop their talents in a profession whose highest ideal is the amelioration of the lot of mankind!

Prof. Silvanus P. Thompson referred to the work Prof. John Perry had done to improve mathematical teaching. At the same time he pointed out that all the modern changes were not necessarily improvements; Euclid might not have been the best method of teaching practical geometry, but nothing had been found to take its place for the purpose of teaching logical deductive reasoning.

SOME GRAPHICAL METHODS, INCLUDING THOSE THAT DETERMINE THE SITUATION OF CARDINAL POINTS

By Archibald Stanley Percival, M.A., M.B. Cantab.

(*Monday, June 24th.*)

In elementary geometrical Optics there are two fundamental formulæ which are universally applicable—for reflection and refraction whether at single spherical surfaces, or any combination of them. They both apply to lenses, whether thick or thin, or any complex system composed of them. For this reason, I think that the student's attention should be primarily drawn to them, instead of having different formulæ referring to reflection and refraction at single spherical surfaces, and again other formulæ for lenses. The two formulæ are, of course, the familiar ones—

$$\frac{F'}{p} + \frac{F''}{q} = 1 \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{i}{o} = \frac{F'}{F' - p} = \frac{F'' - q}{F''} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Now it is mainly the first formula that I propose to consider now. It will be observed at once that it is in precisely the same form as the ordinary expression for a straight line in terms of the intercepts on the axes—

$$\frac{x}{a} + \frac{y}{b} = 1.$$

This at once suggests the following obvious graphical device.

We adopt the usual convention regarding signs—that directions from left to right, or from below upwards are positive; the reverse directions are negative. Draw two rectangular axes and indicate the origin by the letter H. On the horizontal axis mark off the point F′ at such a distance from H that F′H = the first focal distance considered. From the point F′ erect a vertical equal to the second focal distance F″ in height, if this have a negative value; for the distance F″F′ represents the second focal distance, and of course F″F′ is measured downwards, and

is, therefore, negative. When the second focal distance is positive, the point F″ is below the point F′ at a distance representing the second focal distance in value. The diagram (Fig. 1) represents Donders' schematic eye, of which the first focal distance is + 15 mm., and the second focal distance is − 20 mm. Consequently F′H = + 15 and F″F′ = − 20.

Now suppose that we require to find the position of the image formed by such an eye of an object at the distance of 90 mm. We make PH = 90 on the horizontal axis, and from P we draw a straight line through F″ meeting the vertical axis in Q. Then QH is the distance of the cornea from the image. On measuring this we find that QH = − 24. (QH is negative, of course, because it is measured downwards, and the negative sign shows that the image is behind the cornea.) As the normal length of the schematic eye is 20 mm. we infer that the eye cannot focus the object without using an excessive focusing effort. If without using his focusing muscle our subject can see the object distinctly, while the curvature and the refractive indices of the media are normal, we infer that his eye must be 4 mm. longer than normal. Indeed, he would require a concave lens of focal distance, PF′ or 75 mm. when placed at F′ to enable him to see distant objects. Such a lens is, of course, − 13·3̇ D.

Fig. 1.

Further, since $\frac{i}{o} = \frac{F'}{F' - p}$, and as F′P = F′H − PH we see that F′P = F′ − p, so that—

$$\frac{i}{o} = \frac{F'H}{F'P} = \frac{15}{-75} = -\frac{1}{5}.$$

Consequently the size of the image is $-\frac{1}{5}$ of that of the object. The negative sign shows that the image is inverted. So we see that this ridiculously simple little diagram gives us all the information we can possibly require to know about the image from the data given.

In the case of a lens, we know that F′ = − F″, so that a similar diagram may be used in the case of a convex lens with the exception that F′H = F′F″. If the lens be concave we know that now F′ is negative, while F″ is positive. Consequently Fig. 2 represents this condition: for F′H is measured in the negative direction, and F″F′ is measured in the positive direction—upwards—and is numerically equal to HF′, in this case 2. Now if an object is placed at a distance of 8 from this lens we make PH = 8, and we find that the line PF″ cuts the vertical axis through H at a point Q such that QH = + 1·6. The image, therefore, is on the same side of the lens as the object, and it is situated at a distance of 1·6 from it. The size of the

image as compared with that of the object is as before $\frac{F'H}{F'P} = \frac{-2}{-10} = +\frac{1}{5}$. As the sign is positive the image is erect.

Fig. 3 represents the case of a concave mirror with focus + 2. As now F′ = F″, F″F′ and F′H are both measured in the positive direction. If the object be at F distance of 8, we make PH = 8, and QH is found to be $+ 2\frac{2}{3}$, while—

$$\frac{i}{o} = \frac{F'H}{F'P} = \frac{2}{-6} = -\frac{1}{3}.$$

The image is, therefore, situated on the same side of the mirror as the object, and $2\frac{2}{3}$ from the mirror; the image is inverted and $-\frac{1}{3}$ the size of the object.

The method is perfect in its simplicity, and in the wealth of information it imparts. There is, however, a further development of this method that I owe to Professor Sampson, who has kindly given me leave to bring it before you. It is a most ingenious method that he has devised to find graphically the position of the cardinal points of a thick lens. The method will, I think, be quite clear if we take

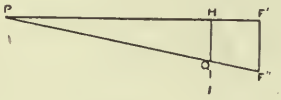


Fig. 2.

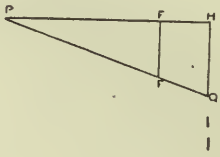


Fig. 3.

one illustration. Consider the case of a thick converging meniscus, the radius of curvature of its first surface r_1 being 4, and that of the second surface r_2 being 2, while the thickness t is 3 and $\mu = 1{\cdot}5$.

For the surface A—

$$f_1' = \frac{-r_1}{\mu - 1} = -8, \qquad f_1'' = -\frac{\mu r_1'}{\mu - 1} = 12,$$

and for the surface B—

$$f_2' = \frac{\mu r_2}{\mu - 1} = 6, \qquad f_2'' = \frac{-r_2}{\mu - 1} = -4.$$

In the lower diagram let $f_1'A = -8$, $f_1''A = 12$, $AB = 3$, $f_2'B = 6$, $f_2''B = -4$.

Complete the parallelograms at K_1 and K_2. Join K_1K_2 and produce, cutting Af_1' at H′ and Bf_2'' at H″. Then H′A represents the distance of the first principal point from the first surface $(-2\frac{2}{3})$, and H″B gives the distance of the second principal point from the second surface $(-1\frac{1}{3})$. It will be noticed that both H′A and H″B are in this case negative, being respectively both measured in the negative direction. Join K_1f_2' and produce to meet H′A produced in F′. Join $f_1''K_2$ and produce to meet BH″ produced in F″. Then F′H′ represents the first principal

focus, and F″H″ gives us the second principal focus of the thick meniscus. It will be noticed that in this case F′H′ is positive, and F″H″ is negative, and they will be found to be numerically equal, being $5\frac{1}{3}$.

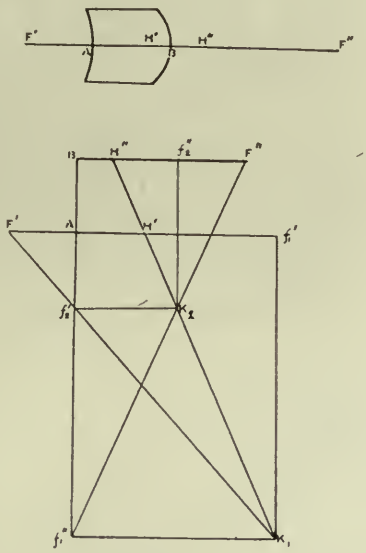


FIG. 4.

The cardinal points are now all found by this purely graphical method, and it will be easy to mark off their positions in the upper figure of the meniscus.

To avoid any error I may point out that f_1'' and f_2' must be in the same straight line as AB, and that K_1 completes the parallelogram referring to the first surface, as its other points are A, f_1' and f_1'', while K_2 completes the parallelogram B, f_2' and f_2''. If, then, it is remembered that K_1 is drawn through f_2', and K_2 through f_1'' to find the positions of F′ and F″, no mistake can be made.

In order to prove the generality of the method we must consider the ordinary algebraic formulæ for the cardinal points.

Let $t + f_1'' - f_2'$ be denoted by D, then we have—

$$H'A = \frac{tf_1'}{D},\ H''B = \frac{tf_2''}{D},\ F'H' = \frac{-f_1'f_2'}{D},\ \text{and}\ F''H'' = \frac{f_1''f_2''}{D}.$$

Now in the diagram D is represented by $AB + f_1''A - f_2'B$ or $f_1''f_2'$.
And it will be observed that—

$$\frac{F''B}{K_2f_2'} = \frac{f_1''B}{f_1''f_2'} \text{ or } \frac{F'' + h_2}{f_2''} = \frac{f_1'' + t}{D};$$

and that—

$$\frac{F'A}{K_1 f_1'} = \frac{A f_2'}{f_1'' f_2'} \text{ or } \frac{F'H' + H'A}{f_1'A} = \frac{AB - f_2'B}{f_1'' f_2'}, \text{ } i.e. \frac{F' + h'}{f_1'} = \frac{t - f_2'}{D},$$

so as before we have—

$$F'' + h_2 = \frac{1}{D}(f_1'' f_2'' + t f_2''), \text{ and } F' + h' = \frac{1}{D}(-f_1' f_2' + t f_1').$$

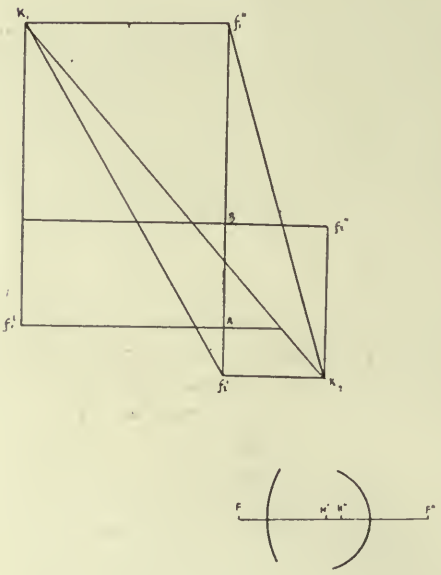


Fig. 5.

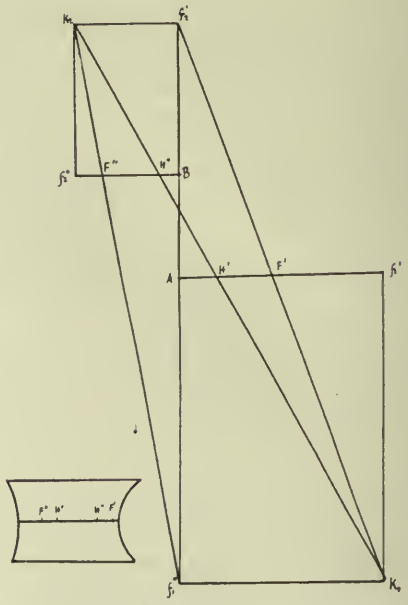


Fig. 6.

Figs. 5 and 6 show the graphic method of determining the cardinal points of a thick biconvex and biconcave lens.

(If the final medium differs from the original medium, the points H′ and H″. determined by the intersections of the line K_1 K_2, will denote the nodal points of the system.)

THE MICRO-SPECTRA METHOD OF COLOUR PHOTOGRAPHY

(with special reference to some optical problems arising therefrom)

By Julius Rheinberg, F.R.M.S.

(*Thursday; June 20th*, 1912.)

Micro-spectra Colour Photography possesses the peculiarity that it is a process which calls for no particular skill on the part of the photographer other than he is accustomed to exercise in his ordinary black-and-white photography. It is a one-plate, one-exposure method, the negatives and positive transparencies for the process are black and white and are developed and made by the usual familiar methods.

For the faithful rendering of colour by the micro-spectra method we rely not upon colour screens, or coloured pigments or particles, but upon optics alone; and my purpose to-day is not to enter into any lengthy account of the method or the apparatus, since these matters may be found in detail in the recent paper before the Royal Photographic Society,* but rather to confine myself to a discussion of some of those numerous optical considerations which have arisen in developing this method, and some problems which still await solution if further progress is aimed at.

Fig. 1.

A brief outline of the theory of the method may, however, not be out of place. If an image of a slit is formed by a lens with a prism in front of same, the slit is, of course, imaged as a spectrum. If in place of a single slit, we substitute a line screen with hundreds of slits (Fig. 2), then we obtain hundreds of spectra, and if we further take the precaution of having the slits on our line screen narrow as compared with the intervening bars, and use a prism of very small dispersion, then we can contrive that each slit of the line screen forms a distinct and separate spectrum.

Now we use for our purpose a line screen having nearly 400 slits per inch, the ratio of the slits to the bars being as 1 to 3 (Fig. 3), and the dispersion of the prism is so calculated that the spectra lie side by side without any gaps between them, but also without any overlap (Fig. 4), other than a very small amount introduced

* Reported in the April number of the *R. P. S. Journal*, 1912.

purposely for special reasons. And since, in the camera, the objective used in conjunction with the prism magnifies the line screen on the focusing screen to the extent of about four times, the result is a surface consisting of a series of regularly repeating spectra, 100 per inch, which, to the unaided eye, appears white by optical fusion, as the individual colours cannot be separately distinguished.

FIG. 2.—General optical arrangement shown diagrammatically.

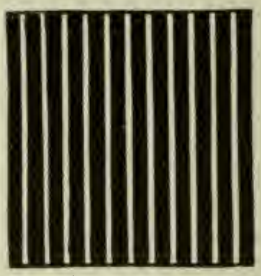


FIG. 3.—Enlarged image of part of the line screen or grating.

Now, just as when using a spectroscope, any object placed before or focussed upon the slit has its colour decomposed into its individual spectrum colours, so, when the image of any coloured object is focused upon the line screen in our camera, each slit of that line screen takes the part of a spectroscope slit and any colour focused upon it is analysed into its separate spectrum colours. The camera being provided with an objective in front which forms an image of the object to be photographed on the line screen, the whole apparatus, in fact, acts as a multiple spectroscope which analyses every tiny portion of the picture.

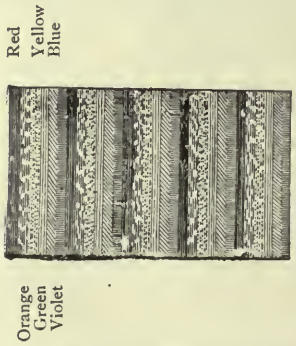


FIG. 4.—Diagram of regular repeating spectra.

When a photograph is taken, each element of the picture impresses its own record on the developed negative in black and white, for, since every spectrum colour has its own fixed position in a spectrum, only such parts of the spectrum are darkened which correspond to the spectrum colours contained in the particular element of the picture in question, and these are wholly or partially darkened in accordance with their comparative intensity. The positive transparency from the negative of course bears the same record, only in this the portions of the spectra corresponding to the colours that have acted on the negative are wholly or partially transparent, while the spectrum colours that were not present in the colours of the object photographed are darkened (Fig. 5). We have now only to place the positive in the camera in the *exact* position occupied by the negative when the photograph was taken, and to transmit white light through the camera in order to form the surface composed of the repeating spectra,

and the positive transparency then acts as a mask that blocks out those components of the spectra that were not present in the colours of the object, whilst allowing those to pass that were. And so we have our picture appearing in its natural colours.

As I hope you will have gathered from this outline, the general theory of the method is simple enough—it is in the carrying of it into practice and constructing a suitable camera that a great number of difficulties and novel problems are encountered.

To start with, an optical system which will yield not merely a good spectrum of a single slit in the axis of the system, but at the same time good spectra of hundreds

Fig. 5.

of further slits which are obliquely disposed as regards the system, and which have to cover a quarter-plate negative from end to end, is a problem quite new to optics.

Equally new, I believe, is the problem of forming a non-distorted image of an entire picture with an optical system, as wide-angled as possible, in which perforce a prism—and unfortunately a thick and strongly refracting prism—has to be interposed. Add to this, that, in order to keep the camera within convenient and portable dimensions, we are somewhat tied as to the means employed, for the optical system has to be of as compact a description as possible.

These are but one or two of the " novelties " which have come up for solution, and, although we have found it possible to solve them to a degree sufficient to get good results, it has only been by dint of sacrificing certain other qualities—I may

instance loss of light attendant on considerable cutting down of aperture—which we would fain have avoided.

Before, however, throwing out some suggestions as to advances in this direction, I would like to touch on one or two other interesting optical questions that have presented themselves, especially as they also have a bearing on the screen-plate methods of colour photography in which so much work has been done of late.

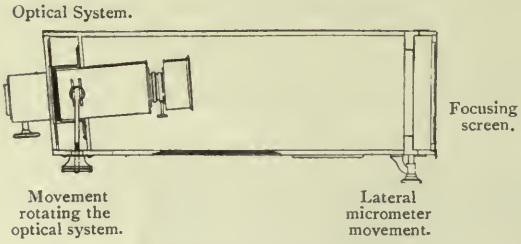


FIG. 6.—Section of micro-spectra camera.

A spectrum may be regarded as a set of different colour points. How many such sets per inch are necessary in a colour photograph to secure a satisfactory result from the point of view of optical fusion and avoidance of any marked appearance of stripes or dots? It would have been as easy for us to produce 200 spectra per inch as 100 spectra per inch, the actual number was determined for us by the behaviour of the photographic plate. For the plate must, within the width of a single spectrum, record the correct differences of intensity of all the component colours of the spectrum that happen to be present, and we found in practice that the differences of intensity were not sufficiently shown when we went much beyond the 100 per inch, with the result that faint or delicate hues in the picture began to suffer. Now, fortunately, 100 spectra per inch, which on first thoughts might seem a comparatively small number, are quite sufficient for optical fusion, and the point I wish to bring out is that various factors enter into the question of optical fusion—which is only the obverse way of regarding the separating power of the eye—when we are dealing with a colour photograph obtained with the aid of juxtaposed spectra or with the help of a screen plate with colour lines or dots, that do not enter into the treatment of the separating power of the eye in the ordinary way. For, in the usual way, we deal with some such simple factors as black and white lines, or bright slits and dark bars having a definite ratio of width to one another, and all slits being equally bright and of the same colour. But in regard to colour photos we have to deal with various colours in various light intensities separated by bars of variable width; also we have to distinguish between the optical fusion of all points of the same colour in the consecutive spectra

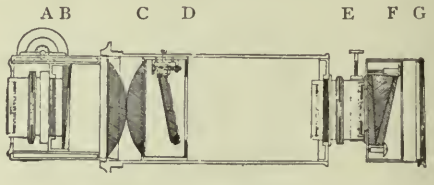


FIG. 7.—Section of optical system.

A Zeiss 75 mm. microplanar objective on focusing mount.
B Spectacle prism.
C Field lens.
D Line screen or grating in adjustable frame.
E 75 mm. microplanar objective.
F Compound prism.
G Cylinder spectacle lens 120″ focus.

or colour sets, and the optical fusion of the different colours in the same spectrum or colour set.

A few illustrations will, I hope, make it clear how these things affect matters.

Consider, firstly, the portion of a micro-spectra picture seen in the camera as white. Here we have repeating sequences of the spectrum colours of equal transparency. But, by reason of the chromatic aberration of the eye, these colours are by no means all in absolute focus at the same moment, and since non-focused colours are present to the eye as diffused spots of larger size than if they were in focus, a sort of visual overlapping takes place. This is, of course, an aid to optical fusion. But we must also take into account that the visual luminosity of the colours differs, notwithstanding that they may all be of the normal intensity, so that the result of visual effects might be shown as a line of repeating curves in which the violet is

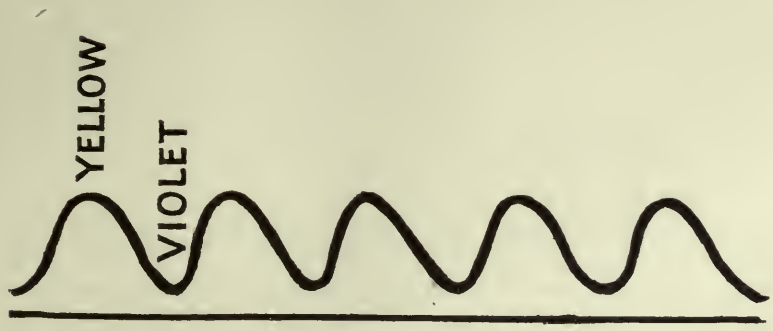


Fig. 8.

at the depressed portion of the curves (Fig. 8). And, in fact, in looking carefully at any white portion of the micro-spectra picture certain slightly darkened stripes may be seen, the centre of location of which, on examination, proves to be in the violet portions of the spectra.* But they are too faint to disturb in any way.

Now consider what occurs when we look at the portion of a micro-spectra picture in the camera which appears in such a colour as, say, sky or turquoise blue. Most of the spectrum colours are present in turquoise blue, but, of course, in quite different proportions than in white light. The yellows and reds, which are strongest in the visual luminosity curve, are toned down most; the upshot is, that if we were to plot out the visual effect of this colour, it would appear as a wavy line in which there is very little variation or contrast between the parts, and that means that optical fusion is immensely benefited (Fig. 9).

The most unfavourable case for us theoretically is one that scarcely ever occurs

* There is also a slight actual depression where the red of one spectrum passes into the violet of the next, occasioned by reasons connected with the impurity of the spectra to which reference is made further on.

in practice. It would be in the case of an object the hue of which is confined entirely to a narrow band of the spectrum, the remainder of the spectrum being perfectly dark. We would then have the simple conditions prevailing that are usually dealt with in treating of separating power, and only a hundred narrow colour lines per inch with darkness intervening would be rather too small a number to be convenient. But the hues of natural objects are practically always composed of various spectrum colours in different intensities and, therefore, much more favourable for our purpose.

It will readily be seen that the factors beneficial to optical fusion and the way in which differences in the colours of the object affect the optical fusion, apply also to screen plate methods of colour photography.

Another curious little conundrum that arose in the course of our experiments was this: seeing that we use a line screen, in which the clear spaces occupy only one quarter of the whole surface (Fig. 3), so that only one quarter of the light reaches the negative, does this fact of itself involve a fourfold increase of exposure (*i. e.* in

Fig. 9.

inverse proportion) in taking a photograph? On first thoughts one would certainly be inclined to say "yes," but the answer is a wrong one. For, although the exposure is increased by the line screen, the increase is not nearly so great as might be anticipated, and is due to causes only indirectly connected with the amount of light cut off by the line screen.

For we have to consider that if a line screen is present, the black bars of which blot out a certain portion of the picture, then only that part of the picture that is passed by the clear part of the line screen is photographed, and the light that falls on those clear parts is precisely the same amount that would have fallen there had no line screen been there at all. Therefore the line screen of itself does not *directly* affect the length of exposure. To understand how the line screen affects the exposure *indirectly*, let us compare exactly what happens when a photograph is taken on a panchromatic plate in the ordinary way and then with our camera, and for simplicity's sake we will fix our attention only on the colours red, green and violet, assuming the plate to be equally sensitive to these colours. Now, in the ordinary way, white light will act on this plate three times as quickly as red light only, since all three colours act on it simultaneously. Similarly a purple object, which can be

considered as a combination of red and violet, would act on the plate twice as rapidly as a red object alone. In short, the density and contrast developed on the negative depends not only on the question of light and shade, but to a very high degree on the question of hue. But in a micro-spectra photograph, in which the various colours are spread out, the time of exposure does not vary at all with the hue; white light, a purple object, or a red object all act on the plate equally quickly or slowly, because compound colours instead of acting cumulatively on the same point of the plate, now merely produce their effect on a greater or smaller surface of the plate.

Now in ordinary photography we should be exposing for an object of some average hue, which would probably be a composite colour, that would, if analysed, show colour components of different intensities, but which ranged over a considerable portion of the spectrum, whereas in micro-spectra colour photography a hue, the colour components of which occupied only a narrow band of the spectrum, would serve as the measure of the time of exposure. How narrow would depend upon how wide the slits of the line screen were compared with the width of a slit + a bar. To a certain extent, therefore, the length of exposure would be increased, but it is nothing like as great as the inverse ratio, for that would only be the case if in photographing in the ordinary way without the line screen, we exposed for the brightest white light in the whole of the picture.

The way how this same consideration bears on screen plate colour photography will readily be seen.

Let me next refer to a novel optical point in the construction of the camera. The spectrum obtained by an ordinary glass prism suffers as is well known from the fact that the colours are not evenly distributed. The red and yellow are unduly compressed, the blue and violet unduly spread out. At a very early stage in our experiments it became evident that as we had to obtain an efficient record of all the colours in the respective intensities they might happen to be present in, within the space of every one-hundredth of an inch, we could not afford any undue proportion of that space for the violet end of the spectrum. A spectrum with evenly distributed colours such as is afforded by a diffraction grating was an essential, and Mr. Conrady of Messrs. W. Watson & Sons, Ltd., computed a prism for us to give this result. Its dispersion, as is required, is very small, amounting to one-twentieth of a degree, or three minutes. It is composed of a suitable crown glass prism of about 25° cemented between two flint glass prisms each of about 5° with their angles turned the opposite way and refracts the light about $6\frac{1}{2}°$ (Fig. 7).

I do not know whether a prism which yields a " rational " or normal spectrum has ever been called for or employed previously, but I hazard the suggestion that, on account of the very great light saving, as compared with the use of a grating, such prisms might well find other fields of employment in certain kinds of spectroscopic work, and that the matter is one which may be found worthy of attention.

Perhaps the most interesting problem we had to deal with in the course of our experiments, was how to get a true and faithful colour rendering with the spectra and the plates at our disposal. When one speaks of colour reproduction effected by

analysing the colours into their spectrum components and subsequently recombining them, the matter sounds so delightfully simple and convincing, that it unconsciously conveys the impression that such a method must yield perfect colour representation. That there are some considerable inherent difficulties does not at first strike one. They managed to worry us a good while, however.

The chief difficulty is this. The purity of the spectrum increases the greater

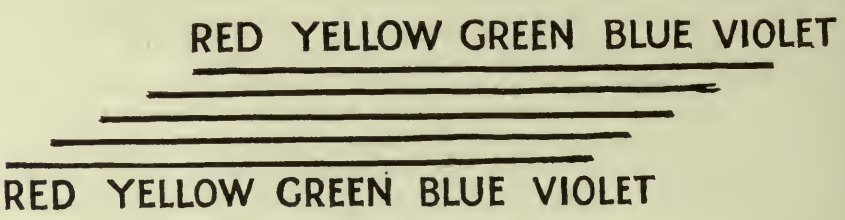


Fig. 10.—The impure spectrum of a wide slit is represented by five overlapping spectra.

the dispersion and the narrower the slit. But the fact that we must be able to pass as much light through the camera as possible, compels us to utilize spectra of small dispersion and relatively wide slits. The spectrum of any slit on the focusing screen, in fact, only covers four times the width that the slit alone would cover, and is consequently very impure. Impure spectra mean for us that when taking a photograph, the line of density which the developed negative should show for any spectrum colour is broadened out into a band, and it also means that in the negative spectrum colours

Fig. 11 (diagrammatic).

that are near to one another overlap. But the most serious thing that it occasions will be understood if we think of what happens suppose we are photographing white light. The overlap is then the cause of greater density on the negative throughout the middle regions of the spectrum than at the ends, for the simple reason that there are no further visual colours to overlap towards the ends of the spectrum (Fig. 10). Now that seriously interferes with proper rendering of colour when viewing the positive, as it means that we weaken the effect of such colour components as may occur at or near the two ends of the spectrum. To meet this difficulty we had to cut right away from our original idea that the panchromatic plate should be *equally*

sensitive throughout the spectrum and arrange that our panchromatic plates should have an increasing sensitiveness towards each end, viz. in the red and in the violet (Fig. 11). It was not easy to find a suitable plate for this, for it should be remembered that the requirements of panchromatic plates for use in the ordinary way are just the reverse of this. For ordinary photography the panchromatic plates should be sensitive to colours in accordance with the visual luminosity curve of the spectrum, *i. e.* strongest in the yellow and pea-green and falling off towards both ends (Fig. 12); any extra sensitiveness in the red or the violet has to be damped down by a suitable colour filter. And so panchromatic plates that are very sensitive to the red end whilst maintaining good sensitiveness to all other parts of the spectrum seem to be scarce. As regards the violet end, of course, most plates are extra sensitive. The best plates for our purpose that we have found as yet are the Ilford Panchromatic Plates.

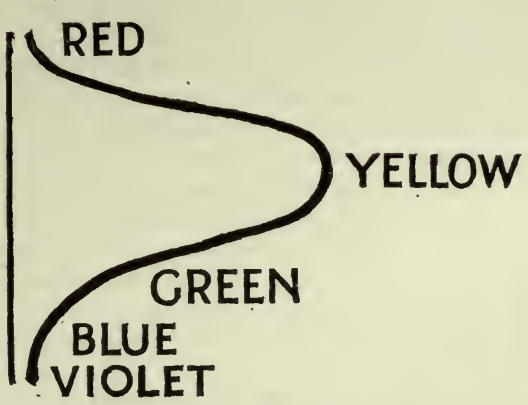


Fig. 12.—Diagram of visual luminosity curve of the spectrum.

However, the above is not the only trouble we had to deal with. When we view a positive, the apparently pure spectrum colours that are producing the general colour impression, are, as we have seen, not really pure spectrum colours at all, but an admixture of overlapping colours. That the visual hue of any pure spectrum colour need not be sensibly altered by the addition of neighbouring colours on each side is well known, only we must pay due attention to the luminosity of the added colours, otherwise the balance will be destroyed. Here it is that the visual luminosity curve comes in (Fig. 12) and interferes with us, for what we are doing is to mechanically mix with each spectrum colour, neighbouring colours on each side of it—so far as they exist on each side—paying no regard to their visual luminosity. Worst of all, at the ends of the spectrum there are no neighbouring visual colours on one side to obtain a colour balance. To take concrete examples, yellow does not suffer by the admixture of an equal amount of orange and pea-green on either side, but a middle green has a tendency to turn out too yellowish when an equal amount of yellowish-green and bluish-green is added to it, whilst with a full red, where we get an addition of orange and yellow on one side of it and nothing to compensate on the other side, what then? The latter difficulty has been met as far as possible by purposely arranging that each spectrum should slightly overlap the next one. In this way we manage to get an addition of violet light to the red which neutralizes the added yellow and restores the balance, whilst the slight addition of red light when we view a violet

counteracts the excess of blue that has come in. The smaller difficulty of correcting for the disturbance of colour balance which the visual luminosity curve introduces in some other regions of the spectrum is adjusted as far as possible by the suitable preparation of the colour filter, which, in the absence of a panchromatic plate perfectly corrected for our purpose, we have, of course, to use in taking a photograph. It is a rather fortunate circumstance that after a great amount of work in connexion with plates and filters, the result was that an extremely weak solution of a single dye, viz. chrysoidin, in conjunction with the Ilford Panchromatic Plate so far effected our purpose, that we do not fear comparison as to faithful colour rendering with any other method.

The last question I propose to enter upon to-day relates to further improvements that can be attained as regards the apparatus for Micro-spectra Photography.

Just consider the natural method of developing and working out an apparatus of this description, where a considerable number of quite novel problems, as has been indicated in the earlier part of this paper, have to be carried into effect. One puts together an optical system embodying the general conception of the ideas, and then concentrates on the most essential features, improving the function of the apparatus in these respects one by one. As is almost invariably the case, any important alteration to improve matters, in any one respect, introduces new difficulties in other directions, and these have then again to be remedied. In the end, it is not surprising that, although all the more essential features may have been achieved, this piecemeal method does not achieve them in just the best and most simple way possible, and that it leaves a residuum of desiderata less important, perhaps, from the point of view of successful accomplishment of the original problems, but sufficiently important from the point of view of greater simplicity and convenience, which are only capable of being tackled if with full knowledge of previous difficulties and all requirements the entire system is planned out *de novo* as a whole.

Now we had, of course, first and foremost to concentrate upon everything that directly affected the fidelity of colour rendering, and as regards this, little further seems to be required to put on the final touches than the obtaining of panchromatic plates similar to those at present employed, but still a little more red sensitive.

But the optical system that has been evolved has left two subsidiary inconveniences outstanding. The optical system employed (Fig. 7) consists as the figure herewith shows of the 75 mm. microplanar objective A, which focuses the small picture on the line screen D, which in its turn is focused enlarged on the photographic plate by the second 75 mm. microplanar objective E with the compound prism F in front of it. Then there is the field lens C to divert the light from the first objective towards the second one, the cylinder lens G, which corrects the astigmatism caused by the compound prism, and the spectacle prism B near to the first objective, the sole function of which is to bring both sides of the picture into sharp focus, as this has been upset by the compound prism.*

* This spectacle prism, it may be mentioned in order to avoid any possible confusion, does not take part in, or in any way affect, the production of the spectra, owing to its position *before* the line screen.

It will be seen that the system is a complex one with a great number of surfaces, at which light scattering takes place. That, and still more so the fact that the aberrations and distortions introduced by the passage of cones of light through a prism—which are great under all circumstances, and which are increased the wider the cones of light we employ, and the more obliquely that we pass them through the prism to extend the field covered—has necessitated a considerable cutting down of the aperture of the optical system in attaining the results we aimed at, with a somewhat undue loss of light in consequence.

Secondly, the exigency of obtaining all the spectra on the focusing screen of the camera in sharp focus at the same time has led to the line screen being set at an angle to the lenses, and the whole optical system being set at an angle to the focusing screen (Fig. 6), with the result that the spectra formed on the focusing screen are very slightly wider on one side of the plate than on the other, because there is a slight inequality between the magnification of the two sides of the line screen. Apart from this the lines of spectra instead of being absolutely vertical have a very slight curvature, which also differs as we proceed from one side of the plate to the other. This is caused by the compound prism. The same thing will doubtless have often been noticed in photographs of spectra taken with an ordinary spectroscope. Although this curvature in our case is far too slight to be directly noticeable, it can, on a careful examination, be detected; and it first revealed itself to us by reason of the untoward effects it is able to occasion, viz. a more or less marked alteration of colours in viewing a picture if the positive is not registered as nicely as it should be. For, supposing all our spectra were *absolutely* vertical and *absolutely* of the same width, then the maximum amount of lateral movement of any positives placed in the camera in order to secure the best colour rendering would not exceed the width of one spectrum, or quarter of a millimetre, because the spectra being all alike, it would be immaterial whether the record on the positive was superposed over the one or the other of them. Whereas what happens is that if the positive is placed in the camera two or three millimetres to one side or the other, it has to be moved up by the lateral micrometer movement (Fig. 6) into more *exact* registration, so that the record of any spectrum is superposed either on to the same spectrum as when the photograph was taken, or at any rate on to a spectrum not more than three or four removed away from it. The somewhat rigorous exactitude in registration entailed can scarcely be termed a fault, and in practice the adjustment is made quickly enough, but it would certainly be more convenient if less care in registration were necessary; moreover, if a camera were made in which the optical system was further simplified, and in which the spectra were all absolutely vertical and of exactly the same width all over the plate, it would promote interchangeability, *i. e.* it would more readily enable the pictures taken in one camera to be used in any other camera of the same kind.

Now it seems to me that there is one thing which, if attainable, would largely help to solve all these outstanding difficulties—which would at one and the same time avoid the necessity of stopping down the lenses so much, help in getting vertical spectra of even width, and generally simplify the construction and reduce the number

of lenses—and that is, if it were possible, to compute a combination of prisms to give a " rational " spectrum of the slight dispersion necessary, with the outer surfaces of the combination parallel to each other. Whether that problem is capable of solution I do not know, at all events I am not equal to it myself. I hope it may be, and that some one will be able to solve it.

If that is impossible there would seem to be various smaller steps in the right direction possible. One of these would be to compute a prism with curved surfaces to reduce errors. Another to use objectives which, for the purpose in view, might well be of a simpler type than the microplanars, and have surfaces suitably calculated and corrected in the first place instead of using supplementary lenses with them. A further improvement might be to obtain a more symmetrical construction than at present by designing the prism in two parts, one placed just in front of and the other just behind the objective. Or, as has been suggested to me, it would be better still to arrange, if possible, that the objective is in two parts sufficiently far apart to allow of the introduction of the prism between them. That undoubtedly would be theoretically the best position for the prism, and it might also help in keeping down the thickness and angle of the prism to their smallest limits.

As will be seen, these outstanding problems are chiefly of a mathematical nature, and I think I am not going too far in saying that should any one competent to solve them be stimulated to take them up and bring them to a successful conclusion, he will not only earn the satisfaction of helping to bring into more general use a singularly fascinating method of Colour Photography, but also to have assisted in making more available for students of Optics an apparatus which, I think it will be admitted, holds a place of its own for the instruction it is able to afford in a most varied assortment of problems connected with that branch of science.

Mr. Chapman Jones in inviting discussion pointed out, that it was a great advantage that the method had nothing to do with either dyes or colour screens; the author had overcome the many difficulties in very ingenious ways.

Prof. S. P. Thompson remarked that the camera of Messrs. Rheinberg was one of the novelties of the Exhibition. No such thing had been suggested seven years ago at the first Convention. The difficulties of the method were enormous. Everything depended upon perfect registration; a displacement of $\frac{1}{1000}$ of an inch would throw the whole out, and superhuman lenses were apparently required to avoid all distortion. The new prism producing a normal spectrum was a distinct novelty. In one of the landscapes which Mr. Rheinberg had thrown on the screen he had noticed that the vertical lines were not quite straight, sometimes deviating a little to the right or left; this might be due to colour effect, and it might be possible, by means of exact measurements, to determine the colour of the respective spot of the picture. The method of Mr. Rheinberg reminded him of an experiment of Newton's which was hardly mentioned in textbooks. Newton wished to recombine the colours of his spectra; he cut a piece of cardboard like a saw; through the teeth of this screen

he perceived many little spectra side by side in which, when the cardboard was moved to and fro, the different colours could no longer be distinguished, but white light was produced.

Mr. T. Thorp asked whether the apparatus could not be simplified by substituting a diffraction grating for the black-line screen. He had prepared a grating in which three-quarters of the light was concentrated in one order. Mr. Conrady was certainly to be congratulated upon his prism; but did it not complicate the apparatus?

Mr. S. D. Chalmers, M.A., said that he was surprised at the excellence of Mr. Rheinberg's results; a certain amount of chromatic or spherical aberration might perhaps be left in the first system so that it would not be necessary to allow so much deviation in the prism; this would simplify the optical corrections required.

Mr. T. Smith wished to ask the author whether the overlapping of the spectra to a suitable extent—possibly a rather different amount from that so far tried—would not do away with the necessity for using a plate more sensitive to the extreme parts of the spectrum than to the central portions. He would further suggest the use of an optical system which gave a field as flat as possible for vertical lines, while the surface containing images of horizontal lines might without much disadvantage be rather curved. The effect of this would be to keep the definition as good as it could be in the direction in which it was inevitably disturbed by the prisms, at the cost of some deterioration in the direction in which the prisms exerted no effect, thus giving an impression of better and more uniform definition on the whole. By freely admitting the use of cylindrical surfaces in the system, such correction should be possible, and it might well be found feasible by this means to simplify the apparatus described by the authors.

In replying, Mr. Rheinberg said that in connection with the interesting observation made by Prof. Sylvanus Thompson he would only like to mention that the perfect registration of the positive over the spectra to secure correct colours over the whole plate was much easier than might have been anticipated, because the picture when out of register always afforded easy *visual* indications of the adjustment required to put it right. It was, of course, important that the number of spectra which were recorded on the positive should coincide *exactly* with the number of spectra projected upon it, and since even so slight an alteration of the distance of the positive from the lens as $\frac{3}{4}$ mm. would add or subtract one complete spectrum to the number projected upon it, the viewing frame in which the positive was placed was not only movable laterally by a micrometer screw, but could also be bodily moved outwards or inwards to the extent of $\frac{1}{2}$ mm. by a sliding-wedge device which was actuated by a little lever. The lever had to be turned through 80° to secure the whole half-millimetre movement and afforded easy means of very rapid and accurate adjustment of fractional parts of a spectrum.

Referring to the " rational spectrum " prism, he remarked that although the desirability of such a prism was his own suggestion, the credit of producing the same was due to Mr. Conrady, who had made the computations.

Regarding Mr. Thorp's remarks, Mr. Rheinberg said he had not overlooked the possibility of utilizing " prism " or " saw " gratings of the kind first suggested by Lord Rayleigh to replace the prism, and had in fact experimented with some beautiful specimens of these, having 1000 lines or serrations per inch, which had been given him by Mr. Thorp himself at the last Optical Convention in 1905.

He had, however, to abandon the idea, for although the most of the light was concentrated in a single spectrum, a certain amount of light scattering took place and there was a not inconsiderable balance which was transmitted as a direct or dioptric beam. He feared that a *complete* avoidance of this, which would be a necessity for the use of such gratings for the purpose in question, was more than could be practically achieved.

He might mention, also, that a suggestion had been put forward some time ago that a photographic plate complete in itself might be made by having a diffraction grating on one side of it to produce micro-spectra on the other emulsion-coated side. He had pointed out elsewhere * that from a strictly theoretical point of view " saw " gratings might be employed for this, but from a practical standpoint the plan was an entire impossibility, for apart from many other difficulties, it would not only involve having such gratings of absolute perfection, but also in a degree of fineness impracticable for " saw " gratings, as in order to produce a spectrum only one-hundredth of an inch wide at a distance from the grating corresponding to the thickness of an ordinary photographic plate, the same would require to have about 15,000 lines per inch.

As to Mr. Chalmers's suggestion Mr. Rheinberg said that in evolving new apparatus such as that in question, it seemed the best plan at the outset to start with perfectly corrected lenses and to find out desirable modifications as experiments proceeded; but if it were afterwards found that any such could be attained by using simpler uncorrected lenses in the first place, then that was obviously in the nature of an advantage. He thought the suggestion made deserved very careful consideration and hoped that Mr. Chalmers himself might be induced to work it out in a concrete form.

Mr. Chapman Jones finally remarked that he wished to add his testimony as to the extreme beauty of the results obtained by Mr. Rheinberg; the difficulty of adjusting the apparatus was not so great as one might perhaps imagine.

* *British Journal of Photography*, " Colour Supplement," April 7, 1911.

SOME POINTS IN THE USE AND DESIGN OF REFRACTOMETERS

By Thomas Martin Lowry, D.Sc., F.C.S.

(*Friday, June 21st*, 1912.)

1. *Choice of Wave Lengths.*—During many years it has been customary in measuring the refractive indices of solids and liquids to select as the standard wave lengths the D line of sodium and the C, F and G lines of hydrogen. This choice was probably based solely upon expediency, the sodium flame and the hydrogen vacuum tube forming part of the most commonplace laboratory equipment; the four lines also appear in the form of absorption bands as Fraunhofer lines in the solar spectrum, and served as convenient points of reference at a time when sunlight was still relied upon as one of the main illuminants in optical work. On other grounds the choice would be difficult to justify, but the force of tradition and the necessity for keeping in touch with previous measurements have up to the present sufficed to compel the majority of experimenters to adhere to the traditional standards. At the present time, however, there are already many signs of revolt amongst pioneer workers, and the time will soon be ripe for an authoritative inquiry as to the best standards to select for general use in this and in other branches of optical work.

2. *Disadvantages of the Present Standards.*—The disadvantages of the existing standards are sufficiently obvious. The principal standard of refraction is a doublet, and, of the three hydrogen lines used to express the dispersion, only the first (the red C line) is of any merit, whilst the third (the violet G line) is so weak as to be difficult to use even under favourable conditions. The best proof of the inadequacy of the existing standards is to be found in the fact that they have been so widely abandoned in all work outside the ordinary routine. Thus, in the case of *optical glasses*, trade specifications are still given in terms of the C, D, F and G lines, but Mr. Gifford in his recent investigations (*Proc. Roy. Soc.*, 1902, **70**) has shown that these are far from being the most suitable lines for use in designing achromatic lenses, and has found it desirable in practice to substitute the series—

He 7066, Na 5893, **Pb 5607,** Fe 5270, **Cd 4678,** H 4341, or better Cd 4415, Hg 4046.

Again, in measuring the *dispersive power of gases*, C. and M. Cuthbertson in their most recent work (*Proc. Roy. Soc.*, 1910, **84**, 13), after actual experience of the use of hydrogen lines, have abandoned them altogether in favour of the series—

Li 6708, Cd 6438, Hg 5790, Hg 5770, Hg 5461, Ag 5209, Cd 5086, Cd 4800.

Previous workers in this field have used lines in the spectra of Li, Na, Tl, Hg and Cd; but hydrogen lines have been conspicuous only by their absence. Turning to other branches of *interferometry* it may be noted that whilst Tutton in his "elasmometer," (*Proc. Roy. Soc.*, 1903, **72**, 193) made use of the red hydrogen line, his wave-length

comparator (*Phil. Trans.*, 1909, A, **210**, 1) has been used chiefly with the red cadmium line, Cd 6438. In this respect he has followed the example of Michelson, who selected as the basis of his *standard measurements of wave length the* lines—

Cd 6438·4722, Cd 5085·8240, Cd 4799·9107.

Finally, it may be noted that Dorn and Lohmann in their measurements of the *refractive indices of liquid crystals*, by a total reflexion method (*Ann. Physik.*, 1909, **29**, 535), have used the series—

Li 6708, Na 5893, Hg 5461, Hg 4359.

This series is identical with that which the writer, after several years' experience with nearly thirty wave lengths, has found to be the only satisfactory series for routine work in the measurements of *optical and magnetic rotatory dispersion.*

3. *Suggestions for New Standards.*—It may be doubted whether it will be possible to find any one series of standard wave lengths which will satisfy all the diverse requirements of the different branches of optical measurement. In particular, the standards selected for optical glasses, as being most helpful in the design of achromatic lenses, may well be different from those used to characterize the optical qualities of an organic liquid or of a solution. But in the latter case *it is obviously of great importance, in correlating the different optical properties of a substance, to be able to use observations made with the same series of wave lengths.* This must, indeed, be one of the decisive factors in choosing standards that shall be of permanent utility. It leads at once to the conclusion that the light must be both pure and intense, in order that it may be used, not only for measurements of refraction, but also for measurements of optical and magnetic rotatory power and for measurements by interferometer methods. The ideal series would undoubtedly include Michelson's three cadmium lines, but unfortunately, in spite of its occasional use in special work, the enclosed cadmium arc is still a curiosity which has been seen and handled by very few workers; the open silver-cadmium arc described by the writer in 1909 (*Phil. Mag.*, **18**, 320), has rendered valuable service in making the cadmium spectrum more accessible, but is not altogether suitable for general laboratory use. At the present stage of development, therefore, there can be little doubt that the mercury spectrum must figure very largely in any new standard series of wave-lengths. The green mercury line Hg 5461 has, indeed, already displaced the sodium doublet as principal standard in a considerable variety of measurements; it is the principal standard in all Cuthbertson's later work on the dispersive power of gases, is rapidly displacing sodium for everyday use in polarimetery and saccharimetry, and has proved to be the only possible standard in the author's experiments on the optical rotation of long columns of quartz (*Proc. Roy. Soc.*, 1912); its spectral purity is but little inferior to that of the cadmium lines, whilst its extreme steadiness and ease of production * render it capable of far more widespread acceptance.

* Some new mercury lamps specially designed for polarimetric work have been described in the *Trans. Faraday Soc.*, 1912, and are exhibited in the Loan Collection in connection with the Convention.

The violet line Hg 4359 must also receive attention, since it is the only line in the blue or violet region of the spectrum with which polarimetric measurements can be easily and satisfactorily made. In refractometry it has the advantage that it lies so close to the hydrogen line H 4341, that it is scarcely possible to distinguish the two lines; but in a vacuum tube containing both hydrogen and mercury * the mercury line, being slightly less refrangible, dominates the hydrogen and gives very satisfactory readings. The two more refrangible satellites of the violet mercury line obviously do not affect its use in refractometry : in polarimetry they increase the readings by about one part in 10,000. In my own practice, I have used the green and violet mercury lines for all essential measurements; auxiliary measurements have been made with the sodium doublet, in order to keep in touch with earlier measurements, and with Li 6708 as the best and deepest of the red lines at present available. The series Li, Na, Hg, Hg is identical with that arrived at quite independently by Dorn and Lohmann, and appears to be the most suitable for general use at the present time.

4. *Form of the Dispersion Curves.*—The difficulties outlined above would, to a very large extent, disappear if the form of the dispersion curves were accurately known, and measurements could be made systematically, in order to determine the constants in some general equation expressing the variation of the property with wave length. The writer has found that in the case of a large number of simple organic compounds, the optical and magnetic rotatory dispersions may be expressed by simple formulæ of the type $\alpha = \frac{\alpha_0}{\lambda^2 - \lambda_0^2}$. In this equation the absolute optical or magnetic rotatory power of the substance is expressed by a single constant α_0, whilst the rotatory dispersive power is expressed by a second constant λ_0, which differs in magnitude according as the optical or magnetic rotation is considered. When a formula of this sort has been established, the actual wave lengths employed in determining the constants are of very little importance, as any suitable pair should give the same results. In practice it has been found that the ratio $\frac{\alpha\ 4359}{\alpha\ 5461}$ affords a convenient and a sufficient gauge of the rotatory dispersive power of a substance of simple type; but in breaking new ground it is advisable to make measurements with a third (or a fourth) wave length, *e. g.* Li 6708, in order to be sure that the dispersion can actually be represented by the simple formula given above. Refractive dispersion is obviously a more complex property than rotatory dispersion, and must involve at least three constants and measurements with three wave lengths. But in the opinion of the writer it is almost a waste of time (to which he himself must plead guilty) to multiply observations of $n_C n_D n_F n_G$, or any analogous series, until serious work has been undertaken on the dispersion-law for typical organic compounds in the infra-red, visible and ultra-violet regions of the spectrum. Work in this direction has been carried out in the case of crystals, but no service-

* Such tubes are very convenient in providing a mercury spectrum for use with the refractometer.

able range of data is yet available for the organic compounds which form the chief subject of refractometric investigations.

5. *Calibration of a Refractometer.*—The experiments of Mr. Gifford have shown that marked variations of refractive index occur in prisms of the same glass, even when taken from different parts of the same melting. It is, therefore, unwise to accept the tabulated data for a refractometer without checking the actual performance of the instrument. The most convenient method of checking the instrument is by means of a piece of quartz, cut so as to give both the ordinary and the extraordinary ray; fluorite, calcite and silica may also be used. Mr. Gifford, who had already published (*Proc. Roy. Soc.*, Feb. 13, 1902; March 3, 1904) measurements of the refractive indices of these substances for twenty-six wave lengths, very kindly undertook, at the author's request, to add values for those lines in the spectra of lithium, cadmium and mercury, which promise to be of greatest importance as standards in future measurements; the values obtained are reproduced in the following table to five places of decimals—

Table of Refractive Indices

Wave Length.	Fluorite.	Quartz (ordinary ray).	Quartz (extraordinary ray).	Calcite (ordinary ray).	Calcite (extraordinary ray).	Vitreous Silica.
6708 Li . .	1·43226	1·54146	1·55047	1·65367	1·48427	1·45607
6438 Cd . .	1·43272	1·54231	1·55135	1·65503	1·48468	1·45677
5461 Hg . .	1·43499	1·54617	1·55534	1·66165	1·48789	1 46015
5086 Cd . .	1·43619	1·54824	1·55748	1·66527	1·48954	1·46190
4800 Cd . .	1·43691	1·55013	1·55945	1·66861	1·49110	1·46357
4359 Hg . .	1·43952	1·55380	1·56323	1·67518	1 49410	1·46674
4046 Hg . .	1·44153	1·55715	1·56671	1·68134	1·49691	1·46968

The actual calibration of a Pulfrich refractometer with quartz gave the following figures—

	Li 6708	Na 5889	Hg 5461	Hg 4357
Ord. ray.	1·61687	1·62210	1·62635	1·64454
Ext. ray.	1·61687	1·62212	1·62634	1·64450
Mean.	1·61687	1·62211	1·62634	1·64452

The nominal refractive power of the prism for sodium light was 1·62197, an error of 0·00014.

6. *Adaptation of Refractometer Tables to New Series of Wave Lengths.*—From the data given above it is possible to deduce the refractive power of a liquid from the angular readings of the refractometer for the new series of wave lengths. The trouble of calculation may, however, be avoided by combining the existing tables with a series of correction curves (Fig. 1). Thus for—

Lithium, *take* μ for Na + H red corr. (neg.) + curve corr. (pos.).
Sodium, *take* μ for Na + curve corr. (pos.).
Hg. green, *take* μ for Na + H red corr. (pos.) * + curve corr. (neg.).
Hg. violet, *take* μ for Na + H violet corr. (pos.) + curve corr. (neg.).

For these curves, and for the data on which they are based, I am indebted to Mr. F. Tinker, B.Sc.

7. *Temperature Regulation.*—The water-jacketing usually supplied with the Pulfrich refractometer, is very inefficient and far below the standard of modern

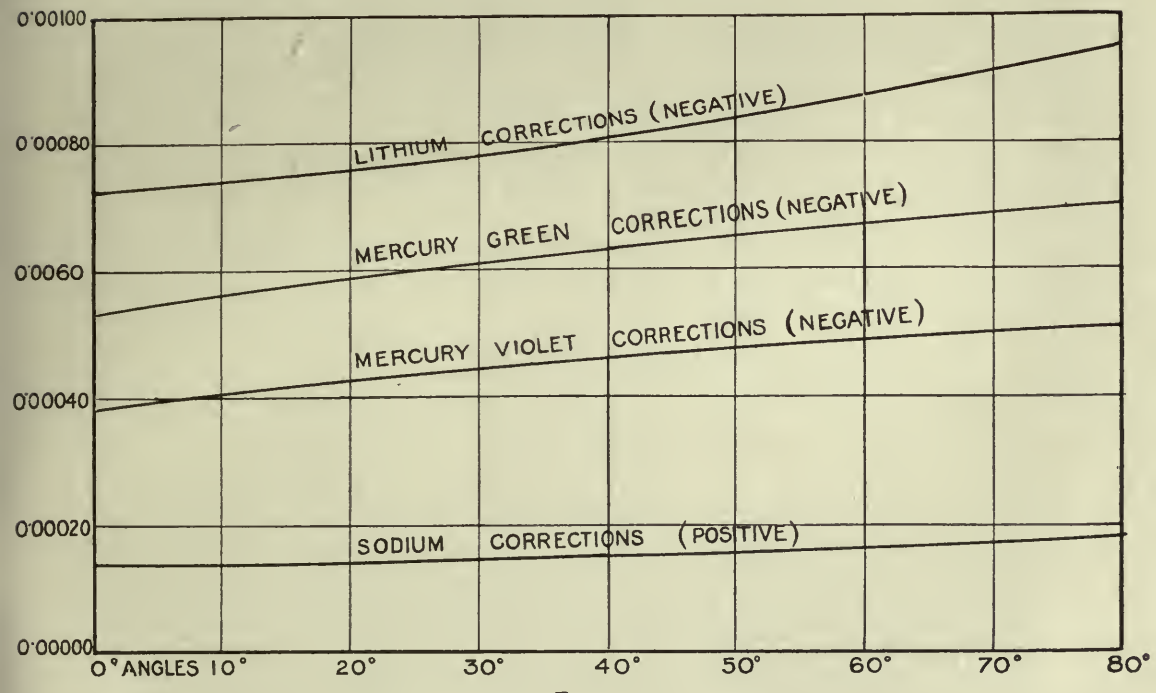


FIG. 1

requirements. All the leads are of such narrow bore that it is impossible to drag any adequate flow of water through them even with the help of a powerful suction pump. It is, therefore, impossible to keep the temperature of the flow even approximately the same as that of the water in the thermostat, unless the room be heated practically to the same temperature and the bath kept cool by a flow of cold water acting in opposition to the usual regulated heater. The necessity of working at room temperature is also obvious from the fact that there is no external jacketing to the liquid, which is merely surrounded by a piece of wood. The inadequacy of this

* Note the reversal of the sign.

arrangement was proved by actual tests. With the room at 20° C. and the water circulation at 20° a micrometer reading of 1° 26·3′ was obtained for one of the octyl alcohols: with the whole apparatus at room-temperature at 16·1° C. the reading was 1° 7·6′, a change of 4·8′ per degree C. With the room at 16·1° and the bath at 20°, the highest temperature that could be reached in the refractometer was 19·7°; but the reading of the instrument was then 1° 22·9′, corresponding with a temperature of 19·3°, *i. e.* the liquid was actually 0·4° below the temperature of the water-flow in the refractometer, for a temperature difference of only 4°. From this test it appears very probable that at 60° there would be a discrepancy of 4° and at 100° a discrepancy of 8° between the temperature of the liquid under test and that recorded for the water circulation by which its temperature is nominally controlled; certainly all measurements that have been made with this instrument in the higher ranges of temperature must be looked upon with very grave suspicion. A refractometer prism, with a jacket designed to overcome these defects, has been constructed; but, at the time of writing, the instrument has not yet been completed, and it is not possible to describe any actual tests of the efficiency of the jacketing.

In the discussion Dr. C. E. K. Mees endorsed the value of the mercury lines, especially in the ultra-violet, for the further reason that so much energy could be concentrated in the quartz lamps. The line 3650 was the most powerful in the ultra-violet, the line 2566 was also very useful, and they gave practically a monochromatic light. The cooling of the instruments had been much improved of late by the introduction of small centrifugal pumps.

Dr. W. Rosenhain said that the Optical Society had some years ago appointed a committee for the consideration of new standards of wave-lengths; he had found the preference for the old system of lines deeply rooted in Jena, however.

Dr. Schuster reminded the meeting that the structure of the much-praised green Hg-line was very complicated and quite unsuitable as a standard; the Cd-lines alone seemed suitable, because they remained homogeneous under all optical powers.

Dr. M. von Rohr remarked that they had at Jena stood by the Na- and Hg-lines, because their accuracy was sufficient for the purposes of the manufacturing optician. They should bear in mind that a great many meltings had been measured in this old way, and it seemed a pity to throw the data of refraction, dispersion and relative dispersion away; the reduction from one system of measurements to another would be very troublesome.

Mr. S. D. Chalmers thought that the adoption of a secondary series of spectrum lines would be an advantage, and such a series should extend into the extreme ultra-violet; there was a great gap between the Cd- and the Al- lines, and textbooks differed widely as to the wave-lengths which had been used in measuring standard refractive indices.

Dr. SCHUSTER considered that the question should be left to the Committee on Wave-lengths of the International Union for Solar Research. The sodium lines were quite hopeless, of course, except in vacuum tubes; as regards the hydrogen lines, much depended upon the circumstances under which they were produced.

Dr. LOWRY, in replying, pointed out that the mercury lamp was of value not only in the visible spectrum and (as Dr. Mees had suggested) in the ultra-violet, but also in the infra-red. In calculating a constant-deviation spectroscope from wave-lengths 7000 to 17,000 the infra-red line at 10,140 had proved most valuable as a principal fixed point on the calibration curve, whilst the other mercury lines had provided a check upon the accuracy of the course of the curve. Two mercury lamps especially designed for spectroscope work were shown in the Loan Exhibition.

THE TREND OF GEOMETRICAL OPTICS

By SILVANUS P. THOMPSON, D.Sc., F.R.S.

(*Friday, June 21st*, 1912.)

THE purpose of this communication is to explain certain points in the methods of geometrical optics, to describe certain models based thereon, and to raise some questions as to notation and nomenclature.

The method of Gauss for treating the properties of an optical system of centered lenses by means of his conception of points and planes is now all but universally accepted. It presents a scheme for demonstrating the performance of an instrument on the assumptions not only that the aberrations of definition of the image are absent, but also that the well-defined image is devoid of curvature and distortion—in other words, that the optical system is both stigmatic and collinear. It furnishes the means of finding the image-point which is conjugate to any object-point, without any consideration of the particular refracting surfaces or thicknesses of the lenses, for it reduces all centred optical systems, whether the refracting surfaces be spherical or aspherical, to a species of equivalent lens, of which it is needful merely to know the two principal foci and the two principal points (Haupt-punkte), together with the respective focal planes and principal planes drawn orthogonally through them. The width between the principal planes (constituting, so to speak, the reduced thickness of the equivalent lens) being known, and the focal lengths as measured from the principal planes to the respective principal foci being also known, the whole performance of the optical system can be stated in the simplest way in terms of these data. For, the width between the principal points and planes simply does not

count in the calculations; and if the distances of object-plane and image-plane measured from the respective principal planes are reckoned, as in ordinary geometry, as positive if measured to the right and negative to the left, the following relations hold good—

$$\frac{1}{u}+\frac{1}{f}=\frac{1}{v}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$-pq=f^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\mathrm{M}=v/u; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where f is the principal focal length (assuming the first and last media to be alike both air); u the distance of the object-plane from the first principal plane; v the distance of the image-plane from the second principal plane; p the distance of the object-plane from the first principal focal plane; q the distance of the image-plane from the second principal focal plane; M the linear magnification. In either form the equations (1) and (2) are simply an expression of collinear relationship.

To these the original features of Gauss's scheme Listing added the two nodal points which coincide with the two principal points only if the first and last media are alike (both air), but otherwise are displaced toward the side on which the end medium is the denser. To the system of points thus conceived Casorati assigned the name "cardinal" points. To them Toepler added another pair, which he termed "negative principal points," but which, for reasons presently to be discussed, I prefer to call the "symmetric" points. These points are situated at distances respectively equal to one focal length beyond the principal foci on the two sides. They are conjugate to one another. Planes drawn orthogonally through them are such that they are planes of unit magnification, an object in one symmetric plane having its image in the other, of equal size, but inverted. Toepler also showed that if, on the image side of the lens, planes are drawn at successive distances apart equal to the focal length, they constitute a family of planes in which the magnifications of the images formed therein differ by unity successively from one another. Mr. Blakesley has further developed this plan, and as he attributes negative signs to the magnifications of inverted images, he would write the successive magnifications, beginning with the plane through the principal focus, as 0, − 1, − 2, − 3, etc. The two principal planes of Gauss are also conjugate to one another, and are planes of unit magnification without inversion. The principal focal planes are, of course, respectively conjugates to planes at $+\infty$ and $-\infty$. The name symmetric planes is apt, not only because these two planes lie symmetrically right and left of the system, but because of the symmetry which there exists between object and image. Any point in the first symmetric plane, having co-ordinates y,z in that plane, will have as its image a point having co-ordinates $-y, -z$, in the second symmetric plane. This circumstance lends itself to the graphic solution of the problem how to find the position, after refraction, of any given oblique ray; and is applicable even in cases where the given ray is askew in direction and never meets the axis of the system. Let Fig. 1 depict the system, having principal points E_1 E_2, principal foci

F_1, F_2, and symmetric points S_1 S_2. And let six planes be drawn orthogonally through these six points. Then let it be required to find the course of a ray of which the part PP′ is given. Produce PP′ both ways, intersecting the first symmetric

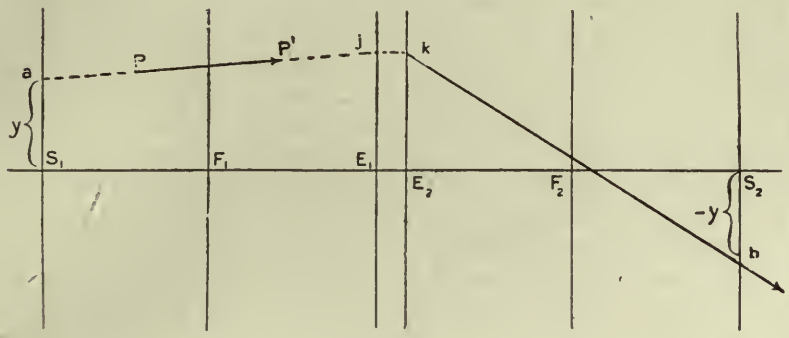


Fig. 1.

plane in *a* and meeting the first principal plane at *j*. The ordinate of *a* being *y*, set off the conjugate ordinate $-y$ in the second symmetric plane, thus fixing the conjugate point *b*, through which the ray must pass. Transferring the ray across

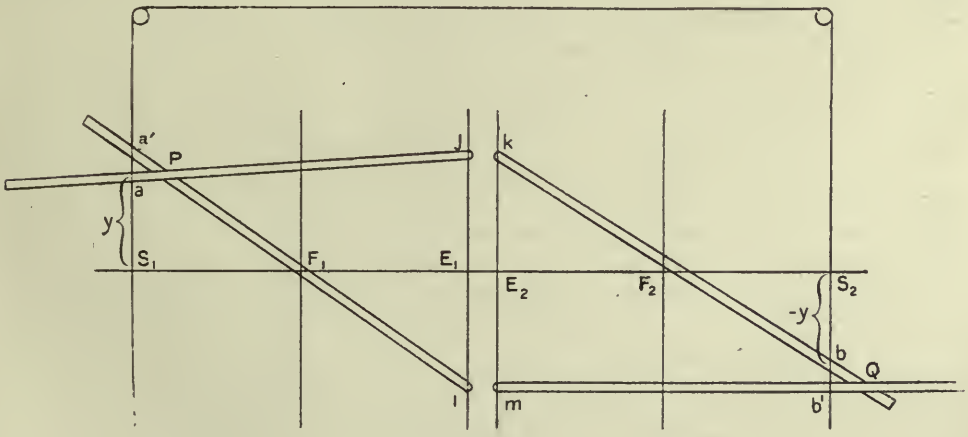


Fig. 2.

the equivalent thickness *jk* between the principal planes, join *k* to *b* by the line *kb*, which is the refracted ray sought. A model based on this property of the symmetric planes is shown in Fig. 2. It has done duty since 1886. Two pairs of wooden laths are articulated at *jk* and *lm* respectively. By a cord passing over pulleys, the distance that *b* is let down in the second symmetric plane is made equal to the

distance that a is raised in the first symmetric plane. By a second cord, the same thing is done for the points $a'b'$ for the other pair of laths. If the two laths are caused to cross one another at the point P, the other portions at once adjust themselves to cross at the conjugate point Q.

The property of the symmetric planes being at the same distances from the principal foci as these are from the principal planes was made use of in the Focometer which I described to the Royal Society in 1891 for determining the true focal lengths and positions of the principal planes of microscopic objectives. Two transparent micrometers suitably mounted are set independently at the two principal foci. They are then clamped to carriers on a long right- and left-handed screw which is parallel to the axis of the lens. By turning this screw the two micrometers are moved away from one another through equal distances until they arrive at the symmetric planes, as ascertained by their then being the image one of the other. The distance through which they have thus been displaced is the true focal length, and the positions of the principal planes are found by measuring back the same distance from the focal points. The instrument is adapted also to determine the cardinal points of camera lenses.

The existence of the family of planes of magnifications of successive integral values furnishes a useful means of readily solving a number of lens problems. For instance, if it be required to find the position where an object must be placed before a lens-system of given focal length in order that the image may be magnified m times, one has only to remember that the image must be formed in the m^{th} plane counting from the focal plane as zero, and will therefore be at $m + 1$ times the focal length from the second principal plane: and dividing this distance by the magnification m will give $\frac{m+1}{m}$ times the focal length as the distance of the object from the first principal plane.

The construction which enables one to draw directly the continuation of any incident ray suggests naturally that in a certain sense the emergent ray, taken as a whole, is the image of the incident ray, seeing that it consists of a range of points, each one of which is the image of a corresponding one in the range of which the incident ray may be considered to consist. Similarly the part of the axis itself, which lies on one side of the lens-system, may be considered in the same sense as image of the part which lies on the other side. If we confine our attention to a linear object placed in the axis, its image will be also linear, lying in the axis at some other part and, in general, of a different length. Thus in Figs. 3, 4 and 5, the object PP', an arrow, will have its image at QQ'. Whatever the distance of the arrow from the lens it will point always in the same sense, and will never be inverted, that is, pointing backward. The image may be real; as in Fig. 3, or virtual, as in Fig. 4, or partly real and partly virtual, passing through $\pm \infty$, as in Fig. 5.

It is, of course, well known that the longitudinal magnification, or "elongation" as Maxwell called it, does not follow the same rule as the transverse magnification, but is, for a small object, placed at any position, proportional to the square of the

transverse magnification for the same position. There are, for example, only two positions in which the image of a small cube can be also a cube, namely, where object and image are in places of unit magnification, namely, either at the principal points, or at the symmetric points. In all other positions the image of the cube

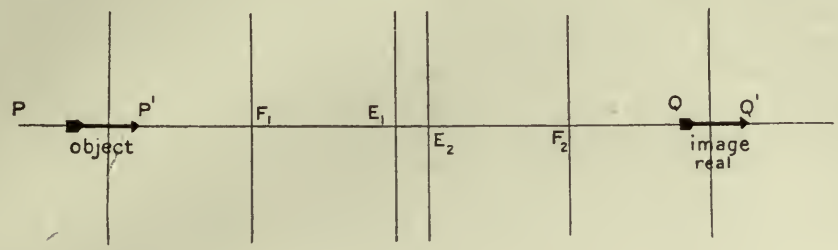


FIG. 3.

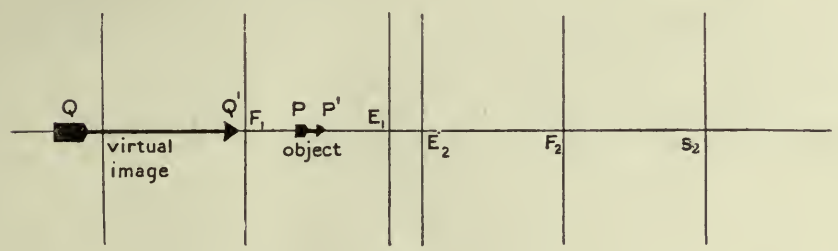


FIG. 4.

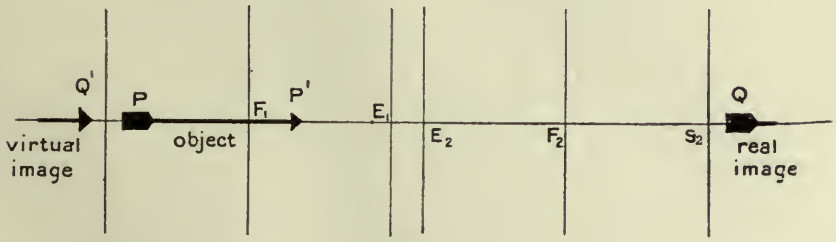


FIG. 5.

will be distorted, becoming elongated axially if the image is enlarged and compressed axially if the image is diminished. We may consider the case of a series of objects all of the same height, placed along the axis like a row of telegraph poles, spaced equidistantly as in Fig. 6. Their images will constitute a harmonic range of lines of diminishing length. The object at the symmetric point will have an equal,

inverted image, and all the others to infinity will have their images crowded together, their heads lying on the sloping line running through the second focal point.

This unequable crowding together of the images of more distant objects is responsible for the curious effect noticeable in modern binocular field-glasses of high power, but seen also in opera-glasses and telescopes, in which the successive planes of landscapes seem exaggerated, and flattened almost like the flat scenery of the theatre. Thin trees and hedges, for example, seem to occupy definite planes; and the more distant objects appear to be compressed up toward those in front of them. This phenomenon, to which Dr. von Rohr has given the name of *porrhallaxia*, is not due, as some suppose, to any binocular hyperstereoscopic effect, but is the consequence of the law of longitudinal magnification.

It was stated above that the system of Gauss is now all but universally accepted for the treatment of lens systems. The exceptions are two. In the case of the formation of images upon the retina of the eye the collinear relation ought not to

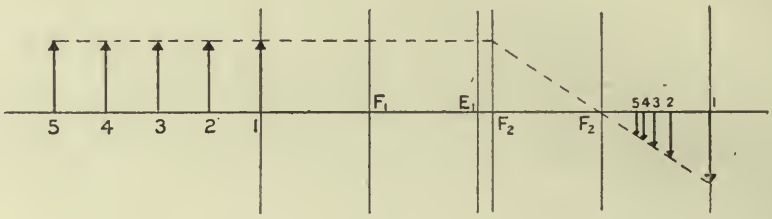


FIG. 6.

be fulfilled, since the images are not required to be thrown upon a plane, but on the interior of a globe; and as Listing, and more recently Gullstrand, have pointed out, the fact that the centre of rotation of the eyeball does not coincide with the second nodal point of the eye, introduces into the problem of free vision considerations outside those of instruments for which the eyeball may be regarded as fixed in direction.

The other exception is the system proposed by Biot (1844) and Bosscha (1876), and adopted by Cornu (1877), and more recently by Sissingh (1900), in which no reference is made to principal planes. In this system the distances of object and image are measured, as are also the focal lengths, from the vertices of the first and last refracting surfaces of the instrument, and the performance is expressed in terms of four constants: the angular magnification; the focal power; the angle subtended at the first vertex by unit length placed at the last vertex; and the "dilatancy" of the system, this last term meaning the ratio of the diameter of the emerging conical pencil as it traverses the last surface to the diameter of an incident cylindrical pencil where it traverses the first surface. It is claimed for this method of treatment that the distances of object and image are more readily measured than in the system

of Gauss; but on the other hand, the mathemetical relations are less direct, and do not lend themselves with equal facility to extended use when proceeding to consider the various aberrations.

In connexion with the relation of the magnification of lenses as dependent on the relative distances of the conjugate points, mention may be permitted of a simple process communicated by the author to the Optical Society in 1899 for deducing the power of a (positive) lens from its magnification. From equation (2) preceding, we have $\frac{M}{v} = \frac{1}{u}$. If we substitute this value in equation (1) and remember that for the production of a real image u and v must be of opposite signs, we obtain for the power P of the lens—

$$P = \frac{1}{f} = \frac{M + 1}{v}.$$

If, therefore, we so adjust the distance of the object from the lens that $v = 1$, we have the relation—

$$P = M + 1.$$

The model proposed by the author to carry out the method consisted in a frame wherein the lens whose power was to be measured was set up at a distance of 1 metre from a vertical scale graduated in centimetres. As object was used an aperture 1 centimetre square placed in front of the flat flame of a paraffin lamp. The object was moved to such a distance from the lens that its image was sharply focussed on the scale, enabling the size of the magnified image to be read off directly; when the addition of 1 to the linear size of the image at once gave the power of the lens in *dioptries*.

It is germane to the subject to suggest that some improvement is needed in defining the different uses of the term magnification. Most commonly the term is applied to the *transverse* (or linear) *magnification*, the ratio of a vertical ordinate in the image to that of the object, as depicted in the ordinary diagrams of the text-books. But there is also a horizontal transverse magnification, identical indeed with the vertical magnification if the optical system consists of spherical surfaces only. But the optical system may be an astigmatic one of special design to give different magnifications in azimuth and altitude. Secondly, we have to consider the *longitudinal magnification* or *elongation* already alluded to; and, thirdly, there is the so-called *angular magnification*, the ratio of the magnitude of an angle in the image to that of the corresponding angle of the object. Let there be an optical system, Fig. 7, such that P Q are two of its conjugate points. At P, an object, let there be placed in the plane of the diagram a small rectangle of height y and length x; its image at Q will have height y' and length x'. The transverse magnification $M = y'/y$; the longitudinal magnification $L = x'/x$. The diagonal of the object rectangle makes an angle $\alpha = \text{arc tan } y/x$, and the image of that angle is the angle $\alpha' = \text{arc tan } y'/x'$. The ordinary statement of the angular magnification is to define it as the ratio α'/α. It would be better to adopt the plan, certainly in use by more than one Continental

writer, of defining the term *angular magnification* as tan α'/tan α. For then, since $L = M^2$, we have the relation—

$$\text{Angular magnification} = \frac{\tan \alpha'}{\tan \alpha} = \frac{y'}{y} \cdot \frac{x}{x'} = \frac{M}{L} = \frac{1}{M}.$$

In other words, the angular magnification thus defined is the reciprocal of the transverse magnification.

But there are yet other senses in which the word magnification is used, namely, in the term the *apparent magnification* produced by a simple magnifying glass, which is the ratio of the angle subtended by the visual image (transverse)—formed presumably at the *punctum proximum* of the observer's eye—to the angle subtended by the object itself at the place where it is. There is, similarly, an apparent longitudinal magnification, and an apparent angular magnification. Further, there are the extensions of these terms to the cases of telescopic and microscopic vision.

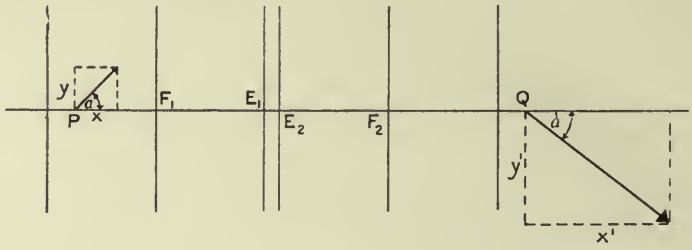


FIG. 7.

It is much to be desired that some general agreement should be arrived at amongst opticians and physicists not only respecting the use of these terms and their proper definition, but also respecting the right use of certain other terms and conventions. At present there are wide differences in usage as to the terms positive and negative. In lens problems some authorities reckon axial lengths positive when measured, as mathematicians always do, to the right of the origin; others reckon lengths positive when measured in the sense opposed to the light. Some take as the origin the surface of the lens, others always reckon from the "principal" points. Some writers call a divergent lens positive, while in the optical trade universally a positive lens is understood as a convergent one.

Further, suitable English terms are much needed for several matters for which at present none are recognized. The central point on the face of a lens is sometimes called its "pole," or even its "vertex"; the German term is *Scheitel*. A name is wanted to denote the distance between the pole of a lens and its principal focus; the German term being *Schnittweite*. We possess no equivalents for the three most useful German terms, *Durchbiegung*, *Durchrechnung* and *Abbildung*. An English term is also needed for the phenomenon called *Porrhallaxia* introduced by Dr. Moritz von Rohr for the phenomenon due to longitudinal magnification, mentioned above.

Mr. T. H. Blakesley took up the challenge of Dr. Thompson upon the subject of the sign of an inverted image, which he contended, with a demonstration, was negative by mathematical necessity, and was not a mere matter of convention like the sign of a radius of curvature or of a focal length.

Another loose point in academical teaching was the determination of the character of a focal length. A Huyghens eyepiece, for instance, had a focal length of the character of a thin double *convex* lens: otherwise it would not give an inverted aspect when used as the eyepiece of a telescope. But if light from a distant object be admitted on the side usually turned towards the eye, the image that results is inside the structure, *i. e.* behind the last surface, and therefore virtual, as would in this respect be the case with a thin double *concave* lens.

Mr. Blakesley derided the academical practice of classifying lenses by the thickness in the middle exceeding or not exceeding that at other points. A double convex lens, if of sufficient thickness between the surfaces, has a focal length of the general character of a thin double *concave* lens. It is a mortal mistake that such errors, and others like them, are being year by year instilled into the minds of technical students, by academical examiners.

Mr. Conrad Beck agreed with Dr. Thompson as to reckoning distances positive or negative; but he also thought Mr. Blakesley's argument sound as to magnification. The signs should indicate whether the image was erect or reversed, and the calculations should be made as mechanical as possible so as to give the full results without re-examination of the whole problem. Converging lenses should be positive in accordance with the practice of the opticians. With regard to nomenclature he would support Dr. Thompson; some new words had to be coined, and a committee might be appointed.

Dr. M. von Rohr: With regard to the etymology of the word *porrhallactic*, it is formed similarly to *parallati*, viz. of *porrho*, πόρρω = far, distant, and of *allatto* ἀλλάττω = I alter. As to the use of the word, I have introduced it to designate a certain property of telescopes, which cannot, I think, be explained by means of the Gaussian theory. This theory takes for granted that the images of the objects are observed, where they are formed, but this supposition does not hold good with every optical instrument. If we suppose, for instance, a telescope properly focused for very distant objects, it will form an image of infinitely distant objects at infinity. An emmetropic observer of such an age that his power of accommodation is gone will be able to see objects lying at a sufficiently great distance with such a properly focused telescope, although he cannot alter his accommodation. If the telescope be of a sixfold magnification, and the two distant objects, *e. g.* houses perceivable in the field of the telescope have in reality a relative distance of 600 yards from one another, they will appear in the telescope as only $\frac{1}{6} \times 600$ yards = 100 yards apart. This effect of altering the distance I have called *porrhallactic* effect.

Mr. T. SMITH expressed his surprise that the use of focal lengths instead of focal powers, and of such a formula as $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$ should have survived till the present time for the purposes of numerical calculation. It was a great deal simpler to measure distances from the principal foci and to use such formulæ as $pq = -f^2$, or what was equivalent, to use the planes of integral magnification ($-\frac{f}{p} = \frac{q}{f}$ = linear magnification), as the President had so strongly urged. Another advantage of measuring from the principal foci was this, that they were points which could be easily and accurately found by experiment; of the principal points, on the other hand, this was not true, and there was the additional disadvantage that an erroneous view was very widely held to the effect that the principal planes were situated at the first and last surfaces of any and every lens. Probably our elementary textbooks were largely responsible for this heresy through the unnecessary amount of attention they gave to the purely hypothetical "thin" lens. While he fully agreed with the President's statements of the great advantage of the system of Gauss, it should not be overlooked that there were limits to its applicability: in the case of telescopic systems, where the focal length was infinitely great and the unit planes were at infinity, it became necessary to revert to the rival method, but it was not perhaps too much to say that in all other cases the Gaussian system was much more convenient. As regards the convention to be adopted for the sign of the magnification, he wished to add another objection to that advanced by Mr. Blakesley. In working out aberrations calculations were made step by step, and on proceeding from one surface to the next the origin of the three-dimensional co-ordinates was changed without altering the directions of the axes. If an inverted image was regarded as having positive magnification, it would be necessary at the end of a set of calculations to arbitrarily reverse the directions of two of the axes, and that would be most objectionable. It was no doubt rather simpler for the mathematician if a converging lens was considered one of negative power, but as the opposite convention was so widely adopted, there was very little trouble involved in defining the fundamental variables to make a converging lens positive and an inverted magnification negative. He would like to add to the list of "names wanted" which the President had mentioned in dealing with optical nomenclature, short and, if possible, descriptive terms for the higher order aberrations.

Mr. S. D. CHALMERS agreed with Prof. Thompson's notation, but accepted Mr. Blakesley's contention that a reversed image should be indicated by a minus magnification. He thought, too, that those writers who wished to call a converging lens negative were not justified in using the term + 1D for a diverging lens, as this term had a well-established meaning.

Mr. LAN-DAVIS wished any committee on notation also to take up the question of symbols like n or ν.

Mr. S. A. WILLMOTT pointed out that the formula $xx^1 = ff^1$ or $-f^2$ for a surface or thin lens was older than the formula of reciprocals, being Newton's Axiom VI. It was true that Halley had shortly before offered the inversion of the reciprocal formula, namely $v = \frac{uf}{u+f}$; but it could be shown that Newton derived his formula independently and previous to Halley's solution.

Prof. SCHUSTER thought the suggestion of a small committee on notation might be taken up by the Convention. As regards symbols they were on more delicate ground, considering that the system of symbols introduced by the University of Berlin was in almost universal use.

Dr. S. P. THOMPSON, briefly replying, said that he was himself convinced by Mr. Blakesley's arguments. He then demonstrated the strange shapes which the shadow of a thin rod would assume when illuminated by an arc lamp and a lens placed obliquely at a distance from the lens, producing a non-homocentric pencil of light.

THE IDEAL PHOTOGRAPHIC LENS

By T. SMITH, B.A., of the National Physical Laboratory

(*Thursday, June 20th*, 1912.)

IT has been known for a long time that an optical instrument cannot produce over a large three-dimensional field the point-to-point correspondence usually associated with elementary geometrical optics. The inconsistence of the two sine conditions at a point on the axis where there is no spherical aberration, $\sin\theta = m \sin\theta'$, for freedom from coma, and $\sin\frac{\theta}{2} = m \sin\frac{\theta'}{2}$ for freedom from spherical aberration at a neighbouring point on the axis, where m is the linear magnification normal to the axis, and θ and θ' are the angles made with the axis by the incident and refracted parts of a ray, illustrates the impossibility of collinear representation. So far as the author is aware, our knowledge stops at the realization of this impossibility, no information being available as to the extent to which an instrument necessarily fails to reach this standard. In the present paper are given the results of an investigation into this question when the simple aberrations—spherical aberration varying as the cube of the aperture, coma varying as the square of the aperture and as the distance of the object point from the axis, curvature and astigmatism varying directly as the aperture and as

the square of the distance from the axis, and distortion varying as the cube of the distance from the axis, all these five aberrations being measured in the image plane—alone are considered.

One of the first questions to be answered is: Can an instrument give perfect definition in two planes separated by an appreciable interval? The two sine conditions show that this cannot be done for two contiguous planes, and, in fact, it is easy to see that with any particular instrument only one plane can be perfectly defined. For suppose that any incident ray meets three planes normal to the axis of the system in (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), (X_3, Y_3, Z_3), and the corresponding refracted ray meets the planes which, on the elementary geometrical theory, are called their image planes in (X_1', Y_1', Z_1'), (X_2', Y_2', Z_2'), and (X_3', Y_3', Z_3') respectively. If the magnifications for these planes are m_1, m_2, m_3, and the axis of Z is along the axis of the system, we have—

$$Z_1 - Z_2 : Z_2 - Z_3 = \frac{1}{m_1} - \frac{1}{m_2} : \frac{1}{m_2} - \frac{1}{m_3}$$

and—

$$Z_1' - Z_2' : Z_2' - Z_3' = m_1 - m_2 : m_2 - m_3.$$

Now the equations of the rays give—

$$\frac{X_1 - X_2}{X_2 - X_3} = \frac{Y_1 - Y_2}{Y_2 - Y_3} = \frac{Z_1 - Z_2}{Z_2 - Z_3}$$

and—

$$\frac{X_1' - X_2'}{X_2' - X_3'} = \frac{Y_1' - Y_2'}{Y_2' - Y_3'} = \frac{Z_1' - Z_2'}{Z_2' - Z_3}$$

and hence—

$$X_1 m_1(m_2 - m_3) + X_2 m_2(m_3 - m_1) + X_3 m_3(m_1 - m_2) = 0$$

and—

$$X_1'(m_2 - m_3) + X_2'(m_3 - m_1) + X_3'(m_1 - m_2) = 0$$

or

$$(X_1' - m_1 X_1)(m_2 - m_3) + (X_2' - m_2 X_2)(m_2 - m_1) + (X_3' - m_3 X_3)((m_1 - m_2) = 0$$

and a similar equation with Y's' in place of X's'.

This equation shows that if for two different magnifications, m_2 and m_3, there is point-to-point correspondence—

i. e.—

$$X_2' = m_2 X_2 \qquad Y_2' = m_2 Y_2$$

and—

$$X_3' = m_3 X_3 \qquad Y_3' = m_3 Y_3$$

then for *any* other magnification m_1—

$$X_1' = m_1 X_1 \qquad Y_1' = m_1 Y_1$$

i. e. if the image is perfect at two magnifications, there will be perfect imagery at all magnifications. It follows, then, that at only one magnification can any optical instrument give a perfect image.

The nearest approach that can be made to the collinear instrument is one which gives a perfect image for some chosen magnification, and also has the nodal points free from spherical aberration, the stop being in the nodal plane. Under these circum-

stances it is evident that there will be no distortion at any magnification, a matter of great importance in such an instrument as a photographic lens, which is used at widely varying magnifications, and has its stop at a fixed position, as distortion is the one defect

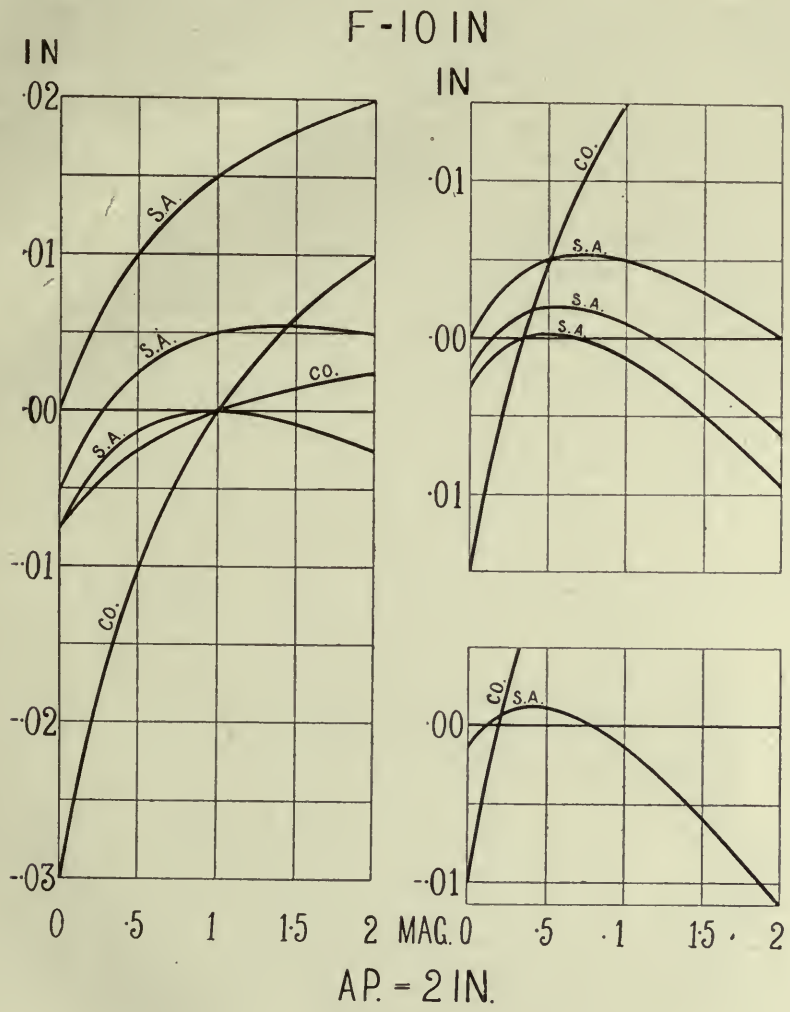


Fig. 1.

the effects of which cannot be decreased by reducing the diameter of the stop. With such a system it is found that if the first and last media are the same, besides freedom from distortion, there will be freedom from curvature and astigmatism at all

magnifications. Thus the only aberrations remaining are spherical aberration and coma, which fortunately are those of which the evil effects are most rapidly cut out by reducing the diameter of the aperture. A lens corrected on these lines may well be regarded as the ideal lens, for it is the best corrected lens for general purposes which it is really possible to make.

On investigating the way in which the coma varies in a lens thus free from distortion, curvature and astigmatism at all magnifications, it is found that the coma may be made to vanish at any given magnification, but that it will not vanish at any other magnification. In fact, the curve giving the relation between the coma and the magnification in these "ideal" lenses is always the same, but the position of the origin from which the magnification is to be measured can be fixed at any place desired. When this origin has been chosen there is one degree of freedom for the spherical aberration, but this is determined if the spherical aberration is made to vanish at the same magnification as that for which there is freedom from coma. The spherical aberration, however, unlike the coma, will as a rule disappear for a second magnification. This fact alone would lead us to expect that the coma in a system corrected on this principle will be the more serious outstanding aberration, and this is actually found to be the case. The diagram on the left of Fig. 1 shows the case of an instrument corrected to give perfect definition at unit magnification. The aperture is here taken to be F/5, and the focal length 10 inches. The vertical scale gives the greatest length of the blurred patch of light which constitutes the image of a point due to aberration. The horizonal scale gives the magnification.

In this diagram are shown two curves marked CO. The one of smaller amplitude denotes the length of the coma opposite the edge of the aperture, *i. e.* at one inch from the axis of the lens. The larger curve shows its amount at what would normally be near the edge of the plate with this focal length, viz. at four inches from the axis. The two curves only differ in the amplitude of the one being four times that of the other. In the other diagrams of Fig. 1 and in Fig. 2 only one curve is shown marked CO, and that only in part; these correspond to the larger curve on the left, and since they are exactly equal to it the values of the coma in these other cases can be ascertained from the large diagram by noting the difference in the position of the horizontal axis.

Three curves are drawn for spherical aberration, corresponding to correction for an object at infinity, minimum spherical aberration over the range of magnifications zero to unity, and correction for unit magnification. It is, of course, only in the latter case that there is perfect definition at unit magnification. It will be noticed that in this case the curve for spherical aberration touches the axis along which the magnification is measured. This is in accordance with the sine conditions, for since there is no spherical aberration or coma at unit magnification—

$$\sin\theta = -\sin\theta' \text{ or } \theta = -\theta'$$

so that—

$$\sin\frac{\theta}{2} = -\sin\frac{\theta'}{2}$$

or there is no introduction of spherical aberration on slightly changing the magnification from unity.

It should be particularly noted that at all magnifications the length of the blurr due to coma exceeds that due to spherical aberration unless the distance from the axis is quite small.

This diagram is particularly interesting on account of the extensive use in photography of double symmetrical lenses—lenses composed of two similar and equal components placed with corresponding parts facing one another and the stop midway between them. Such a lens is necessarily free from distortion and coma at unit magnification, and therefore if it is free from distortion at any other magnification and is generally free from curvature and astigmatism it must have the amount of coma shown by the curves on the left of Fig. 1. It is not, however, necessary for the spherical aberration to follow any of the curves shown, but this is a matter of small importance compared with the inevitably large amount of coma in the images of comparatively distant objects. The fact that such lenses give rise to acceptable images of distant objects with large apertures shows either that the results are appreciably modified on taking higher order aberrations into account, or that the plates by failing to do justice to the fine definition obtainable on the axis, disguise the rapidity with which the definition falls off on departing from the axis.

The upper diagram on the right of Fig. 1 shows the case of an "ideal" lens designed to give minimum coma over the range of magnifications zero to unity. The spherical aberration curves drawn are those when this aberration vanishes for an object at infinity, for minimum spherical aberration over the range zero to unity, and for a perfect image at the magnification for which there is no coma. The lower diagram illustrates the "ideal" system in which the spherical aberration over the range zero to unity, the amount of which depends on the magnification for which coma is corrected, reaches its absolute minimum. Fig. 2 illustrates the case of the lens which has its coma a minimum over the range of magnification zero to one-tenth; the spherical aberration curves drawn are for correction for an object at infinity, and for minimum aberration over the same range as is taken for the coma. In this case there is no magnification at which the imagery is theoretically perfect, but, nevertheless, this probably represents the ideal correction at which to aim for the photographic lens intended to be used for general purposes.

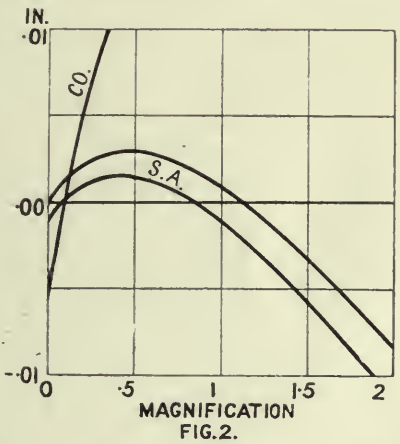


FIG.2.

For the sake of comparison with these curves which show the best results to which lenses can actually attain, Fig. 3 is added, showing corresponding results with a plano-convex lens (plane side to object), an equi-convex lens, and a convexo-plane lens (curved side to object). These show spherical aberration and coma very much greater than the

ideal lenses considered above, and the vertical scale on which the results are plotted has

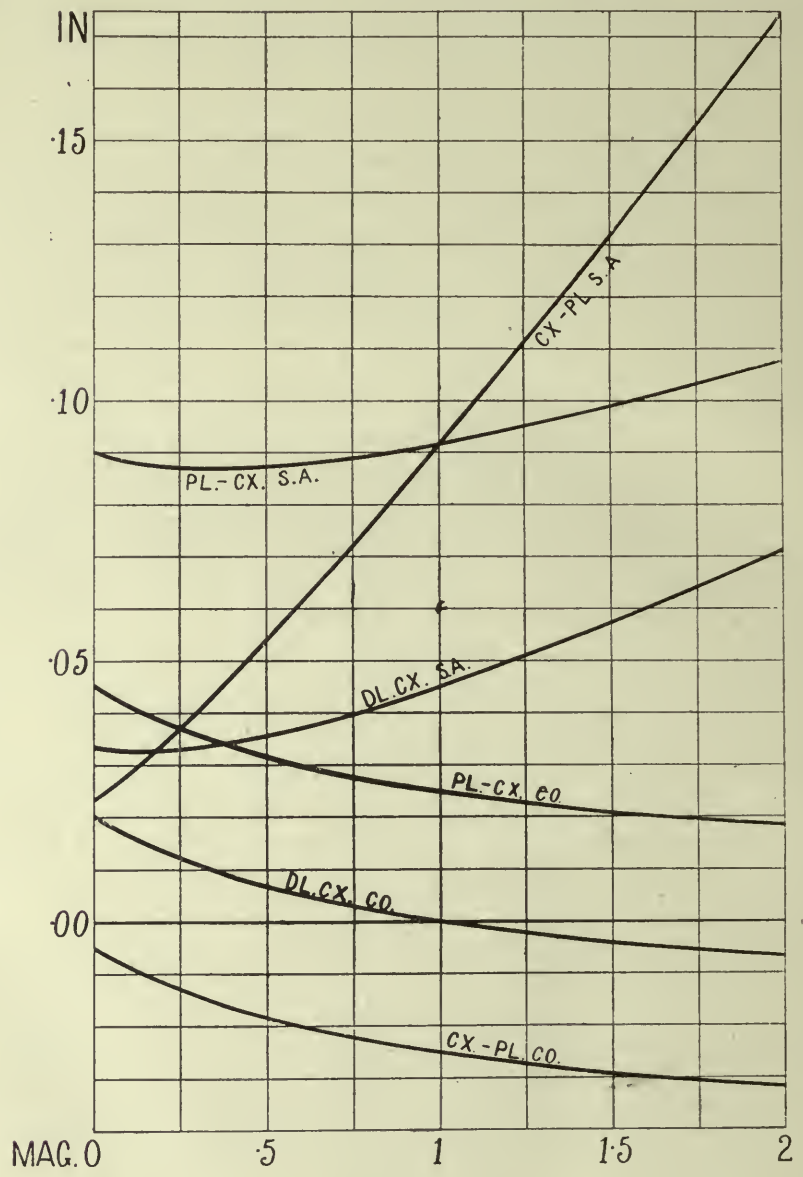


Fig. 3.

therefore been altered. As contrasted with the previous cases the spherical aberration is

large compared with the coma, and the fact that these lenses are those usually considered by students of "Light" perhaps accounts for the exaggerated importance often attached to the correction of spherical aberration, and to the fact that "coma" is quite unheard of by many of them. The coma curves shown are for one inch from the axis.

In addition to these large amounts of spherical aberration and coma these simple lenses also suffer greatly from curvature and astigmatism. Through the kindness of Messrs. Beck and Chalmers, who have supplied the essential figures, it has been possible to add Fig. 4, which shows the corresponding curves for a typical rapid rectilinear lens. It is of interest to notice that these curves show correction for spherical aberration somewhat inferior to that given by a thin double convex lens, but the coma correction, which is here drawn for an image distant four inches from the axis, is very much better than for the single lens. In comparing these lenses it must also be remembered that the rapid rectilinear lens has a much flatter field and much less astigmatism than a simple lens, and the correction of such aberrations is much more important than the elimination of either spherical aberration or coma. It may also be noted that the forms of the curves for the rapid rectilinear lens are intermediate between those of the simple lenses of Fig. 3 and those of the ideal lenses of Fig. 1.

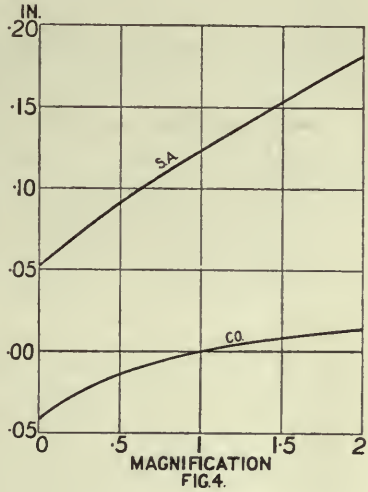


FIG.4.

In all these cases it has been assumed that the stop is placed in the unit plane; this is required for the correction of distortion, as is seen from Figs. 5 and 6, which are photographs of an object consisting of parallel straight lines taken with a plano-convex lens, the only difference between the two being that in the former the stop was in front, and in the latter behind, its proper position.

The results given above may be applied to a lens of any focal length by altering the lengths of aberrations in proportion to the focal length, the curves given being in all cases drawn for a focal length of ten inches. The values for different relative apertures and different distances from the axis are to be derived according to the law given in the first paragraph. An example will make this clear. Suppose that we wish to find the amount of spherical aberration and the coma length at a distance of two inches from the axis for stops F/8 and F/16 with a six-inch lens giving a perfect image at unit magnification. From Fig. 1 we have the spherical aberration for a ten-inch lens at F/5 equal to ·0075 inch; for the same lens at F/8 the value would be $(\frac{5}{8})^3 \times$ ·0075 inch; and therefore for a six-inch lens it would be $\frac{6}{10} \times (\frac{5}{8})^3 \times$ ·0075 inch = ·00110 inch approximately. At F/16 the length would be one-eighth of this, viz. ·00014 inch approximately. Again, a distance of two inches from the axis in a six-inch lens corresponds to a distance $\frac{10}{3}$ inches in a

ten-inch lens, at which distance the length of the coma at F/5 from Fig. 1 is $\frac{10}{3} \times \cdot 0075$ inch. At F/8 the value with a ten-inch lens would be $(\frac{5}{8})^2 \times \frac{10}{3} \times \cdot 0075$ inch, and at F/16 one-quarter of this amount. The values with a six-inch lens would therefore be $\frac{6}{10} \times (\frac{5}{8})^2 \times \frac{10}{3} \times \cdot 0075$ inch $= \cdot 00586$ inch approximately at F/8, and $\cdot 00146$ inch at F/16.

Fig. 5.

Fig. 6.

In conclusion, the author desires particularly to call attention to the fact that the results given in this paper are to be regarded only as approximations to the more accurate results which can only be reached when higher order aberrations are taken into account: he would, however, point out that they take account fully and exactly of all the simple first order aberrations or third order terms. It must also be remembered that in practical cases the results stated are complicated by diffraction phenomena.

Mr. Chalmers thought that aberrations of higher orders might improve the definition in certain cases bringing rays back to more favourable points; but there would be a limit to the definition obtained in the most favourable case.

Mr. Everitt did not understand how Mr. Smith, in altering the magnification, made allowance for the variation of aberration in coma.

THE GREAT OBSERVATORIES OF AMERICA

By Professor H. H. Turner, D.Sc., F.R.S., of Oxford

Dr. Silvanus P. Thompson introduced the lecturer to the audience, remarking that Prof. Turner, who held the distinction of Savilian Professor of Astronomy at Oxford, needed no introduction from him. There were some, however, even in the optical world who had not had the privilege of meeting the Professor before, and they would be glad to see him, and glad also to hear him; for his extraordinary activity and powers of exposition were well known.

Prof. Turner then delivered his lecture. He said: Two years ago, in 1910, there was a meeting of the International Union for Solar Research, the third of its series. One was held in Oxford (in 1905) and another in Paris. This third meeting was remarkable in being held on the top of Mount Wilson in California, 6000 ft. high, a well wooded mountain, and only accessible at first by a 3 ft. track, now happily widened to 5 ft.

The opportunity was taken by the various astronomers from all countries to see something, not only of the Mount Wilson Observatory—but of other great observatories throughout the United States. The speaker, therefore, took as his text for the lecture, "The Great American Observatories," and hoped to tell something of the great development of astronomical appliances and apparatus which the United States had recently witnessed. The advance had not by any means been confined to the United States. Some of those present had that very day been visiting the Greenwich Observatory and had seen the handsome recent additions to it. But it was not the lecturer's intention to refer directly to what had been done in Europe. He would confine himself to progress in America, although he would unblushingly refer to the occasions—and they were numerous—when what had been done in America was only the development of what had been started in Europe, and especially in England.

He hoped that he would not appear to accuse his countrymen of want of enterprise. It was true that they had, in many cases, started an idea which had been, as would be seen, seized upon elsewhere and developed. That development had, however, in many instances, been due to the possession of larger sums of money than were available in England, a point which, without wishing in any way to detract from the merit of our American cousins, should be kept in mind. It was a lesson, nevertheless, that he should like to see learnt; that Englishmen might have profited more by their own ideas had they had a little more of the enterprise and spirit which animated men of business instead of the spirit which inspired the

astronomers, who, he regretted to say, seemed sadly likely to settle down into a groove.

In order to get a skeleton for classifying the rather numerous points of interest, it would be well to look back to the time when Galileo first put the optic tube to his eye. The telescope really consisted of three parts. First the lens or mirror: the increasing size of lenses or of mirrors was very apt to attract a rather large share of attention. But there were other parts. The tube was an important part, and there had been developments which must be classed as developments of the tube, such, for instance, as its attachment to clockwork or its modification in various directions. Beyond that, however, were developments at the eye end of the telescope. The eye of the observer had been replaced by photographic plates, or a spectroscope, a spectroheliograph, or other analyser. The development of the eye end of the telescope had perhaps been the most important part of the history of its development.

For the astronomer, one of the best places to land in America was Boston, as he immediately came into contact with the great Harvard Observatory and the hospitality of Prof. Pickering. The Observatory was composed of a number of detached buildings, in each of which there was some instrument at work: chiefly photographic instruments where the observer was replaced by a photographic plate with the help of clockwork to keep the telescope fixed, and where the only astronomer was the night-watchman, who went round to the different instruments, and changed the plates. That was characteristic of the bulk of the work at Harvard. To it should be added the consequence that they got such a large number of photographs that many of them were put on the shelves without being fully examined. The speaker had, in the early days, been one of those who had thrown up his hands in surprise, and possibly reprobation, at that process. Why take a photograph unless it could be examined or measured properly? He was, however, wrong in this view. Prof. Pickering had shown that in that way a most valuable reference library of photographs could be formed. In that way it became possible, for the first time, after a new object had been discovered to examine its past history by means of this reference library.

For instance, with regard to the new star of 1901 (*Nova Persei*) shown on the slide, it was easy enough to take a photograph of the star after discovery, but at Harvard alone were sufficient plates already in existence to say that the star had not appeared on February 19, but had appeared on February 26. In that way it had been possible to use the great reference library of hundreds of thousands of photographs to identify many new objects after they had been discovered, and to tell their past history, which would have been impossible had not these photographs been accumulated without any special reference to an examination to take place next morning.

The next slide showed a very characteristic selection of such photographs, all taken at Harvard, showing different new stars and the places where they ought to have been, but were not, before. It would be noted that the plate included cases

where stars instead of being round dots were spread out as spectra. Among the instruments at Harvard were telescopes fitted with an objective prism, and the light did not enter as a whole but was analysed into its band of colour, and on the plate appeared a spectrum for each star. A vast amount of classifying work had been done by the use of such plates at Harvard. It was not to be supposed that all the plates passed to the shelves unexamined; an immense amount of work was done in examining them, and the Harvard classification of spectra was now the standard classification.

Nor should it be supposed that Harvard limited itself to waiting until other people had made discoveries and then verifying the past history. An instance of a splendid discovery at Harvard was the eighth satellite of Saturn, now called Phœbe, a body with an astonishing peculiarity. Not only was it very much farther from Saturn than the other seven satellites, but it ultimately turned out to be going round Saturn the wrong way. This fact led to a total subversion of the existing ideas as to the origin of the solar system. The theory was started by Prof. W. H. Pickering, and supported by mathematical analysis, that Saturn itself was originally going round the other way, and in that condition gave birth to an early daughter (Phœbe); and then Saturn turned over by the tidal action, the remaining children going round in the direction with which astronomers were familiar. That was a single instance of many important discoveries that had been made at Harvard Observatory.

One use of photography, in which no very great progress had been made in America, was in the direction of accurate measuring of the positions of the stars. That was, perhaps, more peculiarly the province of the older observatories. The speaker was among those who considered that this use ought to spread further than it had done. It ought to have invaded "meridian work," but had not done so. An American Committee had been formed to consider the matter, and in this case, as in others, we might be outstripped in the race.

As an example of the great value of photography in accurate measuring, he would take a case in which it had been found not exactly to fail, but to possess a source of failure. It was at first thought that there would be distortions in the photographic film which would render the positions of objects photographed liable to small displacements. The fear had fortunately turned out to be groundless, although there had been one or two cases in which systematic errors of that kind had been found.

He would illustrate, first of all, one of these systematic errors.

The case of distortion, he would mention, was a case in which a star happened to be rather near one of the lines [*indicated*], and bent it into a bow. This was an extreme case, but it was quite easy to detect it by studying extreme cases and then to take precautions accordingly for eliminating it.

As he had said, the study of exact measurement of the stars was not, perhaps, so well represented as in Europe, but there was one instance where an observatory had been devoted for some forty years to that particular study. This observatory—the Dudley, in Albany—had indeed a dome and instruments, but these had been

put aside for a number of years, in order to enable Prof. Boss to study very carefully the records accumulated by others. The Professor had produced a classic work of the most fundamental importance. What was known of the motions of the stars, when the last degree of accuracy was required, was due to Prof. Boss, who for forty years had devoted himself to the subject, and as an instance of what he had achieved, the speaker would throw on the screen a discovery the Professor had made of a beautiful cluster of stars in Taurus.

The little arrows representing the motions of the stars, all seemed to be pointing towards one spot in the sky. The little arrows were enormously magnified. They showed the motion in 50,000 years, whereas Prof. Boss had only fifty years to go by. The point that had been brought out was that all the stars seemed to be making for a particular spot in the sky. That did not mean that they were moving towards a collision; it was merely a perspective effect, and meant that the point indicated was a vanishing point of parallel motions. Prof. Boss considered that the cluster was nearest to us about one million years ago, and was now receding; he had also specified the size it would be in a good many years to come. The speaker had been responsible for the suggestion that probably they would come back again; that the whole of our system of stars was a finite and separate system, with a real centre of gravity; that, at the moment, these stars were passing through that centre of gravity, and would ultimately be arrested and come back to us. He regarded the whole of our system of stars as performing oscillations in something of the manner indicated with respect to the ball or cluster to which we belonged. That, however, was a digression from his main subject.

Passing from the Albany to the Yerkes Observatory of Chicago (in a sense in which Oxford might be called a London observatory, as the distance was seventy miles) the dome at the Yerkes Observatory was a large one, containing a 40-in. lens, given by Mr. Yerkes, who gave the name to the whole observatory. The Yerkes 40-in. was completed in about 1898, since when nothing further had been done in the direction of large lenses. The great interest, after all, in the Yerkes refractor was not the telescope, not the lens itself, but the thing hung at the far end of it—the spectro-heliograph. It was at Chicago that Prof. Hale first invented this spectro-heliograph. He first of all put it on his little telescope in the observatory which his father had given him at Kenwood. He got the big Yerkes refractor built in order that he might use a bigger spectro-heliograph, which, after much labour, thought and delay, he got hung on the end of the monster telescope. It was not so much the telescope as the thing to replace the observer, the spectro-heliograph, he wanted to exploit.

The function of the spectroheliograph could be explained very briefly. It was an instrument for taking the sun, or other objects, in one colour. If the sun were taken in the ordinary way (as every day at Greenwich) the spots were obtained, but no trace of many phenomena which were now known to belong to the sun. By the observations in 1868 of Janssen and Lockyer the prominences were shown to be open to detection in full daylight. The spectro-heliograph was the photographic

sequel to that visual observation in 1868, but it had taken twenty-four years to translate the latter into the language of photography, which was, perhaps, a sign that our enterprise as astronomers was not all that it should be. It was left to Prof. Hale to complete the observation of 1868, and show how to photograph in one colour, and so get the prominences, and also much detail over the surface which could not ordinarily be obtained. The speaker would mention, with almost equal honour, Deslandres, of France, who published results almost at the same time; also Mr. Evershed of England.

The function of the instrument could perhaps be more clearly seen by taking a little bit of the sun and magnifying it. The slide showed the region of the big spot, the black portions being the spot, and the rest the ordinary surface of the sun taken in calcium light. The next slide showed the very different appearance when the same region was taken in the light of hydrogen. So different, indeed, was the appearance, that it would hardly be taken to represent the same region. In this way was seen a vista of the immense possibilities of the spectro-heliograph.

It gave for each element the distribution of that element in the region looked at, which might be so immensely different as in the two cases shown. From being simple the study of the sun had become complex, and it was easy to understand why Prof. Hale felt that forces should be joined and energies concentrated by the creation of the International Solar Union. The appearances shown in hydrogen light would remind the audience of the lines of force which iron filings took up under the influence of magnets. The suggestion was by no means illusory. Prof. Hale had shown by beautiful experiments in polarized light, that there was really a magnetic field in the neighbourhood of spots. He had also demonstrated very clearly by photographs showing the vortex action sucking something down.

The discoveries of the vortex magnetic action of the sun were among the most important and recent advances on our knowledge, and were the product of the work done by the spectro-heliograph. Perhaps the latter was the most important feature of the Yerkes Observatory. The work of Prof. Barnard should not be forgotten, however, who used the big telescope for other purposes, and also used his own little photographic telescope to such immensely good purpose in photographing the Milky Way and Comets.

He was the first to point out the extraordinary vicissitudes of the tails of the comets. In one photograph he took the tail had a shattered appearance, and on his showing it to a lady, she remarked that it looked as if it had been out "all night." By getting two photographs on the same night, but not at long intervals, he showed how the shattered appearance was travelling down the tail, not as the smoke of an engine travelled away from the engine, because the comet was moving across, as could be seen by the trails of the stars. (The latter seemed to have moved because the comet was kept steady.) That was only one illustration of the wonderful work that Prof. Barnard had done on comets.

Referring to the scope of the journey, the speaker went on to say that he had started at Boston, visited Albany, then came the Yerkes Observatory. Those

were three Eastern observatories, and he would pass on to show three on the West — in California, all distinguished by being rather of the mountain observatory type, observatories placed especially for their climatic conditions. These were Mr. Lowell's (at Flagstaff), the Mount Wilson and the Lick Observatories.

Mr. Lowell took some very beautiful photographs of the daylight comet of 1910. That illustrated one of the advantages of being a little farther south. In England no photographs could be taken because of the strong twilight. Mr. Lowell, however, got the beautiful picture of the comet shown, and had also shown many other interesting things. As was well known, Prof. Lowell was working very hard at the planet Mars, to establish his idea that it is inhabited.

He would put on the screen a simple little experiment which would bring home the comparative helplessness even of the largest telescope in such a connection. Looking at Mars, it was hardly possible to know the meaning of what was seen. If, however, the observer supposed himself to be on Mars and looking at the earth, he would be more familiar with what was seen. The speaker would assume the best possible telescope, an assumption which was easy to make by a little calculation. He would suppose an observer planted on Mars and looking at the earth, of which he would give two views. To save time in looking, he would point out that everything in the two views was the same with the exception of the British Isles. The British Isles had seriously changed in these pictures, and he would show presently the changes that had occurred. But first of all he would ask the audience to see how much of the change they could detect. As he had said, he was assuming the biggest telescope that could be found on earth, which put the observer into a perhaps more advantageous position than he would be if looking at the earth from Mars. The earth in the slide was, of course, merely a globe hung up in the observatory, and the pictures had been deliberately made, cut out, and pasted on.

One picture it would be seen, was an ordinary picture with wavy lines all round and with ordinary rivers. The other had the coast-line cut into straight pieces. Anglesey had disappeared, islands had disappeared in Scotland, and Ireland had got Home Rule. Two big canals, of a straight kind, had taken the place of the rivers, and the speaker ventured to say that nobody could detect any of the changes mentioned from the planet Mars with the telescope provided. That brought home the fact that even the tremendous optical power of the present day was of little use in studying such a problem as Mr. Lowell had taken up. A recent contribution to the study of the question of the habitability of Mars was, of course, Prof. Campbell's visit to Mount Whitney—what had been called the "top of the United States." He had ascended this bare rock, and had compared very carefully the spectrum of the moon with that of Mars in the infra red in order to see whether there was any trace of water vapour. The infra red of the spectrum was well known to show very strong water bands. Prof. Campbell had come to the conclusion that he could not be sure that there was more water in Mars than in the Moon, though Prof. Lowell was not satisfied with that conclusion.

The Lick Observatory, on the top of Mount Hamilton, 4000 ft. high, had a big

36-in. telescope. When the speaker first went to the Lick Observatory in 1904, that road was ascended by a stage of the old kind pulled by horses, and driven by a man who might have come straight out of Bret Harte. He knew Hank Monk, who drove Horace Greeley in one of Mark Twain's stories. There were 365 turns up the winding path to the Lick Observatory. The driver had said that in Leap Year there were 366; for in that year the stage turned over, which made the extra turn. Now-a-days the ascent was by motor cars, which made light of the journey.

It might again be argued that what was being done at the Lick Observatory was not directly due to the big lens, but to the fact that they were developing the work of a great English astronomer, Sir William Huggins, who showed the way with the spectroscope, not only in classifying the stars by their spectra, but also how the movement of a star might be detected from measurement of the lines in the spectrum. He showed how the lines would move to the left if the stars were moving towards us, and to the right if they were moving from us.

The proof of the theoretical reasons for this phenomena were denied at first, and the speaker recalled that his own fellowship dissertation at Cambridge consisted of an examination of certain objections that had been made to the theory, and an attempt to demolish them. But, at any rate beautiful experimental proofs of the correctness of the theory were obtained in many directions. To take the case of Saturn. This planet consisted of a ball and a ring. The ball was rotating like a solid body, the outer parts fastest, the inner parts slowest. Consequently the outer parts were revolving at a great rate, the intermediate parts at a lesser rate, and so on. The ring, however, was not solid. It had been shown theoretically by Clerk Maxwell that the ring must consist of separate particles obeying Kepler's laws, all the inner ones moving quickest and the outer ones going slowest. Consequently the rate of revolution of the ring was greatest inside and smallest at the outside. Interpreting that in terms of the line of the spectrum, all these motions produced their effect on what ought to be a straight line if the body were at rest. The line practically followed the elongated Z form, as shown on the diagram exhibited.

These Z forms could now be traced in an actual photograph. That was one of the most beautiful experiments which demonstrated the great principle that motion could be detected by means of the spectroscope. The spectrum shown was taken at Prof. Lowell's Observatory, affording another instance of the variety of his work, which was by no means confined to the planet Mars.

A number of observations of movement of the stars had been collected and discussed by Prof. Campbell; and as only one instance of the rich results attained by this work, the speaker would take the marvellous table of figures showing that "the older the star was the quicker it moved." The stars in the table were arranged according to their spectral type, and there was independent evidence that that was probably the order of their age, although it would take too long to go into that point. The average velocity in miles per second was apparently steadily on the increase. The planetary nebulæ had been put in because they seemed to be going faster still, and would therefore be, apparently, still older than the stars. It was a most startling

and very recent discovery, and it was so bewildering, so unlike anything that would have been expected, that the only explanations at first offered were in direct subversion of all pre-conceived notions. It was, for instance, suggested that gravity did not begin to act upon a star all at once; that the newest-born stars were not subject to it at all, gravity being only an "acquired taste."

The speaker believed that the idea which he had put forward earlier in the lecture—namely, that we belonged to an isolated group of stars which had a centre of gravity, and probably a history and a structure—was quite capable of explaining the phenomena. He was not, however, ready with a single explanation; he could offer half-a-dozen different explanations. It would, perhaps, take a long time before this table could be explained in terms of the ordinarily accepted views about gravity. Nevertheless it was a wonderful result which compelled attention.

Coming to the last Observatory on the list, the Mount Wilson Observatory, the view shown was taken from the top, and was not of the heavens but of the earth beneath. The lights of Pasadena and of Los Angeles were pointed out, with the Pacific Ocean beyond. These glittering constellations at night from the top of Mount Wilson were in fact more wonderful than even the brilliant skies. When the speaker was first in the neighbourhood (in 1904) there was no observatory at all.

The top of the mountain was reached by a very narrow track, and most of the materials for the early work of the observatory were carried on the backs of mules. A kind of carriage had been made with automobile wheels. It was called "the automobile" although it required a horse and three men to work it. In this way they got most of the materials right up that very narrow track. Later, however, the track was widened to five feet at enormous expense. The 5-ft. telescope could be seen on the slide going up. That was the reason for widening the track—to accommodate the 5-ft. tube for the mirror which Prof. Ritchey had previously made in Chicago, and which had lain by for some years before the money was provided for mounting. It was now on the top and at work, and the speaker had had the great pleasure of seeing it at work. It was not worked without some ingenuity; there were fourteen separate motors to provide freedom of action and all the things which an observer wanted to do. The view through it was really perfect. The telescope had taken some beautiful photographs, notably of nebulæ. One was of spiral nebulæ showing a good deal of whirling motion. Another picture reminded us that a nebula need not necessarily have much of the whirling motion. This slide represented a globular structure, a spherical form, only seen in a state which was very nearly one of rest, when the gas would collect in approximately a spherical form.

Wonderful as the 5-ft. instrument was, it was not the most wonderful instrument at Mount Wilson, and that brought the lecturer to the last of the points to which he would draw attention—the development of the *tube* of the telescope. He had had much to say about the development at the eye end; the photographs at Harvard, the spectro-heliograph at Yerkes, and the spectroscope at the Lick Observatory. Now he would direct attention for the first time to the problems connected with the tube and the mounting of a telescope.

Before doing so, however, he would remind the audience that the idea of a 5-ft. telescope was not altogether of American origin. The late Dr. Common had made such a telescope in his back garden, and the next slide gave a picture of him contemplating his work. This was another instance of ideas taking root in America which were really sown in England.

In discussing the tube, the speaker would call attention to the work of another English pioneer. The slide shown was an illustration of a page of music, bearing at the top the name "William Herschel." This represented a manuscript of music he had composed in his early years; until the age of thirty-seven he was nothing more than a hard-working musician, first as a soldier, then as an organist at Bath. He ultimately made his great telescope, and the slide on the screen showed him working on it, and indicated the dangers he experienced in getting into a proper position for looking down this great reflecting telescope with its big stage. He would now show an interesting photograph of the great stage. Sir John Herschel (the son) was one of the first to try glass as a medium for the support of the collodion film, and on the first occasion, when looking about for something to take with his first experimental plate, his eye fell on the great stage, then about to be demolished. The first photograph to be taken, therefore, was the great stage. This photograph was still in existence, and some copies had been kindly presented by the family to a few fortunate people.

Herschel had difficulties with the mounting of his telescope, the forerunners of many such difficulties, which were not peculiar to reflecting telescopes; they were equally obvious as the size of the refracting telescope was increased. Although it might be easy enough to look through a small telescope from its position on the floor or from a few steps, when it came to a question of a big telescope, the steps became inconveniently larger and somewhat dangerous. The speaker believed that Caroline Herschel never would mount the curious ladder to look through the 40-ft. telescope. Various devices had been suggested for getting over the difficulty. The most satisfactory method was due to an Irishman—or, as he would prefer to call him, an Englishman—Sir Howard Grubb, who in the early 'eighties suggested the device known as the "rising and falling floor" for the Lick telescope. The Americans had adopted this beautiful device, without, however, having the courtesy to ask Sir Howard Grubb to make it. The same thing happened with the Yerkes. Sir Howard Grubb, therefore, had nothing to do with the point the speaker was about to make. Even the lifting floor was not unattended with dangers. When the first floor was built for Yerkes an admiring crowd of some three hundred people visited it, and had been lifted up and down to their hearts' content; when they had left the building the ropes that were holding the floor pulled through and the floor dropped the whole of its length from top to bottom.

It was appalling to think what might have happened had the ropes pulled through earlier.

It was probably significant that the next step in the improvement of the tube of the telescope originated, so far as regular observations were concerned, at the

Yerkes Observatory; though it had, the speaker was glad to say, an earlier origin first in France and next in England. The Oxford University Observatory had the honour of putting up the first cœlostat for astronomical use; a device consisting of a mirror, which was used to reflect the rays of the sun, or any other object, in a fixed direction, so that a fixed telescope might be used. Under the name of heliostat or siderostat, such arrangements had long been known. But the whole field usually rotated round its centre. The device known as the cœlostat alone had the peculiar merit of allowing the whole field to be photographed steadily just as if it were quite at rest in part of the sky.

Although the lecturer's name had been associated with the use of the instrument, he would take the opportunity of saying that it was not due to him. It was originally invented by August, but was lost sight of until attention was called to it by Lippmann in France and by Dr. Johnstone Stoney in England. The first actual instruments were made by Dr. Common, for use in eclipse expeditions. The one shown was taken out to Japan in 1896, and as eclipse instrument the cœlostat had taken a very firm place. Mr. Reynolds of Birmingham wanted in 1905 to take out a very long telescope, with a mirror of great focal length, to Egypt. All he had to do was to set up the cœlostat and then mount his telescope among the palm trees, a project which would have been impossible if a clockwork were attached to the long tube itself. It was left to Prof. Hale, at the Yerkes Observatory, to demonstrate that the instrument was also of tremendous value in the ordinary way. His "Snow" telescope at the Yerkes Observatory was a long telescope laid along the ground very much as Mr. Reynolds had laid his 100-ft. telescope, the light being reflected by the cœlostat. When a long telescope was desired, however, it was necessary to see that there was enough room to lay it along the ground, which was not always the case on the top of a high mountain. In the case he was discussing a pier had had to be built out for the cœlostat, at a very great expense. And another disadvantage, resulting from the horizontal position, was due to the proximity to the ground for the greater part of its length and the consequent air currents. Mr. H. C. Plummer, the speaker's former assistant, now Royal Astronomer for Ireland, had suggested the use of a vertical telescope going down into a well, with a mirror set at 45° to reflect the rays from the cœlostat. A diagram illustrating this suggestion—which at the time had been received with laughter—was shown. Within two years that suggestion was, in essentials, adopted at the Mount Wilson Observatory. Instead, however, of being put in a well in the first instance, the telescope became a 60-ft. tower. The success of the experiment led Prof. Hale to devise the more ambitious one (equally successful), of having a 150-ft. tower for the cœlostat and telescope, and below that, down into the ground, as Mr. Plummer had suggested, an 80-ft. well.

The telescope was then shown on the slide. There was a single dome made of a pair of half domes, one revolving inside the other. The position of the cœlostat was indicated. The rays were focused somewhere about the ground level, and below the ground was a pit for the spectograph or spectro-heliograph, or the camera, etc., which, as it had been one of the speaker's main points to show, was the most important

part of the telescope. These instruments were arranged in a most ingenious way on a revolving pillar, so that the observer could lift one out and put another in, with something of the ease with which a man might help himself to marmalade or butter from a revolving breakfast tray. The whole length of the apparatus was about 230 ft. The tower had a most ingenious construction to prevent it from being shaken by the wind. It consisted really of two towers, one inside the other. The outer tower was, in fact, a series of tubes; the inner tower was of rods within these tubes, and was the one which carried the cœlostat on the top, the outer casing being supposed to be shaken by the wind without disturbing the inner tower. The speaker believed that in practice the experiment was a success. At the time of the lecturer's visit (to the Conference), however, the work was only just completed. Great exertions had been made to get it ready for the Conference, but at that time the method of getting up to the top was not a very handy one. A bucket was used to draw people up and down, and the lecturer's last slide represented the whole of the Computing Staff at Mount Wilson prepared to make the ascent in a bucket, in perfect harmony. It would be seen that the Americans were utilizing the services of ladies in much of their work, and with very great success. Mrs. Fleming, whose loss astronomers all deplored, had, during many years, been responsible for a great many discoveries and a great deal of classifying work. The idea, however, was not wholly American. Ladies had worked in other countries: and we need only recall the names of Caroline Herschel, Mary Somerville, Agnes Clerke and Lady Huggins. Mrs. Fleming herself was really a Scotswoman, and it was significant that a Scotswoman, who might not have found any such employment in England, when she got into the freer atmosphere of the United States, did such excellent work.

Dr. Silvanus P. Thompson then very briefly addressed the meeting, pointing out that he was only expressing the sentiment of those present in saying that the lecture had been exceedingly interesting and beautiful, and he thanked Prof. Turner for his great courtesy in coming and delivering such a lecture.

Report of Lecture by Professor William Stirling on

OPTICAL ILLUSIONS

(*Thursday, June 20th*, 1912.)

Dr. S. P. Thompson, in introducing the lecturer, observed that those who had had the privilege of hearing lectures by Prof. Stirling would know what to expect. He [Prof. Stirling] had promised a lecture on optical illusions, and when the speaker had received the reply to the request which he had put forward on behalf of the Optical Convention he had thenceforward felt certain that something had been secured which

would make the Convention "go." Those who knew Prof. Stirling would know that the Professor was accustomed to make his lectures "go," and he would therefore no longer interpose between the audience and the treat they were looking for.

Prof. STIRLING then addressed the meeting. He had, he said, no sensational novelties, no investigations of the first magnitude, nothing of the sweet simplicity of Gauss or the mysteries of the anti-point to stir the optical imagination. He had nothing new to put before the audience; everything was old and trite, and he would merely ask them to use their eyes and brains, and "wait and see." Although there was nothing out of the way, they would nevertheless be deceived. There would be no phrases to fill the air; everything would appeal to the visual sense, and baffle the best judgment. Our sense organs were far more numerous than the classic five, and at times they were employed in giving rise to those sensations which were spoken of as "illusions" in contrast to those which characterised certain phases of mental aberration, known as "delusions." There was a widespread opinion that illusions were something abnormal, whereas the very reverse was the fact in many cases. It was unnecessary to go so far as to assume that all men habitually erred, or that illusion was to be regarded as the natural condition of mortals. Most people, however, were occasionally subject to illusions rooted in normal mental life, which fell within the province of the physiologist as well as the psychologist. Attempts had been made to draw distinctions between those illusions which depended on the numerous imperfections of sight, on the eye as an optical instrument, and those which depended upon cerebral factors, where the judgment came in to interpret the impulses sent from the eye. That, however, was not possible, as both factors occurred in most illusions, so that practically all illusions had an optical as well as a physiological side. The vision had been rightly called the "Queen of the Senses." The empire which administered to sight had practically a limitless horizon; it triumphed over both space and time, and in all ages the eye seemed to have been the most honoured of the senses. Though the light of the body is the eye, yet the great majority of mankind did not see one fraction of what was displayed to their organs of vision. So faulty was our power of accurate observation that Carlyle—direct and penetrative—was constrained to say: "The eye sees what it brings the power to see;" also: "I say that the degree of vision that dwells in a man is the correct measure of a man." Ruskin with his poetic temperament, had said: "To see clearly is poetry, prophecy, and religion all in one." All of us, however, the speaker said, saw as through a glass dimly, and even our best judgments were often most fallacious. To quote again: "It is not the eye that sees the beauties of heaven, not the ear that hears the sweetness of music or the glad tidings of a prosperous accident; but the soul, *i. e.* the mind, that perceives all the relishes of sensual and intellectual perceptions, and the more noble and exalted the soul the greater and more savoury are its perceptions." The speaker would also quote from the late W. R. Greg, who in the *Enigmas of Life* had given in picturesque glowing colours, a description of this "allotted home" of ours, which it was the business of our sense organs to in-

vestigate and thereby to cultivate our intellect as well as minister to the gratification of our intellectual and æsthetic sense. This was the passage : " Every sort of beauty has been lavished on our allotted home ! Beauties to enrapture every sense, beauty to satisfy every taste, forms the noblest and the loveliest, colours the most gorgeous and most delicate ; the sunny glories of the day, the pale Elysian grace of moonlight, the lake, the mountain, the primeval forest, the boundless ocean, silent pinnacles of snow in one hemisphere, the marvels of tropical luxuriance in another ; the serenity of sunsets, the sublimity of storms—everything is bestowed in boundless profusion on the scene of our existence." Yet it was rather astonishing how little most of us knew of the world in which we lived. " The wise man's eyes are in his head, but the fool walketh in darkness."

The eye was one of the most perfect instruments which could be conceived of for the manifold purposes for which it was used. Helmholtz had pointed out its many imperfections, but no instrument of human manufacture could be compared with it in applicability to the solution of all kinds of problems involving the action of the other adequately adapted organs that participated with it in the act of seeing. The sense of sight was one in which we believed most implicitly, and the one which gave the largest store of information regarding the world in which we lived. So great was our confidence that what was seen must be true, that the expression " Seeing is believing " had passed into a proverb. The attitude of universal doubt, *De omnibus dubitandum*, expressed by Descartes, was in a certain sense not without its value. Sense perception was a very complex question ; it had both a physical and a mental side. When an object was looked at, besides the effect on the eye there were changes in the brain and their mental concomitants, to which was added something from memory, the result of past experiences. The eye, on account of its almost illimitable range of action and its vast and varied combination of sensations of form, light and colours, of various qualities and intensities, and by its movements, adding the effect of muscular sensation, was an easy first as a field of illusion amongst the organs of sense, the ear coming next. Plato in his *Sophist* remarked : " If artists in *large* works were to give the true proportion of beautiful things the upper parts would appear smaller than is fitting, and the lower parts larger, through the former being seen at a distance, and the latter close at hand." Consequently artists, " bidding farewell to truth, work out not real proportions, but such as will seem to be beautiful in their representations." Pictorial art is based on illusions. In the histrionic art the actor did his best to conceal his art (but did not sometimes succeed), and the illusion was more or less complete. The capabilities of illusionary presentment reached their highest evolution in the stereoscope. The painter working on a flat surface might very successfully imitate both solidity and depth or relief. The position of a shadow would determine whether an object appeared as projecting towards or receding from the observer. Even a regular geometrical figure, such as Necker's cube, gave a marked sense of relief. The sense of depth or distance in a picture might be given by diminishing the magnitude of distant objects desired to be suggested. Entasis, the increment or swelling given to a column in the middle

part of the shaft corrected the unpleasant optical illusion which columns with perfectly straight sides suggested, namely, an attenuated or narrower part in the middle of the shaft. In the columns of the Parthenon, the Temple of Jupiter Olympus at Athens, the Theseum and other Temples, the delicate convex curve from base to neck was added. In their Temples the Greeks, instead of making lines or beams horizontal, made them convex, in order to remove the apparent concave appearance of the lines. It was difficult to imagine any other reason for these deviations than that they were intended as optical corrections. Judgment of form of objects was entirely dependent on education and experience, and judgments were very liable to be influenced by causes inherent in the object itself or by other objects in proximity to it. The columns of the Parthenon were not absolutely vertical. There was evidence that the original designs in ancient sculpture and architecture were corrected. Vitruvius stated: "Beauty being the province of the eye, which if not satisfied by the due proportion and augmentation of its members—correcting apparent deficiencies with proper additions—the aspect would appear coarse and displeasing." The curves of the steps of St. Paul's Cathedral showed some very remarkable effects which some might know; but in appearance they were straight lines. The illusions of motion had long ago attracted the attention of Brewster, Wheatstone and Faraday in this country, and of Plateau in Belgium; more recently we had, in the 'seventies, the beautiful strobic discs and optical illusions which the President of the Convention, Dr. Silvanus Thompson, had invented. Wheatstone studied the remarkable illusions known as "fluttering hearts" which, like many other such illusions, had been used as an advertisement. Among the most interesting of the optical illusions depending on sense impressions were those known as "after-effects of seen movements." To take the case of a waterfall: if, in that case, the eyes were turned to contiguous rocks, the rocks appeared to move in the opposite direction—upwards, with a velocity equal to that of the falling water. This effect could be imitated in the laboratory [as the lecturer then proceeded to illustrate] by fixing a paper or cloth marked with alternate black and white stripes, on a drum. On looking at the drum as it moved, or on suddenly stopping the drum, and while still gazing at it, the stripes appeared to move backwards for a considerable time. Aristotle has recorded in his *Treatise on Dreams* that after looking at objects in motion—*e. g.* a river, and especially swiftly flowing streams—and then at the objects at rest, the latter seemed to be in motion.

The first slide thrown upon the screen showed a text from an old copy of the *Apocrypha*, which as printed, might not be recognizable. It was as follows: "WISEDOME OF SALOMON: And hardly can wee discerne the things which are upon the earth and with great labour finde wee out the things which are before us." In 1868 Helmholtz had said that the eye had every possible defect of an optical instrument, and even some which were peculiar to itself. The slide exhibited offered one of the best proofs of these defects, but the lecturer did not propose to deal on that occasion with the physical defects of the eye.

In proof of the foregoing the stars, as shown on the slide, appeared to have rays, but no star had rays. Owing to the imperfection of the eye, it was telling an untruth.

If a star were photographed a number of things like round spots were seen. That established a marvellous optical defect in the human eye, but as the defect was shared by all it made no difference. The marvel of the eye was the number of instruments it represented. It might be said to represent a common camera, but it was also a microscope and a telescope, the shortest known, being only one inch from front to back; it was a range-finder and a photometer, and it had a self-adjusting lens; and in addition it was an automatic iris diaphragm, a cinematograph, and a stereoscope. With an instrument like that, having the properties of such an enormous number of pieces of apparatus, it was possible to be tolerant of its apparent imperfections.

The next slide illustrated the fact that the eye over-estimated height. A hat shown in the slide appeared taller than it was broad. He had told his laboratory assistant to cut out, as best he could, a piece of paper to represent the height of a hat. Taking another hat (not measured on the screen) and putting one on top of the other, there was seen the marked difference in result. The error of judgment in estimating was, in this case, something like 25 per cent.

Again, in looking at an " S " or an " 8 " the upper half was considerably smaller than the lower half, but no one noticed this. If the slide were turned upside down, then the lower half of the " 8 " seemed a great deal bigger. The upper part was over-estimated.

On the question of depth as apart from height, the lecturer showed figures which gave a suggestion of perspective, although all were on the same level.

Judgment of distance was no better, and in the figures shown, the red would appear nearer than the white. This was easily explained by the fact that the red came forward, white retired. Both were exactly the same height, however.

The lecturer next dealt with cube figures. He said: " *That* is a cube which you can look into. *Here* is the illusion. *This* is a right angle, and *that* is a right angle. When you look at it the other lines have completely altered that angle, so that these two, which are at right angles are altered altogether in their appearance."

Another figure was shown which, Prof. Stirling said, each spectator would see in his own way. He added: " You will see that ring receding, with an effect like a truncated pillar. As you are looking at it it will change." Again: " If you look at a drawing of a book, the back may appear to be projected towards you, or it may be an open book, and you are looking at the stitching of the binding. While looking, it changes."

" The influence of converging lines is very remarkable. There are two circles of exactly the same size, two lines of exactly the same length. *That* appears bigger than that. The second line (nearer the angle) appears bigger than the one further from it."

With regard to the perception of angles, as a rule small angles were, Prof. Stirling said, over-estimated and large ones under-estimated. This the speaker illustrated with a square figure inside a circle. Certain lines were drawn to form certain angles, and it would be seen that distortion made its appearance. The raising of verticals at certain points introduced alteration. In order completely

to deceive, the speaker had introduced a circle and a square, the effect being a remarkable illusion.

A circle was next placed in front of a number of squares having more or less straight lines. This also introduced a completely baffling modification of form.

There was, Prof. Stirling remarked, a great deal of truth other than mathematical truth; life was not all a mathematical problem, and, as an additional means of proving this assertion, segments of circles were shown on the screen, which, although of the same size, did not appear so.

The slides next shown were, the speaker intimated, illusions which were possibly questions of eye movement. Two lines were shown crossing each other, and the illusion was produced by a piece of black paper and a piece of glass. The two whites, it would be seen, looked a great deal bigger than the divided space, where the angles were.

Again a rather curious illusion of judgment in regard to angles and in regard to spaces open or otherwise. The height of a triangle indicated was exactly the same as another, but it appeared taller. Distance of that kind was over-estimated. Similarly the square looked bigger than a circle of the same size. Two open spaces of exactly the same size appeared unequal in size by reason of a difference in the thickness of the bounding lines.

In the case of an incomplete circle or arcs of a circle, it would be seen that the use of the diameter shown had a pulling-in effect.

This touched the problem of architecture. The Greek building was elevated to a certain height, so that at a certain distance it gave the idea of the straight line. The circle on the slide was, in a case indicated, the same size as another, also indicated, but it did not appear so.

Poggendorf's experiments were next illustrated. The speaker said: "In the case of *this* line passing through *this* line, I do not know your idea as to which is the straight line, but you are sure to be wrong. It is *this* one, not *that* one. That shows the influence of the length of a line on your judgment."

Further illustrations were given to show that the length of a line determined the amount of error in the judgment.

Clearly it was impracticable to judge the height of the line, its distance, or its direction. In the illustrations shown, the Poggendorf illusion was again utilised, with the figures of two devils pulling at either end of the line. The addition to the line had a diminishing effect.

The Zoellner effects were then shown. The speaker had had the pleasure of knowing Zoellner, and remarked incidentally that no portrait of him existed; owing to a physical defect in one half of his face no portrait had been taken. The lines shown on the slide constituted an extension of the Poggendorf illusion; they were parallel, but, in effect, did not look so.

The next illustration showed the influence of rotation, the object shown being placed in a carrier and rotated vertically. At a certain angle, the illusion was increased.

This showed how the judgment was affected by direction, and that there was a certain angle at which the effect of the illusion was greatest.

The lecturer then dealt with an illusion illustrated by a number of vertical lines with oblique lines going through. The vertical lines were parallel, but appeared to be inclined. Withdrawal of the oblique lines eliminated the deception.

Another illustration of the effect of angles on the apparent length of the line. Two angles were shown, and on one angle being widened the other appeared shorter, and *vice versa*. The effect of angular position on a short brass bar was also shown.

A slide was then given to show the famous illusion of Dr. James Fraser, the letters of the word " LIFE " appearing to be out of the vertical, although in fact they were vertical.

Irradiation effects were next shown, black and white stripes and black and white gloves being compared, and the white in each case looking a great deal broader than the black stripes and gloves; both were severally of the same size. The white " irradiated " over the black, and in the case of the superposition of white on black and black on white, the latter appeared smaller. In the case of a corporation tramway announcements as to direction, the wrong method of applying this had been adopted. Persons who had attained a certain age and rotundity of figure might take a hint from this effect as to the advantage of black clothes over white.

The next illustration was that of a crescent moon, showing a concentric illusion. In the same direction circles were shown, the spaces between being black. The black spaces introduced irradiation, causing the circles to have a hexagonal appearance.

This slide illustrated admirably the effect of short sight, and an exhibition of it in schools might assist endeavours to correct the defect.

The next picture was one in which the moon had the appearance of a peppermint drop suspended in the heavens, and served to show the artist's misconception as to the size of the moon, the real size being indicated by the speaker.

The next slides showed the new medical building at Owens College, in which the illusion was produced in connexion with the two halves of the doorway. The speaker also dwelt on the unsightly effect of certain pillars of a public building. These pillars were vertical and had been fluted.

A view of the Cathedral at Rouen was next considered, and followed by a photograph obtained from Dr. Lindsay Johnson. In the latter case two verticals had the appearance of being wider at the top than at the bottom.

The next view was one of part of the Parthenon, which indicated the entasis, or swelling of the columns. In the Doric form the slope was so gradual and delightful that the swelling would probably not be seen. Penrose, Penithorne and others had carefully worked out the question of the entasis of columns. Other similar photographs were shown, which emphasized the dictum of Plato to the effect that art was a deviation from truth to please the eye.

Another view showed the Corinthian column, in which the entasis was less marked than in the Doric form. As having a bearing on why the architects introduced this beautiful rotundity, the speaker pointed out that the contours of the human

body suggested many beautiful forms and curvatures in art. A certain rotundity in human beings was more pleasing, for instance, than angularity or straightness. The Greek architect had adopted that view in relation to the columns.

Other people made the letters of an inscription on their buildings the same size, but the Greeks did not; and, as illustrated by the slide, they made the upper parts bigger, and introduced a diminuendo; from a particular point of view all the letters were the same.

Another view in which the entasis or swelling of the column was clearly marked.

The next view was of the peristyle of a house in Herculaneum. The effect of closing one eye presented an enormous vista. The picture was a comparatively flat one, and in looking into the depth one eye gave distance where both eyes did not. One eye looking through a tube would see a greater distance than was otherwise possible.

An illustration was then presented showing the letters of the word " Lehich." This picture illustrated the relief effects of shadow. The same effect was produced in the case of certain shell-like ornaments on the wall of a Spanish villa.

Another illustration of two eyes drawn by Lawrence, touched the question of the direction of eyes. In some portraits the eyes appeared to look at the spectator and follow his movements. In this instance the speaker had arranged to cut off the eyes for the better conviction of the audience. Brewster had long ago explained the phenomenon in his *Letters on Natural Magic.*

The next slide showed how surroundings modified the colour presented to them, long ago discovered by Chevreul.

The lecturer then exhibited a piece of cloth, manufactured in Manchester, which had been returned as not equal to sample, and which nearly led to a lawsuit. The illusion involved required a physiologist to throw light upon it.

A cloth, woven of materials supplied to the manufacturer by the person who required the cloth, showed pinks which were the result of illusion, the material being actually grey. The effect was known as " simultaneous contrast." The speaker then related how the elder Chevreul dealt with a classic instance connected with rejected tapestries.

As showing involuntary eye movement, the speaker illustrated the difficulty of looking steadily at a series of black geometrical figures. A halo was formed round the black, showing eye movements.

The next slide showed the effect on the eye of variations in colour under steady vision and the effects of complementary colours. In this connection the speaker said: " Think of the beautiful and marvellous adjustment of the short muscles that move the vocal cords and the sweet sounds produced, intensified and altered by the different cavities. Observe the steadiness and variety of the note. Compare that with your eyes. While you are looking at *this*, you will see, at this signal, *this* will be taken out. You have paralysed the terminals for green and see only the pink. . . . Fixing your eye on the black spot, you will see that there is a little margin going round it; that a little piece of red is seen round the margin, then a little piece of green

coming on. . . . When you look at this pink for a certain time, you will get the complementary colour—that will be the green-blue."

The lecturer also showed a photograph illustrating the effects known as "after images."

The next photograph showed one of Purkinje's experiments. Purkinje, the lecturer stated, was an eminent physiologist at Breslau and Prague.

Another slide illustrated certain illusive effects of blue and red, one passing over the other.

Further illustrations of motion and after-images were then shown.

Colour shadows formed the subject of the next illustrations: one shadow from the white and another from the red. The white should have been black and grey, but was not. As further evidence of these effects the speaker showed one rod casting two shadows, yellow and blue, in addition to other illustrations.

The lecturer proceeded to show the strobic circles of Dr. Silvanus Thompson, one moving one way and one another.

Plateau's discs were next shown with their expansion and contraction and their "after-movement" impression on the eye, when the gaze was removed. This the lecturer described as one of the most curious illusions of science.

One or two other slides were shown which illustrated the confusion of impressions with regard to colours, pink effects appearing where grey would have been expected, as in the case of the Manchester cloth.

Some further illustrations of reverse motion when the gaze was taken off moving objects, as in the case of the waterfall, or when the movement was stopped, the after-image moving with the same velocity as the original. It would be understood that the stroboscope exhibited led directly to the cinematograph. In one illustration balls were going through a hoop which was spinning. The whole, however, was in one plane, perfectly flat. Finally chromatrope effects were shown, the lecturer remarking that this instrument afforded a beautiful study of after-movement impressions that remained on the brain and eye, impressions that led to many illusions, such as ghosts, etc.

Continuing, Prof. Stirling showed the audience a few effects without the aid of the lantern slide. After dealing with certain effects of height and size illusion, he went on to the question of irradiation. In this connexion the audience was shown two cylinders of exactly the same size, one of which, being illuminated inside, appeared bigger than the other. The same effect was produced in the case of a filament to which an increasing resistance was applied, there being a corresponding illusion of enlargement.

Among other things the speaker repeated with certain objects illusive effects which he had already shown on the screen—illusions with regard to diameter, expansion and contraction effects, waterfall effects, an improved example of Dr. Thompson's disc on a large scale, etc., together with colour-photography effects. Colour was, the speaker said, of enormous importance in showing the true relation of size, and how, in the important art of colour-photography, one colour might affect another.

At the conclusion of the lecture Dr. SILVANUS THOMPSON said that he, no doubt, expressed the views of every one present when he stated that he had been delighted with the optical illusions shown. They had been learning to distrust their own senses, and he could only admire the *aplomb* of the lecturer in showing things in which he did not believe. They had lost faith in themselves and in the lecturer, who had led them such a dance, and so thoroughly deceived them that they did not know whether they were transparent, whether they were standing on their heads, or whether they were afflicted with entasis. They would go back lost in wonder, and endeavour to find a remedy which would restore them to their natural senses. They had, he could venture to say, been delighted with what they had seen, and their knowledge of physiological optics could not have grown smaller under the stimulus of the illusions to which they had been subjected. For himself and for the audience, he thanked Prof. Stirling for his lecture.

Prof. STIRLING then said that it was the eyes of the audience—the thought and suggestion of so many pairs of them—that had stimulated him to do his best. The physiological aspect of the matter recalled to his mind the conviction he had always entertained that the best-trained men were medical men, inasmuch as no man had such a wide outlook on the whole of Nature. No culture—not even instruction in religious affairs, though this compared to a certain extent—could compare with the drastic years that the medical student had to pass through; five years by law, but thirty-seven per cent. took seven years, and many others more. Oliver Wendell Holmes was a medical man, a fact which influenced his writings. As an instance, the speaker referred to the peculiar kinds of tall hats which in Holmes's day prevailed in London, and which he had described as the *ultimum moriens* of human respectability. This allusion was derived from his medical knowledge. Everyone would remember the "One-Hoss Shay," which, after it had run for so long, went to pieces "all at once, and nothing first." When the heart died, however, it did not die all at once—a fact known to the ancients; it died in a particular way, and the last part to die was the right auricle—giving rise to the expression *ultimum moriens*. No one but a medical man would have made that allusion. He would conclude by quoting the following verse—

> "Why should we look one common faith to find,
> When one in every score is colour blind?
> If here on earth they know not red and green,
> Will they see better into things unseen?"

OPTICAL CONVENTION, 1912.

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APPENDIX

TABLE FOR CALCULATING SPHERICAL ABERRATION

Calculated by Horace C. Beck, arranged by H. M. Elder, M.A.

Coddington's expression—

$$A\frac{y^3}{8f^3} = \frac{1}{\mu.\mu-1}\left\{\frac{\mu+2}{\mu-1}X^2 + \overline{4\mu+1}\ \alpha X - \overline{3\mu+2}\cdot\overline{\mu+1}\ \alpha^2 - \frac{\mu^3}{\mu-1}\right\}\frac{y^3}{8f^3}$$

has to be evaluated numerically for various values of μ, α, and X in calculating the curves of lenses.

In the following table the values of the four coefficients $\frac{\mu+2}{\mu.(\mu-1)^2}$, $\frac{\mu+1}{\mu.\mu-1}$, $\frac{3\mu+2}{\mu}$ and $\frac{\mu^2}{(\mu-1)^2}$ are given for values of μ, the refractive index ranging from $\mu = 1{\cdot}4$ to $\mu = 2$.

From $\mu = 1{\cdot}4$ to $\mu = 1{\cdot}5$ and from $\mu = 1{\cdot}66$ to $\mu = 2{\cdot}0$ the values of μ increase by differences of 0·01.

From $\mu = 1{\cdot}5$ to $\mu = 1{\cdot}63$ the differences are 0·001, and from $\mu = 1{\cdot}63$ to $\mu = 1{\cdot}66$ the differences are 0·005.

Difference tables are also given.

Concurrently with the values of the coefficients the values of their common logarithms are given printed in red, together with the differences.

The table has been extracted from a very complete table compiled for his own use by Mr. Horace C. Beck, and by the kindness of Messrs. R. & J. Beck, Ltd. this has been made available for publication. Many of the numbers and logarithms have been recalculated to check their accuracy, and the whole table has been carefully checked by means of the differences.

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·400	15·17857		4·285714		4·428571		12·25000	
		79166		116886		10131		42312
·1461280	1·1812309		·6320232		·6462637		1·0881360	
1·410	14·38691	232634	4·168828	120092	4·418440	9948	11·82688	152556
		73356		111148		9989		39604
·1492191	1·1579675		·6200140		·6452689		1·0728704	
1·420	13·65335	227283	4·057680	117362	4·408451	9829	11·43084	147924
		68073		105818		9850		37135
·1522883	1·1352392		·6082778		·6442860		1·0580780	
		222181		114760		9714		143430

μ	$\frac{\mu+2}{\mu\cdot(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu\cdot\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·420	13·65335	68073	4·057680	105818	4·408451	9850	11·43084	37135
·1522883	1·1352392	222181	·6082778	114760	·6442860	9714	1·0580780	143430
1·430	12·97242	63295	3·951862	100852	4·398601	9712	11·05949	34875
·1553360	1·1130211	217306	·5968018	112272	·6433146	9600	1·0437350	139154
1·440	12·33947	58979	3·851010	96220	4·388889	9579	10·71074	32802
·1583625	1·0912905	212644	·5855746	109890	·6423546	9489	1·0298196	135086
1·450	11·74968	54995	3·754790	91895	4·379310	9447	10·38272	30899
·1613680	1·0700261	208185	·5745856	107612	·6414057	9379	1·0163110	131208
1·460	11·19973	51370	3·662895	87846	4·369863	9319	10·07373	29148
·1643529	1·0492076	203912	·5638244	105428	·6404678	9271	1·0031902	127512
1·470	10·68603	48051	3·575049	84058	4·360544	9192	9·782253	275306
·1673173	1·0288164	199813	·5532819	103331	·6395407	9165	·9904388	123980
1·480	10·20552	45008	3·490991	80500	4·351352	9064	9·506947	260283
·1702617	1·0088351	195882	·5429488	101319	·6386242	9062	·9780410	120606
1·490	9·755440	422107	3·410491	77158	4·342288	8955	9·246564	246544
·1731863	·9892469	192102	·5328169	99382	·6377180	8959	·9659804	117378
1·500	9·333333	40760	3·333333	7541	4·333333	887	9·000000	23940
·1760913	·9700367	19007	·5228787	9834	·6368221	890	·9542426	11566
1·501	9·292573	40504	3·325792	7507	4·332446	887	8·976066	23814
·1763807	·9681360	18972	·5218953	9816	·6367331	889	·9530860	11536
1·502	9·252069	40256	3·318285	7478	4·331559	887	8·952252	23680
·1766699	·9662388	18937	·5209137	9799	·6366442	889	·9519324	11504
1·503	9·211813	39998	3·310807	7446	4·330672	884	8·928572	23558
·1769590	·9643451	18898	·5199338	9778	·6365553	887	·9507820	11474
1·504	9·171815	39755	3·303361	7416	4·329788	885	8·905014	23434
·1772478	·9624553	18866	·5189560	9762	·6364666	887	·9496346	11444
1·505	9·132060	39506	3·295945	7386	4·328903	882	8·881580	23308
·1775365	·9605687	18828	·5179798	9742	·6363779	885	·9484902	11412

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·505	9·132060		3·295945		4·328903		8·881580	
		39506		7386		882		23308
·1775365	·9605687		·5179798		·6363779		·9484902	
		18828		9742		885		11412
1·506	9·092554		3·288559		4·328021		8·858272	
		39262		7357		881		23190
·1778250	·9586859		·5170056		·6362894		·9473490	
		18794		9726		885		11384
1·507	9·053292		3·281202		4·327140		8·835082	
		39010		7324		879		23068
·1781133	·9568065		·5160330		·6362009		·9462106	
		18756		9705		882		11354
1·508	9·014282		3·273878		4·326261		8·812014	
		38784		7295		879		22942
·1784013	·9549309		·5150625		·6361127		·9450752	
		18723		9688		883		11324
1·509	8·975498		3·266583		4·325382		8·789069	
		38538		7265		878		22832
·1786892	·9530586		·5140937		·6360244		·9439428	
		18688		9671		881		11294
1·510	8·936960		3·259318		4·324504		8·766240	
		38298		7237		877		22702
·1789769	·9511898		·5131266		·6359363		·9428134	
		18653		9653		881		11262
1·511	8·898662		3·252081		4·323627		8·743538	
		38070		7207		876		22592
·1792645	·9493245		·5121613		·6358482		·9416872	
		18618		9635		880		11236
1·512	8·860592		3·244874		4·322751		8·720946	
		37832		7177		874		22474
·1795518	·9474627		·5111978		·6357602		·9405636	
		18583		9616		878		11205
1·513	8·822760		3·237697		4·321877		8·698472	
		37598		7148		874		22350
·1798389	·9456044		·5102362		·6356724		·9394430	
		18547		9599		878		11174
1·514	8·785162		3·230549		4·321003		8·676122	
		37374		7118		870		22242
·1801259	·9437497		·5092763		·6355846		·9383256	
		18514		9581		875		11148
1·515	8·747788		3·223431		4·320133		8·653880	
		37144		7093		871		22126
·1804126	·9418983		·5083182		·6354971		·9372108	
		18480		9565		876		11118
1·516	8·710644		3·216338		4·319262		8·631754	
		36916		7062		870		22010
·1806992	·9400503		·5073617		·6354095		·9360990	
		18445		9546		875		11088
1·517	8·673728		3·209276		4·318392		8·609744	
		36698		7034		869		21902
·1809856	·9382058		·5064071		·6353220		·9349902	
		18418		9530		874		11062
1·518	8·637030		3·202242		4·317523		8·587842	
		36470		7005		867		21790
·1812718	·9363644		·5054541		·6352346		·9338840	
		18373		9511		872		11032

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·518	8·637030	36470	3·202242	7005	4·317523	867	8·587842	21790
	·9363644	18377	·5054541	9511	·6352346	872	·9338840	11032
·1812718	8·600560	36248	3·195237	6978	4·316656	866	8·566052	21672
1·519	·9345267	18342	·5045030	9494	·6351474	872	·9327808	11002
·1815578	8·564312	36034	3·188259	6950	4·315790	865	8·544380	21566
1·520	·9326925	18311	·5035536	9477	·6350602	870	·9316806	10976
·1818436	8·528278	35812	3·181309	6922	4·314925	864	8·522814	21458
1·521	·9308614	18277	·5026059	9460	·6349732	870	·9305830	10948
·1821292	8·492466	35602	3·174387	6893	4·314061	862	8·501356	21346
1·522	·9290337	18244	·5016599	9442	·6348862	868	·9294882	10918
·1824147	8·456864	35386	3·167494	6867	4·313199	863	8·480010	21236
1·523	·9272093	18210	·5007157	9426	·6347994	868	·9283964	10890
·1826999	8·421478	35168	3·160627	6840	4·312336	859	8·458774	21130
1·524	·9253883	18174	·4997731	9408	·6347126	866	·9273074	10862
·1829850	8·386310	34964	3·153787	6813	4·311477	860	8·437640	21026
1·525	·9235709	18145	·4988323	9392	·6346260	866	·9262210	10836
·1832698	8·351346	34756	3·146974	6787	4·310617	858	8·416618	20920
1·526	·9217564	18111	·4978931	9375	·6345394	865	·9251376	10808
·1835545	8·316590	34548	3·140187	6758	4·309759	859	8·395698	20810
1·527	·9199453	18079	·4969556	9358	·6344529	865	·9240568	10778
·1838390	8·282042	34344	3·133429	6733	4·308900	855	8·374888	20712
1·528	·9181374	18046	·4960198	9342	·6343664	862	·9229790	10754
·1841234	8·247698	34138	3·126696	6706	4·308045	855	8·354176	20606
1·529	·9163328	18013	·4950856	9324	·6342802	862	·9219036	10726
·1844075	8·213560	33934	3·119990	6681	4·307190	854	8·333570	20500
1·530	·9145315	17980	·4941532	9308	·6341940	861	·9208310	10696
·1846914	8·179626	33732	3·113309	6652	4·306336	853	8·313070	20398
1·531	·9127335	17948	·4932224	9291	·6341079	861	·9197614	10670
·1849752								

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·531	8·179626		3·113309		4·306336		8·313070	
	·9127335	33732	·4932224	6652	·6341079	853	·9197614	20398
·1849752		17948		9291		861		10670
1·532	8·145894		3·106657		4·305483		8·292672	
	·9109387	33538	·4922933	6628	·6340218	852	·9186944	20300
·1852588		17917		9275		859		10644
1·533	8·112356		3·100029		4·304631		8·272372	
	·9091470	33338	·4913658	6602	·6339359	850	·9176300	20202
·1855422		17885		9259		858		10618
1·534	8·079018		3·093427		4·303781		8·252170	
	·9073585	33140	·4904399	6575	·6338501	850	·9165682	20098
·1858254		17851		9241		857		10590
1·535	8·045878		3·086852		4·302931		8·232072	
	·9055734	32944	·4895158	6550	·6337644	847	·9155092	19998
·1861084		17819		9226		856		10569
1·536	8·012934		3·080302		4·302084		8·212074	
	·9037915	32754	·4885932	6525	·6336788	847	·9144528	19898
·1863912		17789		9209		855		10536
1·537	7·980180		3·073777		4·301237		8·192176	
	·9020126	32564	·4876723	6500	·6335933	846	·9133992	19806
·1866739		17757		9193		854		10512
1·538	7·947616		3·067277		4·300391		8·172370	
	·9002369	32370	·4867530	6475	·6335079	846	·9123480	19708
·1869563		17725		9177		854		10486
1·539	7·915246		3·060802		4·299545		8·152662	
	·8984644	32182	·4858353	6450	·6334225	843	·9112994	19606
·1872386		17694		9161		852		10456
1·540	7·883064		3·054352		4·298702		8·133056	
	·8966950	31997	·4849192	6423	·6333373	842	·9102538	19512
·1875207		17663		9144		851		10432
1·541	7·851067		3·047929		4·297860		8·113544	
	·8949287	31808	·4840048	6401	·6332522	844	·9092106	19414
·1878026		17631		9130		852		10404
1·542	7·819259		3·014528		4·297016		8·094130	
	·8931656	31621	·4830918	6374	·6331670	839	·9081702	19322
·1880844		17599		9111		849		10380
1·543	7·787638		3·035154		4·296177		8·074808	
	·8914057	31444	·4821807	6352	·6330821	840	·9071322	19228
·1883659		17571		9098		849		10365
1·544	7·756194		3·028802		4·295337		8·055580	
	·8896486	31241	·4812709	6326	·6329972	839	·9060957	19134
·1886473		17539		9081		848		10317

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·544	7·756194		3·028802		4·295337		8·055580	
	·8896486	31262	·4812705	6326	·6329972	839	·9060968	19134
		17539		9070		848		10328
1·545	7·724932		3·022476		4·394498		8·036446	
·1889285	·8878947	31078	·4803628	6302	·6329124	838	·9050640	19042
		17507		9065		847		10302
1·546	7·693854		3·016174		4·293660		8·017404	
·1892095	·8861440	30902	·4794563	6280	·6328277	835	·9040338	18950
		17477		9050		845		10278
1·547	7·662954		3·009894		4·292825		7·998454	
·1894903	·8843963	30722	·4785513	6255	·6327432	836	·9030060	18860
		17449		9035		846		10252
1·548	7·632230		3·003639		4·291989		7·979594	
·1897710	·8826514	30543	·4776478	6229	·6326586	833	·9019808	18767
		17414		9017		843		10226
1·549	7·601687		2·997410		4·291156		7·960827	
·1900514	·8809100	30377	·4767461	6208	·6325743	834	·9009582	18680
		17387		9003		844		10202
1·550	7·571310		2·991202		4·290322		7·942147	
·1903317	·8791713	30193	·4758458	6184	·6324899	832	·8999380	18587
		17356		8987		842		10176
1·551	7·541117		2·985018		4·289490		7·923560	
·1906118	·8774357	30025	·4749471	6160	·6324057	830	·8989204	18500
		17325		8971		841		10152
1·552	7·511092		2·978858		4·288660		7·905060	
·1908917	·8757031	29853	·4740499	6136	·6323216	830	·8979052	18406
		17296		8957		841		10124
1·553	7·481239		2·972722		4·287830		7·886654	
·1911715	·8739735	29687	·4731542	6114	·6322375	829	·8968928	18328
		17267		8941		839		10104
1·554	7·451552		2·966608		4·287001		7·868326	
·1914510	·8722468	29515	·4722601	6092	·6321536	827	·8958824	18234
		17236		8926		838		10086
1·555	7·422037		2·960516		4·286174		7·850092	
·1917304	·8705232	29345	·4713675	6068	·6320698	827	·8948748	18147
		17206		8911		838		10052
1·556	7·392692		2·954448		4·285347		7·831945	
·1920096	·8688026	29182	·4704764	6044	·6319860	825	·8938696	18065
		17177		8895		837		10028
1·557	7·363510		2·948404		4·284522		7·813880	
·1922886	·8670849	29018	·4695869	6024	·6319023	824	·8928668	17975
		17149		8881		836		10002

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·557	7·363510	29018	2·948404	6024	4·284522	824	7·813880	17975
·1922886	·8670849	17149	·4695869	8881	·6319023	836	·8928668	10002
1·558	7·334492	28852	2·942380	5999	4·283698	824	7·795905	17893
·1925675	·8653700	17117	·4686988	8864	·6318187	834	·8918666	9980
1·559	7·305640	28690	2·936381	5977	4·282874	823	7·778012	17806
·1928461	·8636583	17089	·4678124	8850	·6317353	834	·8908686	9954
1·560	7·276950	28531	2·930404	5956	4·282051	820	7·760206	17727
·1931246	·8619494	17061	·4669274	8836	·6316519	833	·8898732	9932
1·561	7·248419	28367	2·924448	5934	4·281231	822	7·742479	17639
·1934029	·8602433	17030	·4660438	8820	·6315686	832	·8888800	9906
1·562	7·220052	28212	2·918514	5911	4·280410	820	7·724840	17558
·1936810	·8585403	17003	·4651618	8806	·6314854	832	·8878894	9882
1·563	7·191840	28051	2·912603	5890	4·279590	817	7·707282	17478
·1939590	·8568400	16972	·4642812	8790	·6314022	830	·8869012	9860
1·564	7·163789	27896	2·906713	5867	4·278773	817	7·689804	17394
·1942367	·8551428	16944	·4634022	8775	·6313192	829	·8859152	9837
1·565	7·135893	27741	2·900846	5846	4·277956	818	7·672410	17312
·1945143	·8534484	16917	·4625247	8762	·6312363	830	·8849315	9807
1·566	7·108152	27587	2·895000	5826	4·277138	814	7·655098	17236
·1947918	·8517567	16888	·4615485	8747	·6311533	827	·8839508	9790
1·567	7·080565	27432	2·889174	5803	4·276324	814	7·637862	17150
·1950690	·8500679	16858	·4607738	8732	·6310706	827	·8829718	9762
1·568	7·053133	27281	2·883371	5781	4·275510	812	7·620712	17078
·1953461	·8483821	16831	·4599006	8717	·6309879	825	·8819956	9744
1·569	7·025852	27130	2·877590	5761	4·274698	814	7·603634	16992
·1956229	·8466990	16803	·4590289	8704	·6309054	826	·8810212	9716
1·570	6·998722	26977	2·871829	5739	4·273884	810	7·586642	16915
·1958997	·8450187	16772	·4581585	8687	·6308228	824	·8800496	9694

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·570	6·998722	26977	2·871829	5739	4·273884	810	7·586642	16915
·1958997	·8450187	16772	·4581585	8687	·6308228	824	·8800496	9694
1·571	6·971745	26828	2·866090	5718	4·273074	809	7·569727	16840
·1961762	·8433415	16745	·4572898	8673	·6307404	822	·8790802	9672
1·572	6·944917	26685	2·860372	5698	4·272265	809	7·552887	16757
·1964525	·8416670	16719	·4564225	8660	·6306582	823	·8781130	9648
1·573	6·918232	26540	2·854674	5677	4·271456	808	7·536130	16688
·1967287	·8399951	16692	·4555565	8646	·6305759	821	·8771482	9626
1·574	6·891692	26387	2·848997	5657	4·270648	807	7·519442	16600
·1970047	·8383259	16661	·4546919	8631	·6304938	821	·8761856	9600
1·575	6·865305	26247	2·843340	5636	4·269841	806	7·502842	16538
·1972806	·8366598	16635	·4538288	8616	·6304117	819	·8752256	9582
1·576	6·839058	26098	2·837704	5615	4·269035	805	7·486304	16457
·1975562	·8349963	16607	·4529672	8603	·6303298	819	·8742674	9556
1·577	6·812960	25961	2·832089	5595	4·268230	803	7·469847	16378
·1978317	·8333358	16579	·4521069	8588	·6302479	818	·8733118	9534
1·578	6·786999	25821	2·826494	5575	4·267427	802	7·453469	16312
·1981070	·8316777	16553	·4512481	8575	·6301661	817	·8723584	9514
1·579	6·761178	25676	2·820919	5554	4·266625	802	7·437157	16230
·1983821	·8300224	16525	·4503906	8560	·6300844	816	·8714070	9488
1·580	6·735502	25535	2·815365	5535	4·265823	800	7·420927	16157
·1986571	·8283699	16497	·4495346	8546	·6300028	815	·8704582	9466
1·581	6·709967	25402	2·809830	5515	4·265023	801	7·404770	16088
·1989319	·8267202	16471	·4486800	8532	·6299213	815	·8695116	9466
1·582	6·684565	25263	2·804315	5496	4·264222	798	7·388682	16015
·1992065	·8250731	16444	·4478268	8519	·6298398	813	·8685670	9424
1·583	6·659302	25123	2·798819	5474	4·263424	798	7·372667	15937
·1994809	·8234287	16415	·4469749	8504	·6297585	813	·8676246	9398

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·583	6·659302		2·798819		4·263424		7·372667	
·1994809	·8234287	25123	·4469749	5474	·6297585	798	·8676246	15937
		16415		8504		813		9398
1·584	6·634179		2·793345		4·262626		7·356730	
·1997552	·8217872	24990	·4461245	5456	·6296772	797	·8666848	15873
		16391		8492		811		9380
1·585	6·609189		2·787889		4·261829		7·340857	
·2000293	·8201481	24851	·4452753	5437	·6295961	795	·8657468	15797
		16360		8476		811		9356
1·586	6·584338		2·782452		4·261034		7·325060	
·2003032	·8185121	24723	·4444277	5417	·6295150	795	·8648112	15730
		16337		8463		810		9336
1·587	6·559615		2·777035		4·260239		7·309330	
·2005769	·8168784	24589	·4435814	5397	·6294340	793	·8638776	15655
		16311		8449		809		9312
1·588	6·535026		2·771638		4·259446		7·293675	
·2008505	·8152473	24456	·4427365	5379	·6293531	792	·8629464	15590
		16283		8436		808		9292
1·589	6·510570		2·766259		4·258654		7·278085	
·2011239	·8136190	24327	·4418929	5359	·6297723	792	·8620172	15516
		16257		8422		807		9270
1·590	6·486243		2·760900		4·257862		7·262569	
·2013971	·8119933	24195	·4410507	5341	·6291916	791	·8610902	15450
		16237		8410		807		9248
1·591	6·462048		2·755559		4·257071		7·247109	
·2016702	·8103702	24065	·4402097	5322	·6291109	790	·8601654	15379
		16204		8395		806		9226
1·592	6·437983		2·750237		4·256281		7·231740	
·2019431	·8087498	23938	·4393702	5302	·6290303	789	·8592428	15313
		16178		8382		805		9206
1·593	6·414045		2·744935		4·255492		7·216427	
·2022158	·8071320	23808	·4385320	5284	·6289498	786	·8583222	15243
		16150		8367		803		9184
1·594	6·390237		2·739651		4·254706		7·201184	
·2024883	·8055170	23687	·4376953	5266	·6288695	787	·8574038	15180
		16128		8356		803		9164
1·595	6·366550		2·734385		4·253919		7·186004	
·2027607	·8039042	23556	·4368597	5247	·6287892	787	·8564874	15109
		16100		8342		803		9140
1·596	6·342994		2·729138		4·253132		7·170895	
·2030329	·8022942	23430	·4360255	5228	·6287089	784	·8555734	15046
		16073		8328		801		9122

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·596	6·342992	23430	2·729138	5228	4·253132	784	7·170895	15046
·2030329	·8022946	16073	·4360255	8328	·6287089	801	·8555734	9122
1·597	6·319562	23312	2·723910	5211	4·252348	785	7·155849	14979
·2033049	·8006869	16049	·4351927	8316	·6286288	801	·8546612	9100
1·598	6·296250	23187	2·718699	5191	4·251563	783	7·140870	14910
·2035768	·7990820	16023	·4343611	8301	·6285487	800	·8537512	9078
1·599	6·273063	23066	2·713508	5175	4·250780	780	7·125960	14850
·2038485	·7974797	15998	·4335310	8290	·6284687	798	·8528434	9060
1·600	6·249997	22941	2·708333	5156	4·250000	781	7·111110	14783
·2041200	·7958799	15971	·4327020	8274	·6283889	797	·8519374	9038
1·601	6·227056	22822	2·703177	5138	4·249219	780	7·096327	14717
·2043913	·7942828	15946	·4318746	8264	·6283092	798	·8510336	9016
1·602	6·204234	22701	2·698039	5119	4·248439	779	7·081610	14653
·2046625	·7926882	15920	·4310482	8248	·6282294	796	·8501320	8996
1·603	6·181533	22587	2·692920	5102	4·247660	778	7·066957	14589
·2049335	·7910962	15896	·4302234	8237	·6281498	796	·8492324	8974
1·604	6·158946	22464	2·687818	5086	4·246882	775	7·052368	14533
·2052044	·7895066	15871	·4293997	8224	·6280702	795	·8483350	8958
1·605	6·136482	22346	2·682732	5066	4·246107	777	7·037835	14460
·2054750	·7879195	15845	·4285773	8210	·6279909	794	·8474392	8934
1·606	6·114136	22236	2·677666	5051	4·245330	776	7·023375	14402
·2057455	·7863350	15822	·4277563	8199	·6279115	794	·8465458	8914
1·607	6·091900	22116	2·672615	5031	4·244554	773	7·008973	14343
·2060159	·7847528	15795	·4269364	8184	·6278321	791	·8456544	8896
1·608	6·069784	22000	2·667584	5015	4·243781	772	6·994630	14277
·2062860	·7831733	15770	·4261180	8172	·6277530	791	·8447648	8874
1·609	6·047784	21877	2·662569	4999	4·243009	773	6·980353	14213
·2065560	·7815963	15746	·4253008	8160	·6276739	791	·8438774	8852

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·609	6·047784		2·662569		4·243009		6·980353	
	·7815963	21877	·4253008	4999	·6276739	773	·8438774	14213
·2065560		15746		8160		791		8852
1·610	6·025897		2·657570		4·242236		6·966140	
	·7800217	21773	·4244848	4980	·6275948	771	·8429922	14158
·2068259		15721		8146		789		8836
1·611	6·004124		2·652590		4·241465		6·951982	
	·7784496	21663	·4236702	4963	·6275159	770	·8421086	14096
·2070955		15697		8134		788		8814
1·612	5·982461		2·647627		4·240695		6·937886	
	·7768799	21550	·4228568	4947	·6274371	770	·8412272	14034
·2073650		15673		8123		789		8794
1·613	5·960911		2·642680		4·239925		6·923852	
	·7753126	21439	·4220445	4930	·6273582	768	·8403478	13977
·2076344		15648		8108		786		8776
1·614	5·939472		2·637750		4·239157		6·909875	
	·7737478	21326	·4212337	4913	·6272796	767	·8394702	13915
·2079035		15622		8096		786		8754
1·615	5·918146		2·632837		4·238390		6·895960	
	·7721856	21222	·4204241	4896	·6272010	767	·8385948	13853
·2081725		15600		8085		786		8734
1·616	5·896924		2·627941		4·237623		6·882107	
	·7706256	21108	·4196156	4880	·6271224	765	·8377214	13802
·2084414		15575		8071		784		8708
1·617	5·875816		2·623061		4·236858		6·868305	
	·7690681	21000	·4188085	4862	·6270440	765	·8368496	13738
·2087100		15550		8059		784		8696
1·618	5·854816		2·618199		4·236093		6·354567	
	·7675131	20894	·4180026	4843	·6269656	763	·8359800	13680
·2089785		15525		8045		783		8676
1·619	5·833922		2·613356		4·235330		6·840887	
	·7659606	20790	·4171987	4833	·6268873	762	·8351124	13624
·2092468		15504		8035		782		8658
1·620	5·813132		2·608523		4·234568		6·827263	
	·7644102	20682	·4163946	4814	·6268091	761	·8342466	13566
·2095150		15479		8022		781		8638
1·621	5·792450		2·603709		4·233807		6·813697	
	·7628623	20577	·4155924	4798	·6267310	761	·8333828	13510
·2097830		15455		8009		780		8620
1·622	5·771873		2·598911		4·233046		6·800187	
	·7613168	20470	·4147915	4780	·6266530	760	·8325208	13450
·2100508		15430		7997		780		8598

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·622	5·771873	20470	2·598911	4780	4·233046	760	6·800187	13450
·2100508	·7613168	15430	·4147915	7997	·6266530	780	·8325208	8598
1·623	5·751403	20369	2·594131	4767	4·232286	758	6·786737	13398
·2103185	·7597738	15408	·4139918	7986	·6265750	778	·8316610	8582
1·624	5·731034	20264	2·589364	4749	4·231528	759	6·773339	13337
·2105860	·7582330	15384	·4131932	7973	·6264972	779	·8308028	8560
1·625	5·710770	20161	2·584615	4733	4·230769	756	6·760002	13279
·2108534	·7566946	15359	·4123959	7960	·6264193	776	·8299468	8544
1·626	5·690609	20062	2·579882	4717	4·230013	757	6·746723	13233
·2111205	·7551587	15337	·4115999	7949	·6263417	777	·8290929	8522
1·627	5·670547	19954	2·575165	4702	4·229256	754	6·733490	13175
·2113876	·7536250	15310	·4108050	7936	·6262640	775	·8282402	8506
1·628	5·650593	19863	2·570463	4686	4·228502	755	6·720315	13119
·2116544	·7520940	15293	·4100114	7925	·6261865	775	·8273896	8486
1·629	5·630730	19760	2·565777	4670	4·227747	754	6·707196	13064
·2119211	·7505647	15267	·4092189	7913	·6261090	774	·8265410	8468
1·630	5·610970	97321	2·561107	23122	4·226993	3751	6·694132	64519
·2121876	·7490380	75988	·4084276	39385	·6260316	3857	·8256942	42060
1·635	5·513649	94917	2·537985	22741	4·223242	3729	6·629613	63208
·2135178	·7414392	75416	·4044891	39090	·6256459	3836	·8214882	41606
1·640	5·418732	92597	2·515244	22371	4·219513	3707	6·566405	61927
·2148438	·7338976	74854	·4005801	38800	·6252623	3817	·8173276	41152
1·645	5·326135	90346	2·492873	22011	4·215806	3685	6·504478	60692
·2161659	·7264122	74300	·3967001	38515	·6248806	3797	·8132124	40714
1·650	5·235789	88165	2·470862	21657	4·212121	3662	6·443784	59484
·2174839	·7189822	73754	·3928486	38234	·6245009	3778	·8091410	40276
1·655	5·147624	86054	2·449205	21312	4·208459	3640	6·384302	58313
·2187980	·7116068	73216	·3890252	37956	·6241231	3758	·8051134	39850

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·655	5·147624		2·449205		4·208459		6·384302	
		86054		21312		3640		58313
	·7116068		·3890252		·6241231		·8051134	
·2187980		73216		37956		3758		39850
1·660	5·061570		2·427893		4·204819		6·325989	
		166038		41610		7215		113246
	·7042852		·3852296		·6237473		·8011284	
·2201081		144852		75096		7458		78450
1·670	4·895532		2·386283		4·197604		6·212743	
		158344		40345		7128		108936
	·6898000		·3777200		·6230015		·7932834	
·2227165		142793		74034		7381		76826
1·680	4·737188		2·345938		4·190476		6·103807	
		151109		39103		7044		104858
	·6755207		·3703166		·6222634		·7856008	
·2253093		140792		73001		7307		75256
1·690	4·586079		2·306835		4·183432		5·998949	
		144301		37928		6962		100989
	·6614415		·3630165		·6215327		·7780752	
·2278867		138847		71996		7233		73734
1·700	4·441778		2·268907		4·176470		5·897960	
		137887		36800		6880		97325
	·6475568		·3558169		·6208094		·7707018	
·2304489		136956		71020		7160		72262
1·710	4·303891		2·232107		4·169590		5·800635	
		131841		35724		6799		93845
	·6338612		·3487149		·6200934		·7634756	
·2329961		135117		70069		7088		70838
1·720	4·172050		2·196383		4·162791		5·706790	
		126133		34693		6722		90547
	·6203495		·3417080		·6193846		·7563918	
·2355284		133326		69144		7018		69454
1·730	4·045917		2·161690		4·156069		5·616243	
		120742		33700		6643		87391
	·6070169		·3347936		·6186828		·7494464	
·2380461		131579		68239		6948		68114
1·740	3·925175		2·127990		4·149426		5·528852	
		115651		32752		6569		84409
	·5938590		·3279697		·6179880		·7426350	
·2405492		129883		67363		6880		66816
1·750	3·809524		2·095238		4·142857		5·444443	
		110835		31841		6493		81562
	·5808707		·3212334		·6173000		·7359534	
·2430380		128228		66506		6813		65552
1·760	3·698689		2·063397		4·136364		5·362881	
		106272		30966		6420		78852
	·5680479		·3145828		·6166187		·7293982	
·2455127		126612		65670		6746		64330
1·770	3·592417		2·032431		4·129944		5·284029	
		101957		30126		6348		76272
	·5553867		·3080158		·6159441		·7229652	
·2479733		124041		64856		6680		63144
1·780	3·490460		2·002305		4·123596		5·207757	
		97865		29320		6277		73805
·2504200	·5428826		·3015302		·6152761		·7166508	
		123306		63861		6616		61790

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·780	3·490460	97865	2·002305	29320	4·123596	6277	5·207757	73805
·2504200	·5428826	123506	·3015302	63861	·6152761	6616	·7166508	61790
1·790	3·392595	93984	1·972985	28541	4·117319	6208	5·133952	71452
·2528530	·5305320	122009	·2951441	63486	·6146145	6553	·7104518	60868
1·800	3·298611	90301	1·944444	27795	4·111111	6139	5·062500	69205
·2552725	·5183311	120547	·2887955	62528	·6139592	6490	·7043650	59778
1·810	3·208310	86803	1·916649	27076	4·104972	6071	4·993295	67060
·2576786	·5062764	119122	·2825427	61789	·6133102	6428	·6983872	58722
1·820	3·121507	83481	1·889573	26383	4·098901	6004	4·926235	65007
·2600714	·4943642	117727	·2763638	61066	·6126674	6367	·6925150	57690
1·830	3·038026	80321	1·863190	25716	4·092897	5940	4·861228	63033
·2624511	·4825915	116367	·2702572	60360	·6120307	6307	·6867460	56690
1·840	2·957705	77316	1·837474	25073	4·086957	5875	4·798195	61170
·2648178	·4709548	115036	·2642212	59669	·6114000	6247	·6810770	55714
1·850	2·880389	74455	1·812401	24454	4·081082	5812	4·737025	59364
·2671717	·4594512	113738	·2582543	58997	·6107753	6190	·6755056	54768
1·860	2·805934	71731	1·787947	23856	4·075270	5751	4·677661	57633
·2695129	·4480774	112466	·2523546	58336	·6101563	6128	·6700288	53842
1·870	2·734203	69134	1·764091	23279	4·069519	5688	4·620028	55980
·2718416	·4368308	111223	·2465210	57690	·6095431	6075	·6646446	53044
1·880	2·665069	66658	1·740812	22722	4·063831	5629	4·564043	54390
·2741578	·4257085	110007	·2407520	57060	·6089356	6020	·6593502	51966
1·890	2·598411	64297	1·718090	22183	4·058202	5570	4·509658	52867
·2764618	·4147078	108818	·2350460	56441	·6083336	5965	·6541436	51214
1·900	2·534114	62045	1·695907	21665	4·052632	5511	4·456791	51403
·2787536	·4038260	107654	·2294019	55837	·6077371	5910	·6490222	50382
1·910	2·472069	59891	1·674242	21162	4·047121	5453	4·405388	49999
·2810334	·3930606	106513	·2238182	55243	·6071461	5856	·6439840	49572

μ	$\frac{\mu+2}{\mu.(\mu-1)^2}$	Diff.	$\frac{\mu+1}{\mu.\mu-1}$	Diff.	$\frac{3\mu+2}{\mu}$	Diff.	$\frac{\mu^2}{(\mu-1)^2}$	Diff.
1·910	2·472069		1·674242		4·047121		4·405388	
		59891		21162		5453		49999
·2810334	·3930606		·2238182		·6071461		·6439840	
		106513		55243		5856		49572
1·920	2·412178		1·653080		4·041668		4·355389	
		57836		20677		5398		48647
·2833012	·3824093		·2182939		·6065605		·6390268	
		105398		54665		5803		48780
1·930	2·354342		1·632403		4·036270		4·306742	
		55872		20207		5341		47349
·2855573	·3718695		·2128274		·6059802		·6341488	
		104308		54097		5751		48012
1·940	2·298470		1·612196		4·030929		4·259393	
		53992		19753		5287		46106
·2878017	·3614387		·2074177		·6054051		·6293476	
		103234		53539		5700		47256
1·950	2·244478		1·592443		4·025642		4·213287	
		52194		19314		5235		44883
·2900346	·3511153		·2020638		·6048351		·6246220	
		102186		52994		5650		46522
1·960	2·192284		1·573129		4·020407		4·168404	
		50474		18887		5178		43735
·2922561	·3408967		·1967644		·6042701		·6199698	
		101158		52459		5598		45808
1·970	2·141810		1·554242		4·015229		4·124669	
		48827		18476		5128		42620
·2944662	·3307809		·1915185		·6037103		·6153890	
		100152		51935		5559		45108
1·980	2·092983		1·535766		4·010101		4·082049	
		47248		18076		5076		41542
·2966652	·3207657		·1863250		·6031553		·6108782	
		99163		51421		5501		44424
1·990	2·045735		1·517690		4·005025		4·040507	
		45735		17690		5025		40507
·2988531	·3108494		·1811829		·6026052		·6064358	
		98194		50916		5452		43758
2·000	2·000000		1·500000		4·000000		4·000000	
·3010300	·3010300		·1760913		·6020600		·6020600	

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